PHYS 105: Introduction to Computational Physics

Spring 2016

Homework #1

(Due: April 9, 2016)

1. Write a program to determine the function $\sin x$ from the infinite series

$$\sin x = \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)!} ,$$

stopping the calculation when the next term in the sum is less than (say) 10^{-6} in magnitude. Use your program to evaluate $\sin 0.1$, $\sin 1$, $\sin 10$, and $\sin 100$. Compare your results with the correct values, as obtained using the built-in math functions in C++ or Python. Turn in a copy of yor program.

2. Write a program to sum the series

$$f_N(x) \; = \; \sum_{n=1}^{N} \frac{2}{\pi} \left[1 - (-1)^n\right] \frac{\sin(nx)}{n} .$$

This is the so-called Fourier series representation of the step function

$$f(x) \; = \; \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi . \end{cases}$$

Plot $f(x)$, $f_1(x)$, $f_5(x)$, $f_{10}(x)$, and $f_{100}(x)$ on a single graph, for $0 \le x < \pi$ in steps of $\delta x = \pi/500$. Turn in the graph and a copy of your program.

3. Consider the motion of a particle in one dimension under uniform acceleration a. Let's assume that, at time $t = 0$, the particle has position x_0 and velocity v_0. Then, from elementary physics, we know that the position and velocity of the particle at any time t are given by (in C notation):

```
x = x0 + v0*t + 0.5*a*t*t;

v = v0 + a*t;
```

(a) Write a program to evaluate the particle's position x at times $t_i = i\,\delta t$, where δt is some time step. Use `double` variables to represent all quantities, and use the values

```
x0 = 0

v0 = 1

a = -1.5

dt = 0.05
```

Use a `while` loop to continue your calculation until $x < 0$, and thereby estimate the time T_0 taken for the particle to return to $x = 0$. Print out this estimate, and compare your result with the analytic result obtained by solving the appropriate quadratic equation.

(b) Modify your program to print out the values of t and x (one pair per line) after every step (<u>don't</u> print out this output!). Plot the trajectory, using `gpl` or `matplotlib`, and verify that the particle does indeed return to $x = 0$ at the time obtained in part (a).

For part (a), turn in a printout of your program and the output produced when it runs. For part (b), print out and turn in the (clearly labeled) graph.