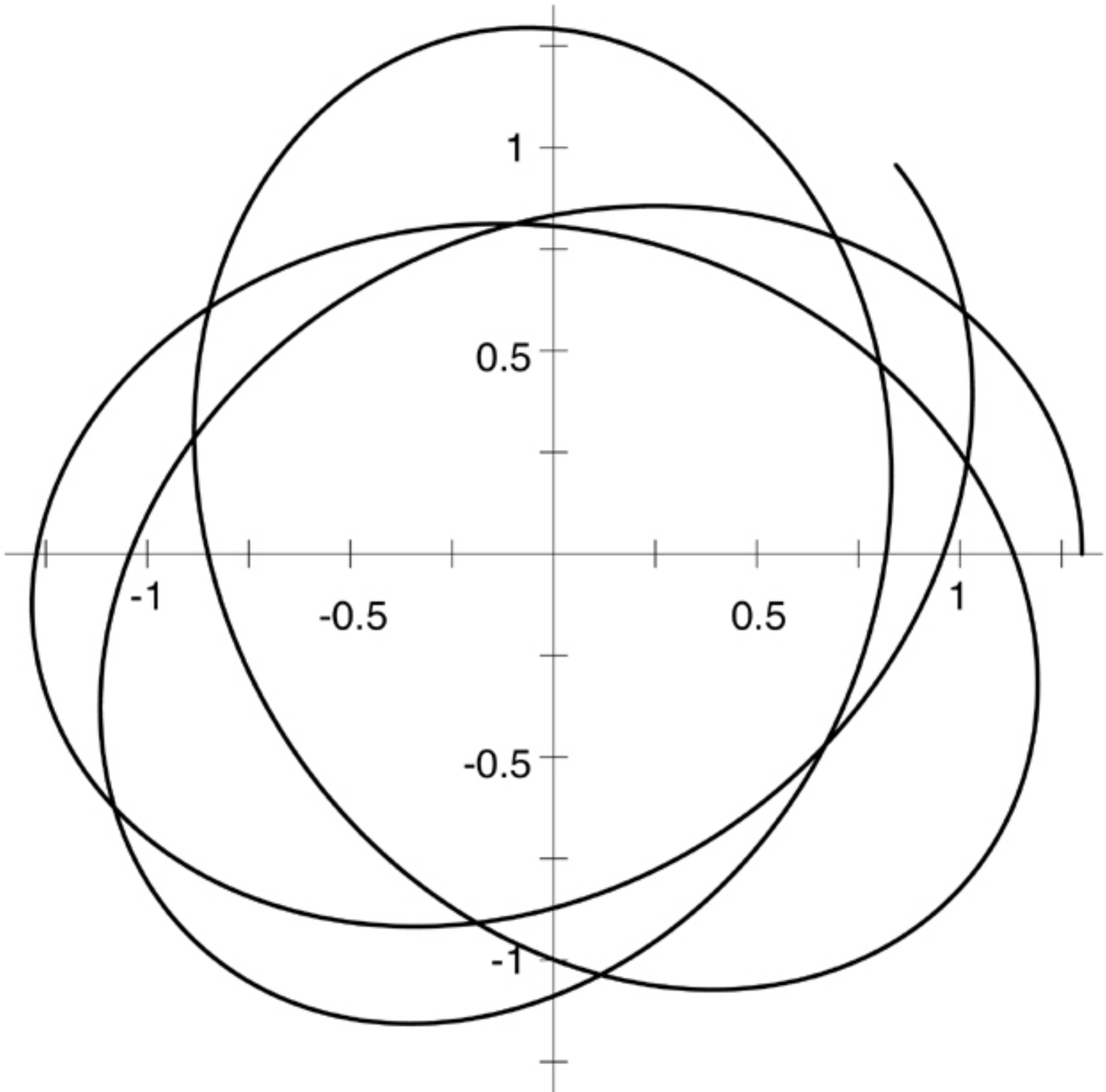
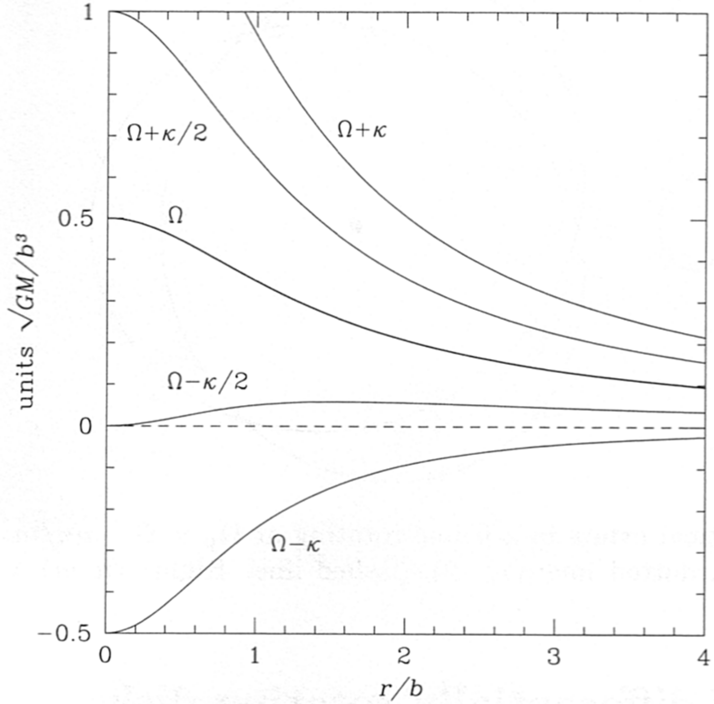
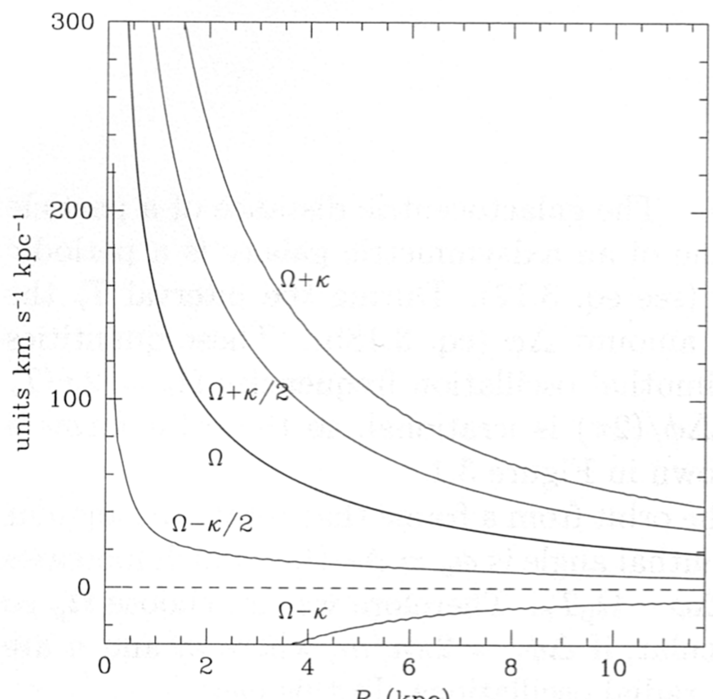
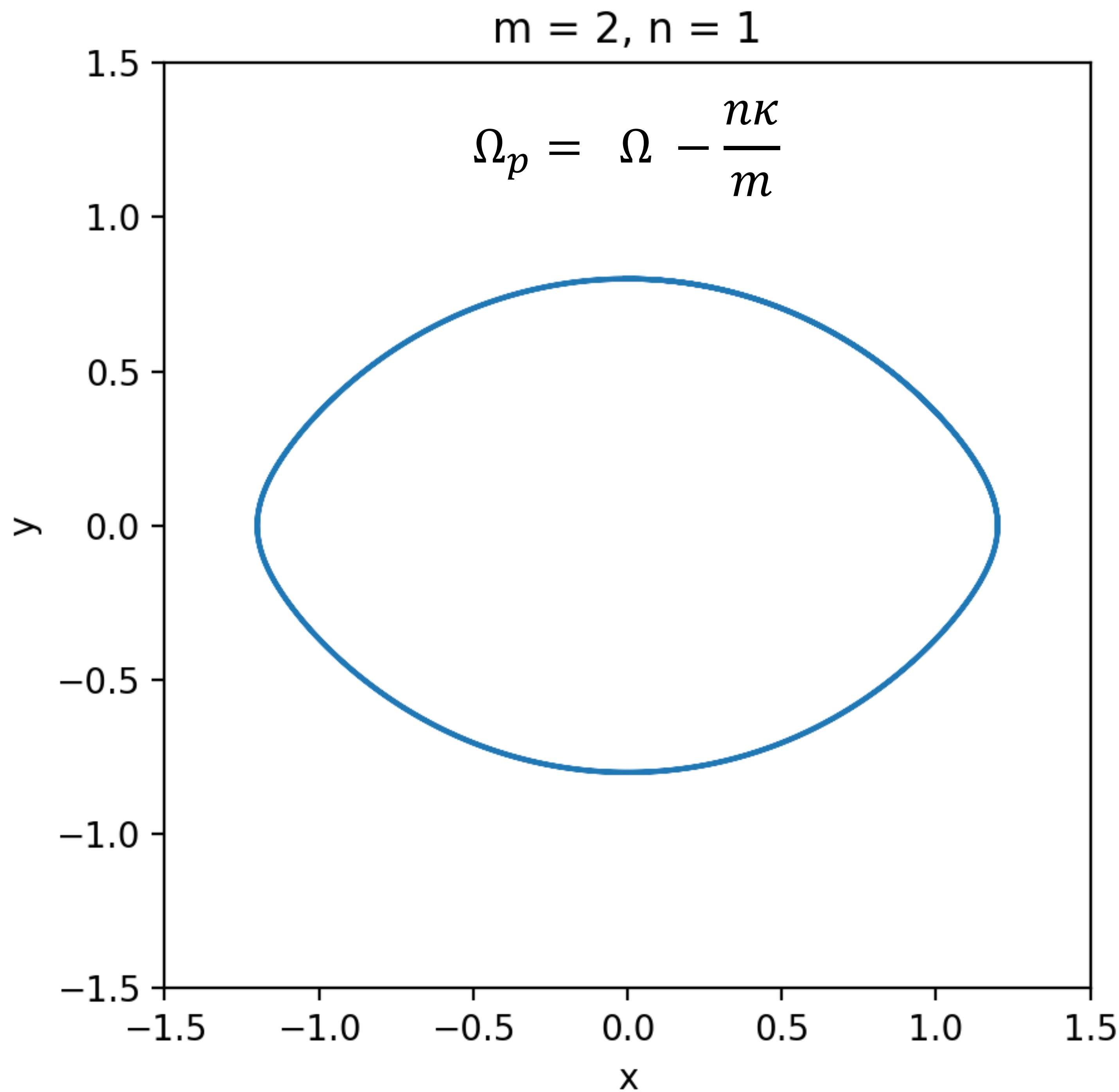


