PHYS 105: Introduction to Computational Physics

Spring 2016

Homework #6

(Due: May 31, 2016)

1. Consider a projectile moving in two dimensions under the combined effects of gravity and air resistance. Initially the projectile is launched from $x = 0, y = 0$ with speed $v_0 = 100$ m/s at an angle of $\theta_0 = 60°$ to the horizontal. The components of its acceleration are

$$\begin{aligned} a_x &= -\beta\,|v|\,v_x\,, \\ a_y &= -g - \beta\,|v|\,v_y\,, \end{aligned}$$

where $\beta = 0.001$.

   (a) Compute the projectile's range and time of flight (take $\delta t = 0.01$, and don't forget to interpolate!). Plot the projectile's trajectory `y(x)`.

   (b) At what angle to the horizontal θ_1 does the projectile hit the ground in either case in part (a)? What would θ_1 be in the absence of air resistance (i.e. $\beta = 0$)?

   (c) By what factor (to within 1 percent) must the launch speed for $\beta = 0.001$ be increased to restore the range to the $\beta = 0$ result?

   (d) By varying the value of θ_0, determine the maximum range of the projectile for $v_0 = 100$ m/s (again with $\beta = 0.001$). To what value of θ_0 (to 1 decimal place) does this correspond?

   (e) For $v_0 = 100$ m/s, plot θ_1 as a function of θ_0.

2. Now suppose that the value of β in problem 1 varies with height y, according to the law

$$\beta(y) = 0.001e^{-y/h}\,,$$

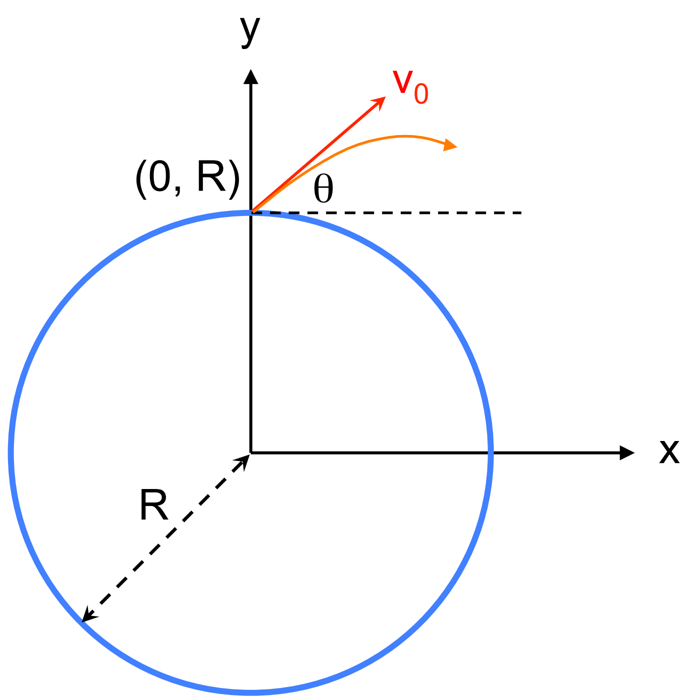
where h = 500 m (*not* a very realistic description of Earth's atmosphere!). How does the maximum range of the projectile (as computed in problem 1d) change as a result? What if h = 5 km (a much better approximation to reality)?

3. A rocket is fired from Earth's surface with speed v_0 = 7.5 km/s at an angle $\theta = 45°$ to the horizontal at the launch point, as illustrated in the diagram below. The gravitational acceleration of the spacecraft due to Earth (of mass M), lying at the origin of coordinates is

$$\mathbf{a_{grav}} = -\frac{GM\mathbf{r}}{r^3}\,,$$

where $\mathbf{r} = (\mathbf{x}, \mathbf{y})$ is the spacecraft's position vector, $r = |\mathbf{r}| = \sqrt{x^2 + y^2}$, and G is the gravitational constant.

Thus, in the two-dimensional coordinate system shown in the diagram, the rocket starts from location $(0, R)$, where R is Earth's radius, with velocity $(v_0 \cos\theta, v_0 \sin\theta)$. Take $GM = gR^2$, where $g = 9.80\,\mathrm{m/s^2}$ and $R = 6400\,\mathrm{km}$. Neglect both air resistance and the effect of Earth's rotation.



(a) Write a program to determine the trajectory of the spacecraft. Stop the calculation when the spacecraft hits Earth's surface (i.e. when r becomes less than R). Choose a time step δt of 1 second. Plot the spacecraft's trajectory `y(x)`.

(b) For v_0 = 7.5 km/s and $\theta = 45^\circ$, what is the maximum height reached by the rocket (relative to Earth's surface), and what is its range (in kilometers, measured along Earth's surface)?

(c) For *fixed* v_0 = 7.5 km/s, find, to within 1 degree, the *minimum* value of θ needed to hit a target on the surface at location $(R, 0)$, i.e. where the x-axis intersects the surface. What is the maximum distance above Earth's surface reached by the rocket in this case, and what is the rocket's time of flight?