PHYS 105: Introduction to Computational Physics

Homework #7

(Due: Tuesday, June 7)

Problem # 1

A useful model to understand the dynamics of the interaction between two biological species, a predator and a prey, is expressed as a system of two Ordinary Differential Equations (ODEs)

$$\frac{dx}{dt} = \alpha x - \beta x y \tag{1}$$

$$\frac{dy}{dt} = -\gamma x y + \delta y \tag{2}$$

where $x(t)$ is the number of preys, $y(t)$ the number of predators, $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are the rates of change in the population of the two species, t is the time in arbitrary units and α, β, δ and γ are positive constants. They should be made accessible from the command line.

1. Write a program to solve this two species model, or modify for this problem one of the codes that we wrote earlier to solve the Harmonic Oscillator or the pendulum models.
2. Use a Predictor - Corrector method to solve the ODEs.
3. Make the constants accessible via the command line and default their values to accessible via the command line and default them to the following values: $\alpha = 4.0$, $\beta = 2.0$, $\delta = 3.0$, and $\gamma = 3.0$.
4. Do the same for $x(0.0) = 5$ preys and $y(t) = 12.5$ predators.
5. Solve the model for $x(t)$ and $y(t)$ and plot both functions simultaneously (one graph) as functions of time t.
6. Plot a Phase Space Portrait of the solution, i.e., $y(t)$ *vs* $x(t)$ Explain this graph.
7. Calculate the periods for the prey and predator populations. Are these periods time dependent?
8. Find analytically the locations of the fixed points.
9. Plot symbols on the Phase Space Portrait to clearly mark the location of the fixed points.

Problem # 2

Find the stability of the Fixed-Points. That is, launch a large number of trajectories originating from an equally spaced distribution of points on a small circle centered on each fixed points. Use the program you built in problem # 1 to evolve these trajectoriess over a tiny time interval.

What do you conclude?

Problem # 3

Solve the following system of three ODEs

$$\frac{dx}{dt} = -y - z \tag{3}$$

$$\frac{dy}{dt} = x + ay \tag{4}$$

$$\frac{dz}{dt} = -cz + xz + b \tag{5}$$

1. Modify the program that you wrote (or adapted) for problem # 1 to solve for the functions $x(t)$, $y(t)$and $z(t)$.
2. Assign the following values to the parameters: $a = 0.432$, $b = 2.0$, and $c = 4.0$.
3. Use the following initial values: $x(0.0) = y(0.0) = z(0.0) = 1.0$
4. Use $dt = 0.001$
5. Use $tMax = 500$

Enjoy the picture!!!!

Best of luck in all your courses!