Fig 3.10 'Galaxies in the Universe' Sparke/Gallagher CUP 2007


Ω+κ/2
Ω+κ
Ω
Ω−κ/2
Ω−κ
units $\sqrt{GM/b^3}$
r/b
1
0.5
0
−0.5
0
1
2
3
4


Ω+κ
Ω+κ/2
Ω
Ω−κ/2
Ω−κ
units km s^{-1} kpc^{-1}
R (kpc)
300
200
100
0
0
2
4
6
8
10



m = 2, n = 1
$\Omega_p = \Omega - \frac{n\kappa}{m}$
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
y
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
x

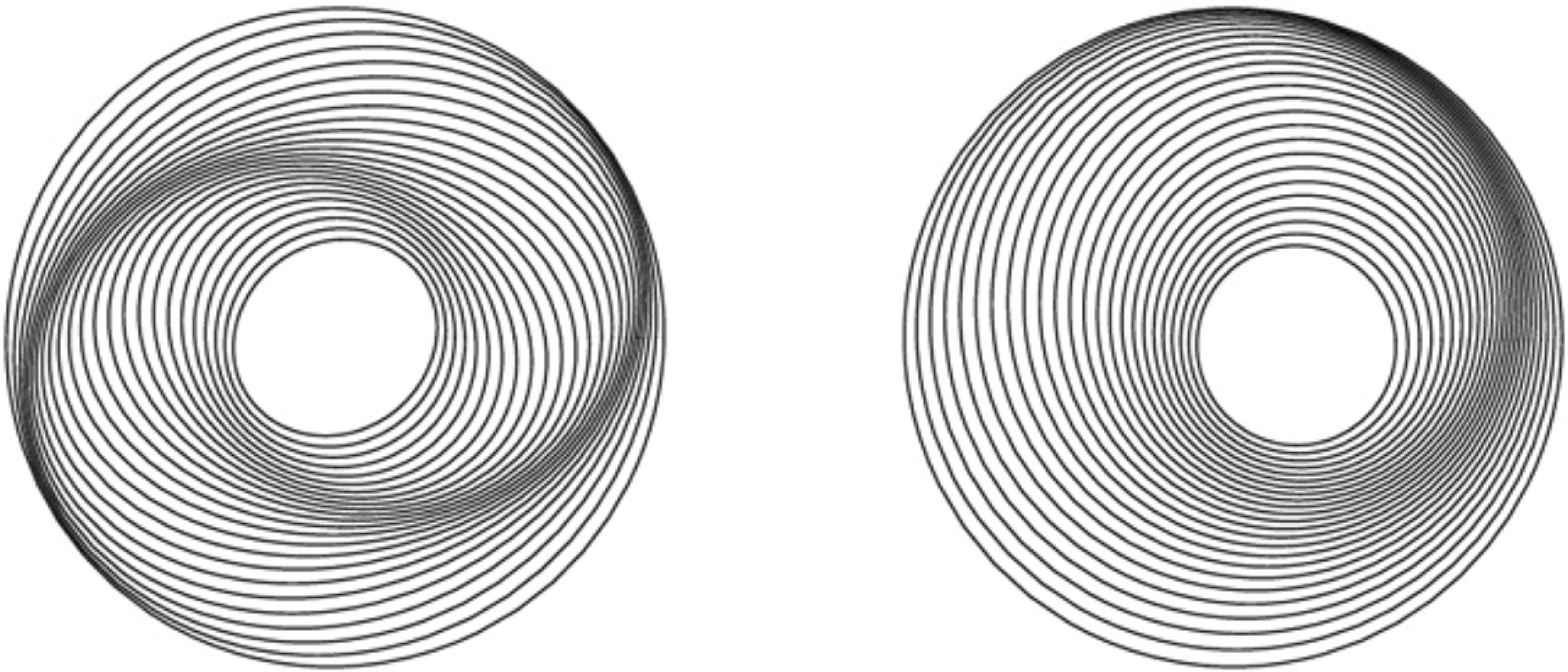
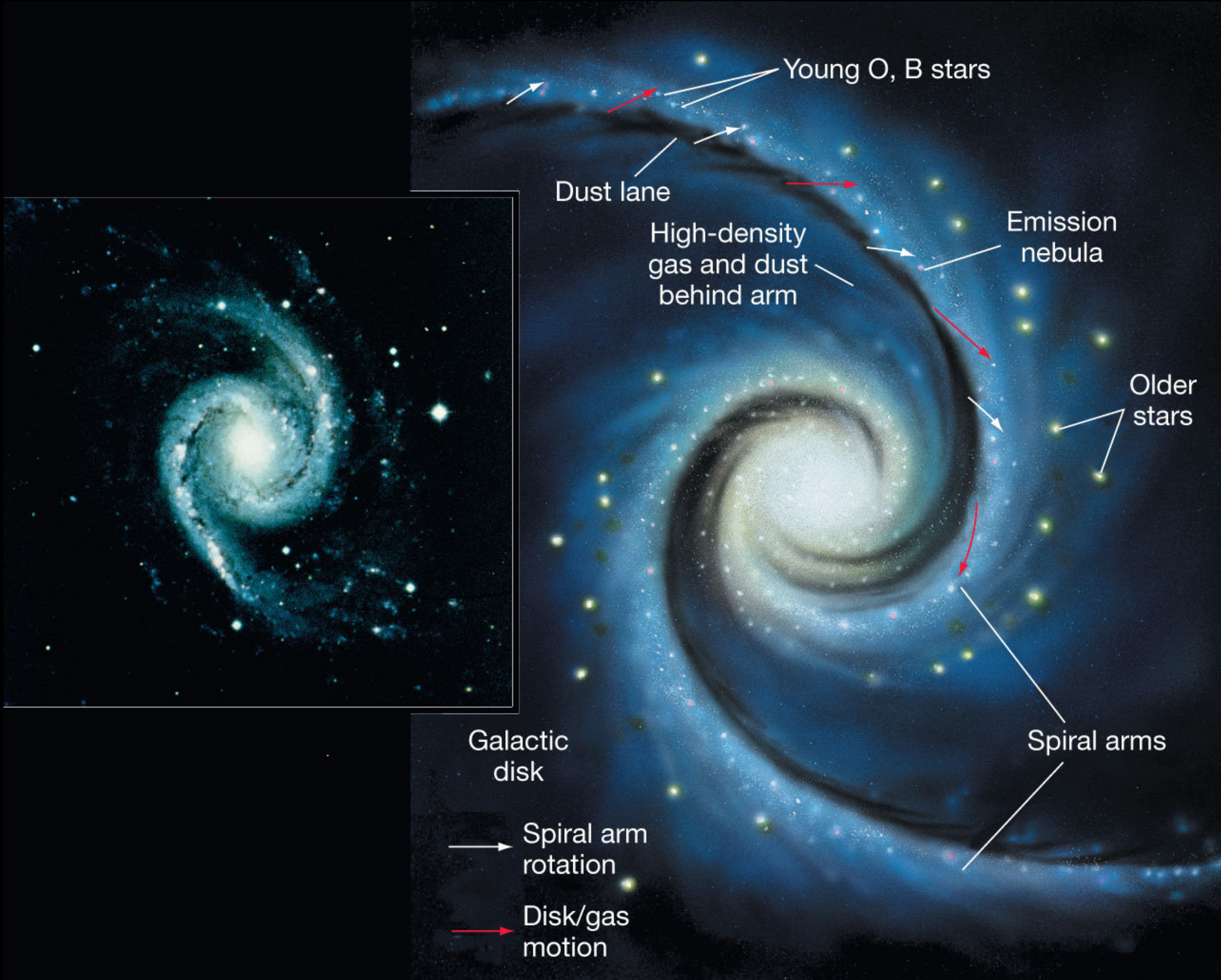


Fig 5.29 'Galaxies in the Universe' Sparke/Gallagher CUP 2007


Young O, B stars
Dust lane
High-density gas and dust behind arm
Emission nebula
Older stars
Galactic disk
Spiral arms
Spiral arm rotation
Disk/gas motion

Damped forced harmonic oscillator

$$\ddot{x} - \alpha\dot{x} + \Omega_0^2 x \;=\; f(t) \;=\; Ae^{i\Omega t}$$

Look for steady-state solutions $x_0 e^{i\Omega t}$

$$\Rightarrow\; x_0 = \frac{A}{\Omega_0^2 - \Omega^2 - i\alpha}$$

<u>resonance</u> for $\Omega \approx \Omega_0$

formally infinite response as $\alpha \to 0$

Near-circular orbits of disk stars near radius R_0 with a weak rotating perturbing potential

$$\phi_1(R, \theta, t) \;=\; \phi_p(R) \cos m\big(\theta - \Omega_p t\big)$$

Response of stellar orbits $(R = R_0 + R_1)$ satisfies

$$\ddot{R}_1 + \kappa^2 R_1 \;=\; -\frac{1}{\Delta}\left[\frac{d\phi_p}{dR} + \frac{2\Omega_0 \phi_p}{R\big(\Omega_0 - \Omega_p\big)}\right]_{R=R_0} \cos m\big(\Omega_0 - \Omega_p\big)t$$

