PHYS 105: Introduction to Computational Physics

Spring 2016

Homework #4

(Due: May 5, 2016)

Use the physical problem of computing the motion of a particle with initial position $x = 0$ and velocity $v = 1$ subject to an acceleration

$$a(x) = -4\,\sin^3 x$$

as a test-bed for solving the following questions.

1. Use the modified integration scheme including the "jerk" term analytically:

$$\begin{aligned}
a_i &= \texttt{acc}(x_i) \\
j_i &= \texttt{jerk}(x_i, v_i) \\
x_{i+1} &= x_i + v_i\,\delta t + \tfrac{1}{2}\,a_i\,\delta t^2 \\
v_{i+1} &= v_i + a_i\,\delta t + \tfrac{1}{2}\,j_i\,\delta t^2 \\
t_{i+1} &= t_i + \delta t
\end{aligned}$$

   where the acceleration a_i and jerk j_i are recomputed at the start of each step. Start with a time step $\delta t = 0.5$, and decrease δt by factors of 2 until $\delta t < 0.001$. For each choice of time step, compute the energy error $\Delta E(\delta t) \equiv \max_i |E(t_i) - E(0)|$, where $t_i = i\delta t$ and the maximum is over the entire integration (as in in-class exercises 5.1–5.4).

   Plot $\log_{10} \Delta E$ versus $\log_{10} \delta t$, and hence determine the order of the method — that is, the exponent n in the relation $\Delta E \propto \delta t^n$. Turn in your program, the plot, and the order you measure.

2. Repeat the previous question for the *predictor-corrector* scheme defined by:

$$\begin{aligned}
a_i &= \texttt{acc}(x_i) \\
x_{i+1} &= x_i + v_i\,\delta t + \tfrac{1}{2}\,a_i\,\delta t^2 \\
v_p &= v_i + a_i\,\delta t \\
a_p &= \texttt{acc}(x_{i+1}) \\
v_{i+1} &= v_p + \tfrac{1}{2}\,(a_p - a_i)\delta t \\
t_{i+1} &= t_i + \delta t
\end{aligned}$$

3. (a) Show analytically that, if the acceleration depends on position only, a single step of the predictor-corrector scheme (problem 2) is *time reversible*, in that a forward step followed by a backward step (time step $-\delta t$) using the same algorithm is guaranteed to return the particle to the original position.

   (b) Demonstrate this result numerically by computing the trajectory of the particle in our standard example from $t = 0$ to $t = 10$ using a time step of 0.0625, then reversing the motion and integrating back to $t = 0$. What is the final value of x (at $t = 0$) thus obtained?

   (c) Demonstrate using the same numerical approach that the "analytic jerk" method (problem 1), while also second order, is not time reversible.