Steady-state solution

$$R_1 = -\frac{1}{\Delta}\left[\frac{d\phi_p}{dR} + \frac{2\Omega_0 \phi_p}{R\big(\Omega_0 - \Omega_p\big)}\right]_{R=R_0} \cos m\big(\Omega_0 - \Omega_p\big)t$$

where $\Delta = \kappa_0^2 - m^2\big(\Omega_0 - \Omega_p\big)^2$

Near-circular orbits of disk stars near radius R_0 with a weak rotating perturbing potential

$$\phi_1(R, \theta, t) = \phi_p(R) \cos m\left(\theta - \Omega_p t\right)$$

Response of stellar orbits $(R = R_0 + R_1)$ satisfies

$$\ddot{R}_1 + \kappa^2 R_1 = -\frac{1}{\Delta}\left[\frac{d\phi_p}{dR} + \frac{2\Omega_0 \phi_p}{R(\Omega_0 - \Omega_p)}\right]_{R=R_0} \cos m\left(\Omega_0 - \Omega_p\right)t$$

Steady-state solution

$\Omega_p = \Omega_0$ — corotation resonance

$$R_1 = -\frac{1}{\Delta}\left[\frac{d\phi_p}{dR} + \frac{2\Omega_0 \phi_p}{R(\Omega_0 - \Omega_p)}\right]_{R=R_0} \cos m\left(\Omega_0 - \Omega_p\right)t$$

where $\Delta = \kappa_0^2 - m^2\left(\Omega_0 - \Omega_p\right)^2$

$\Delta = 0$ — Lindblad resonance: $\Omega_p = \Omega_0 \pm \kappa_0/m$

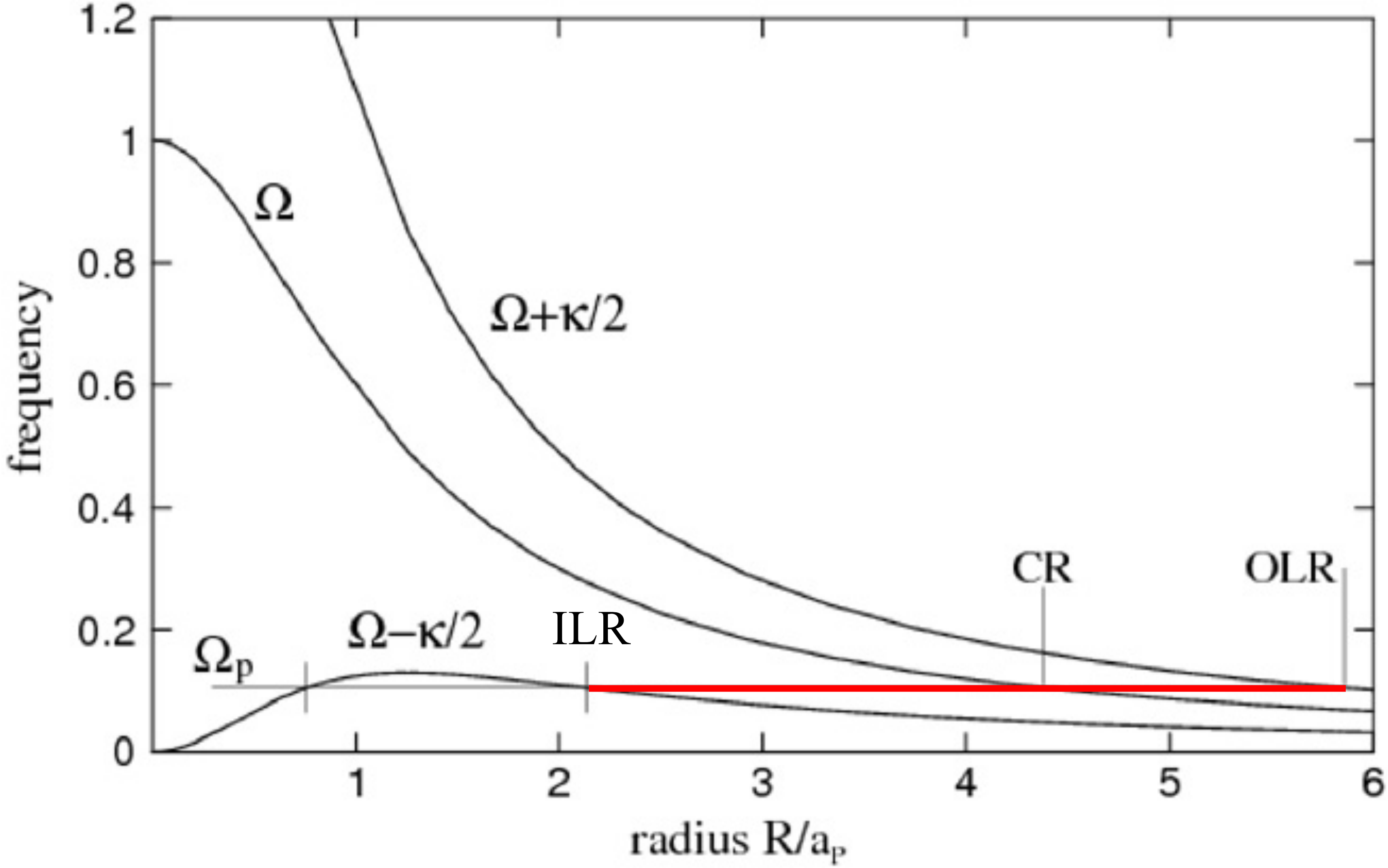
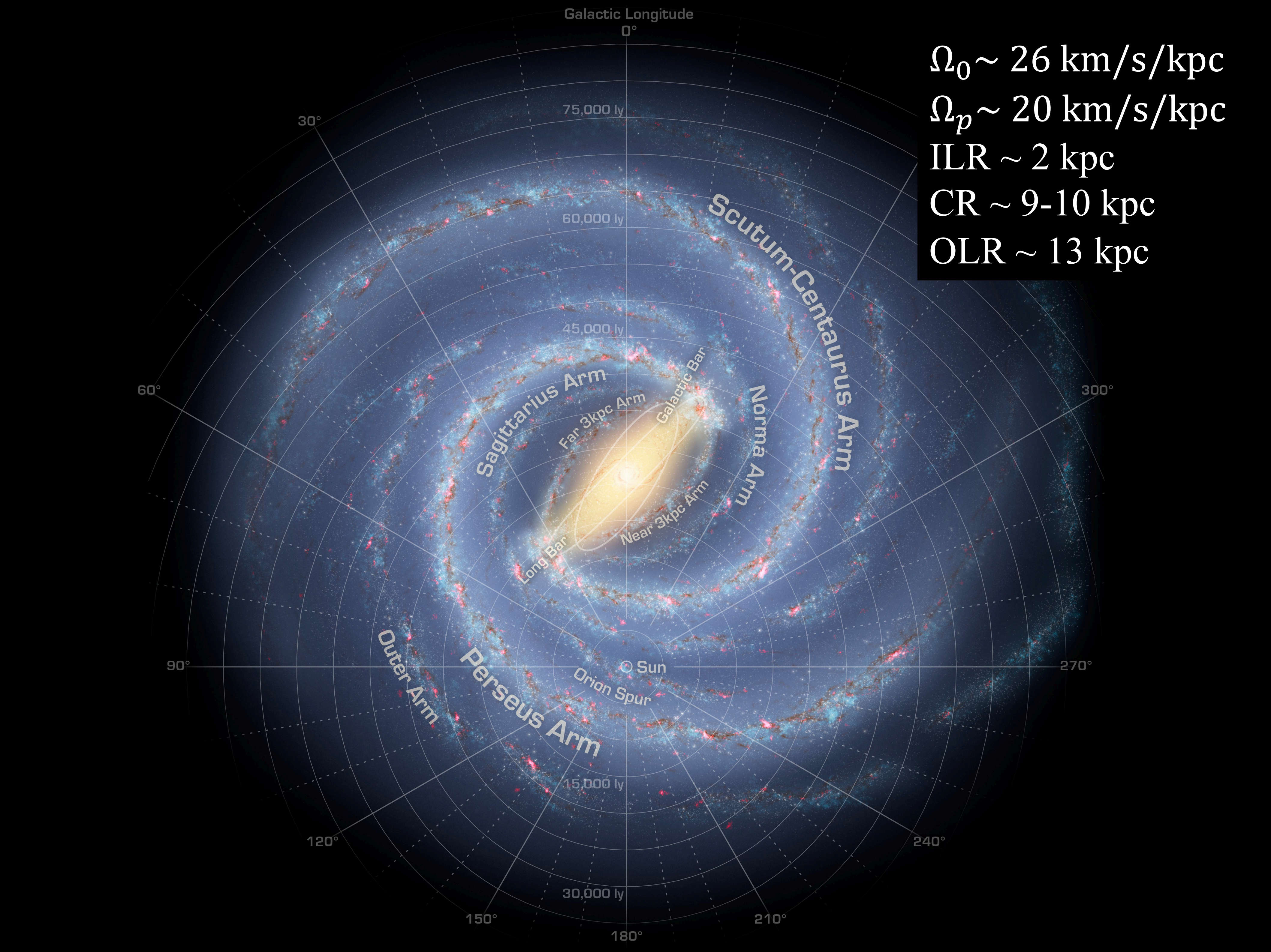
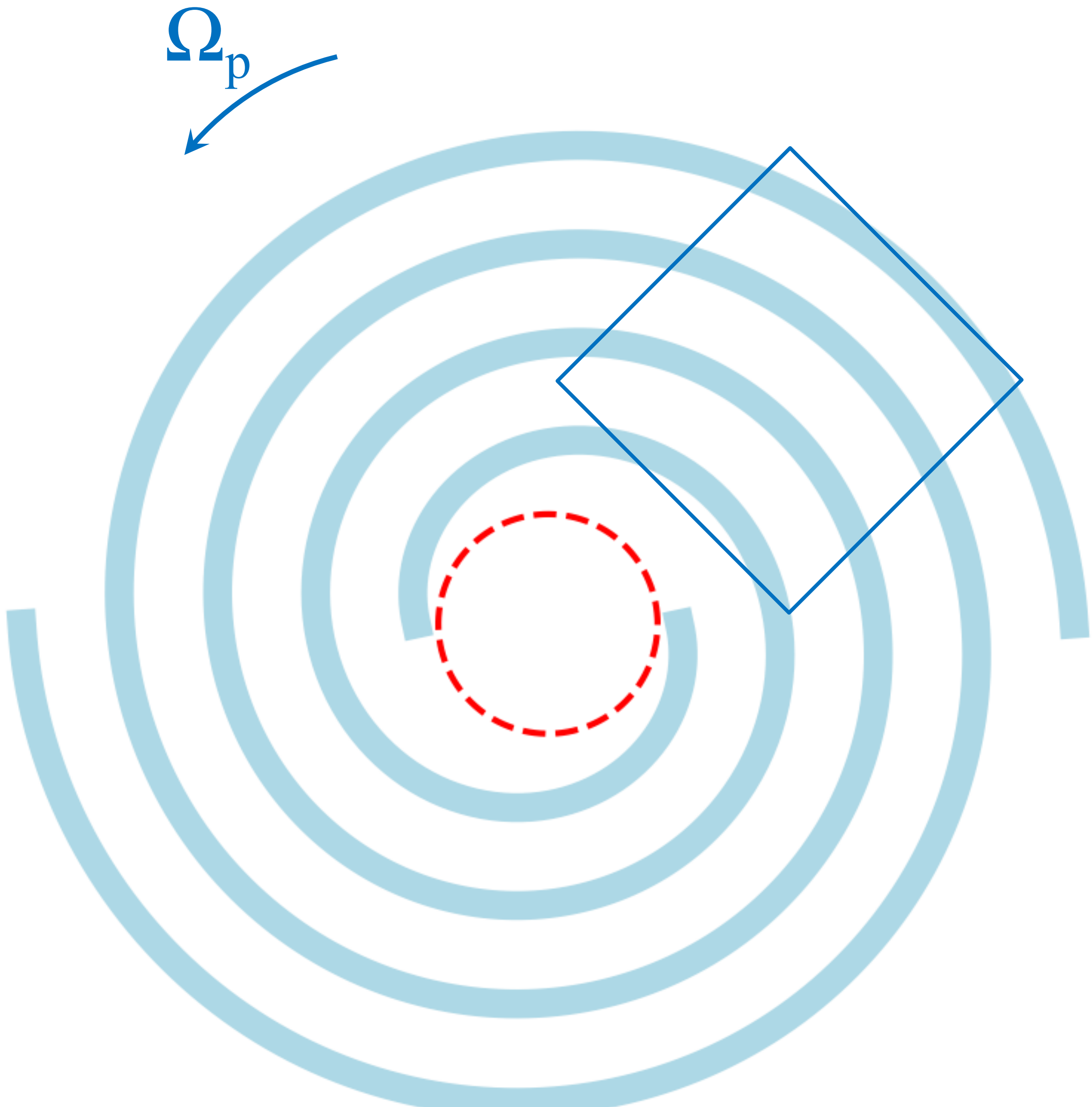
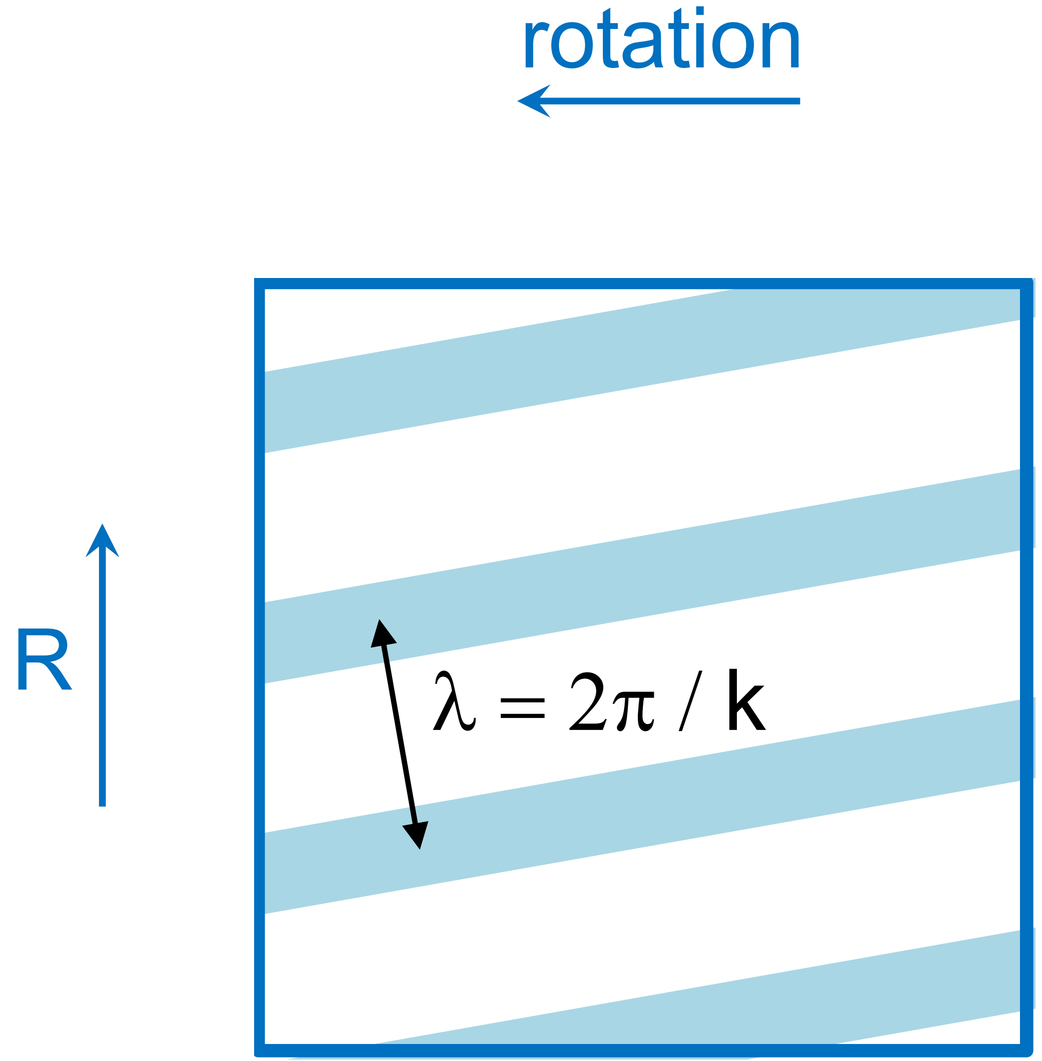


Fig 5.30 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

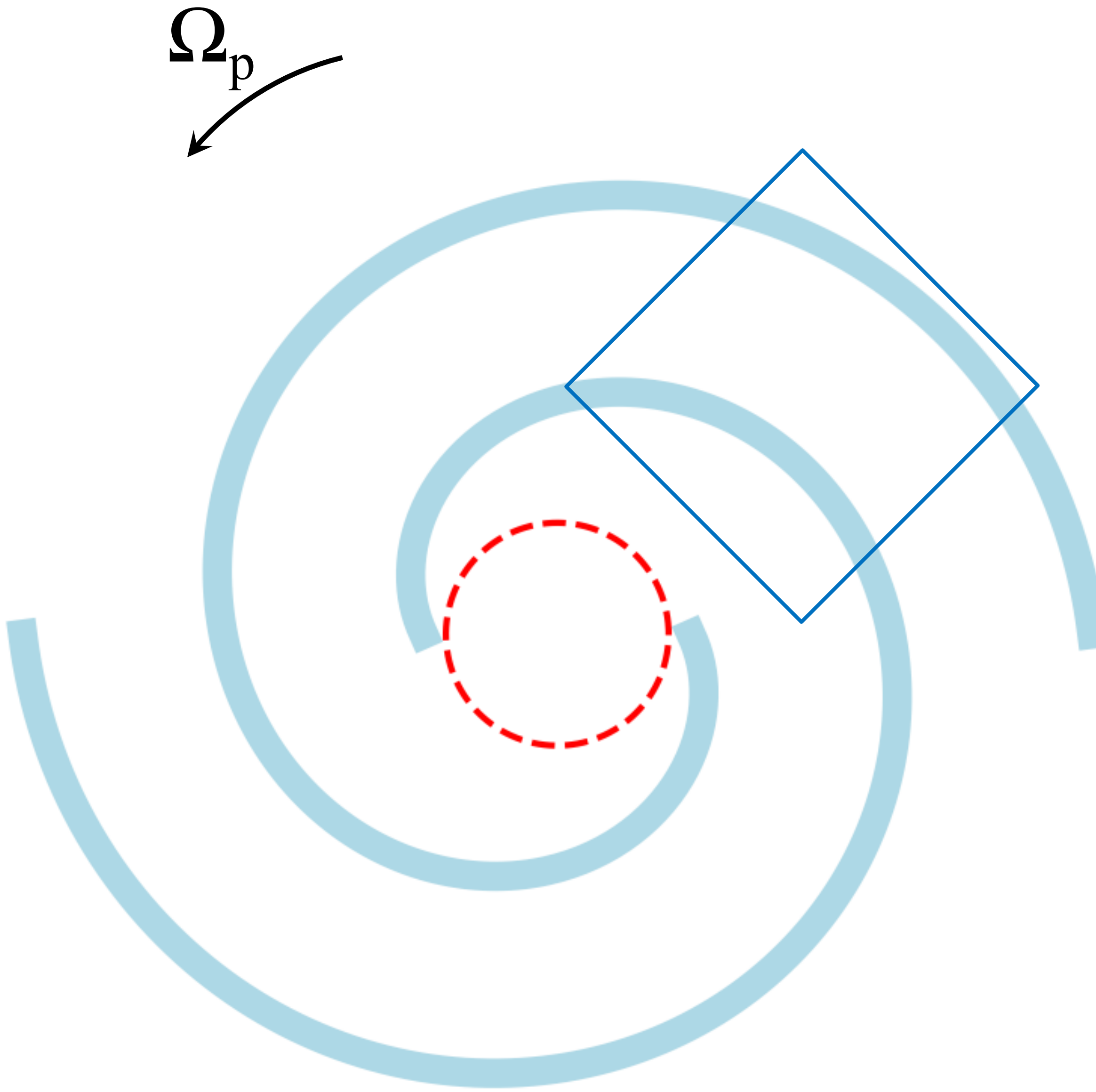


$\Omega_0 \sim 26$ km/s/kpc
$\Omega_p \sim 20$ km/s/kpc
ILR ~ 2 kpc
CR ~ 9-10 kpc
OLR ~ 13 kpc

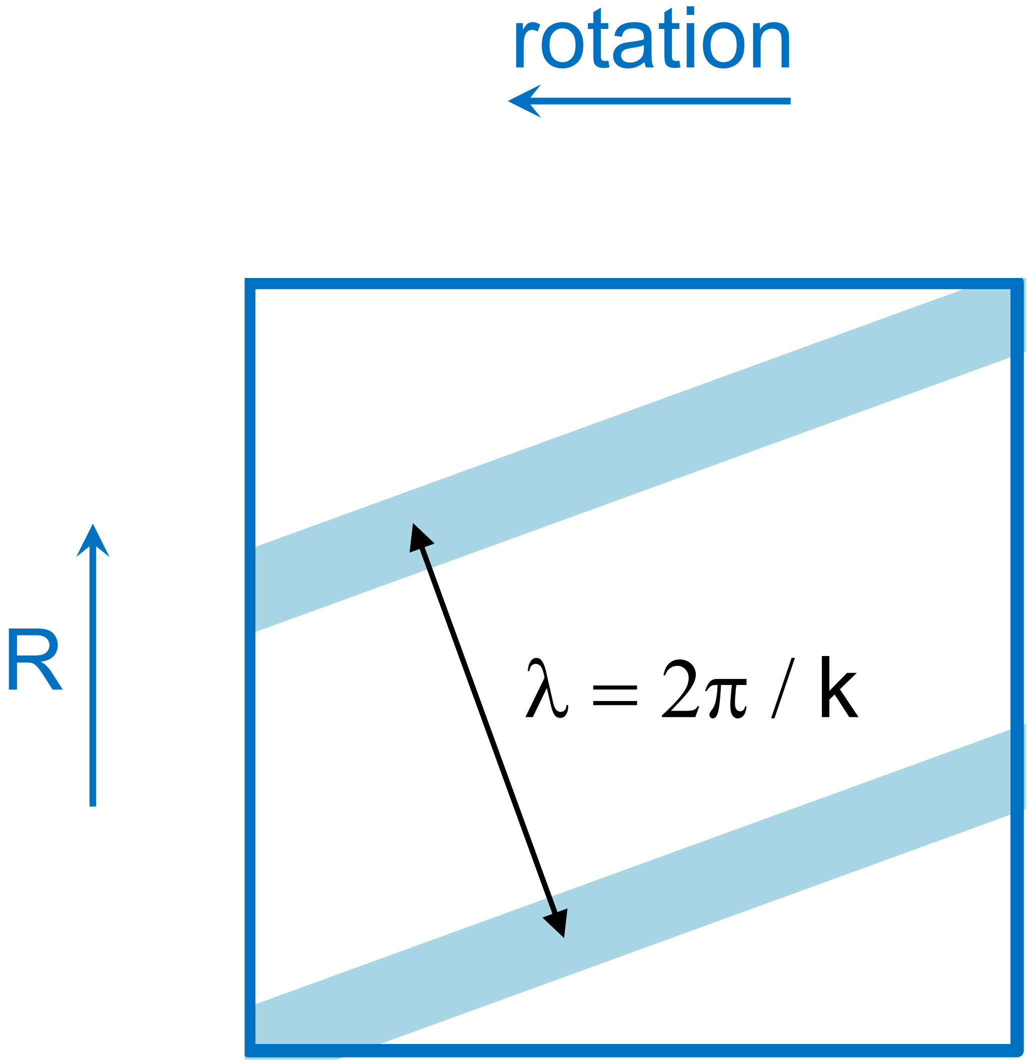

Ωp



Convention:
k > 0 for a
trailing spiral



Ω_p

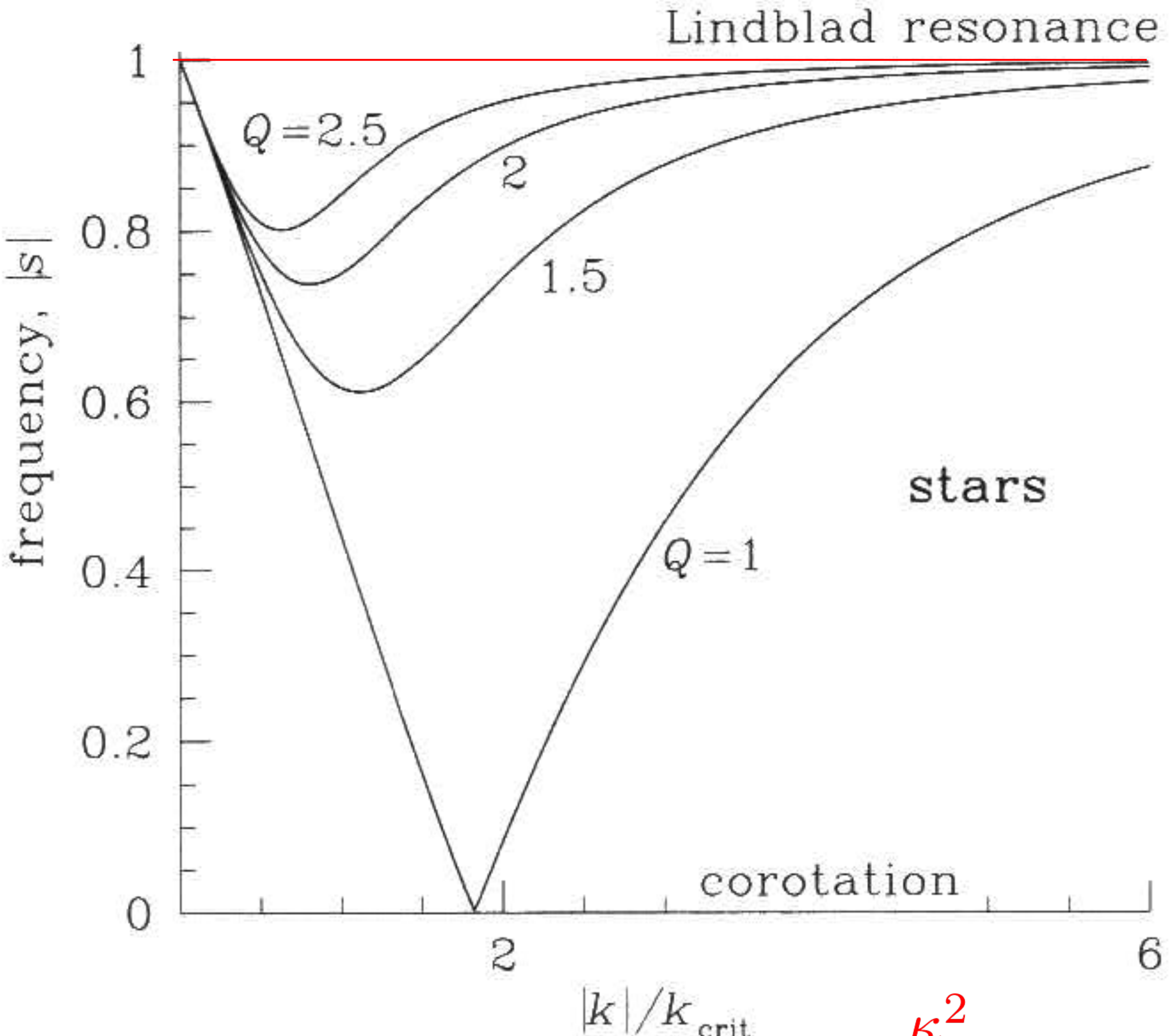

rotation
R
λ = 2π / k

$$
\begin{aligned}
\omega^2 &= v_s^2 k^2 \\
\omega^2 &= v_s^2 k^2 - 4\pi G\rho \\
\omega^2 &= v_s^2 k^2 - 4\pi G\rho + 4\Omega^2
\end{aligned}
$$

$$
\begin{aligned}
\omega^2 &= v_s^2 k^2 - 2\pi G\Sigma|k| + 4\Omega^2 \\
\omega^2 &= v_s^2 k^2 - 2\pi G\Sigma|k| + \kappa^2
\end{aligned}
$$

$$
(\omega - m\Omega)^2 = v_s^2 k^2 - 2\pi G\Sigma|k| + \kappa^2
$$

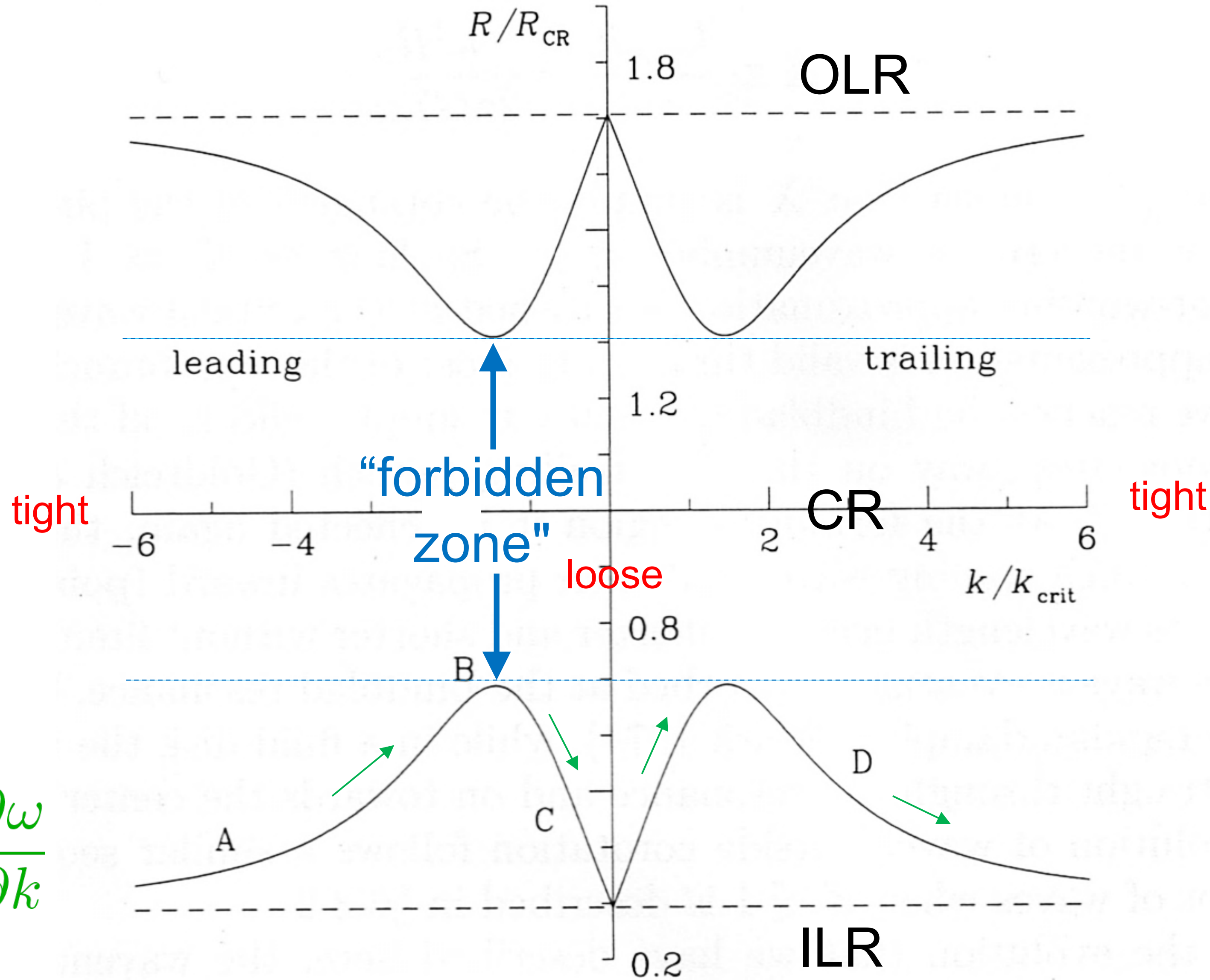
$$(\omega - m\Omega)^2 = v_s^2 k^2 - 2\pi G\Sigma|k| + \kappa^2$$



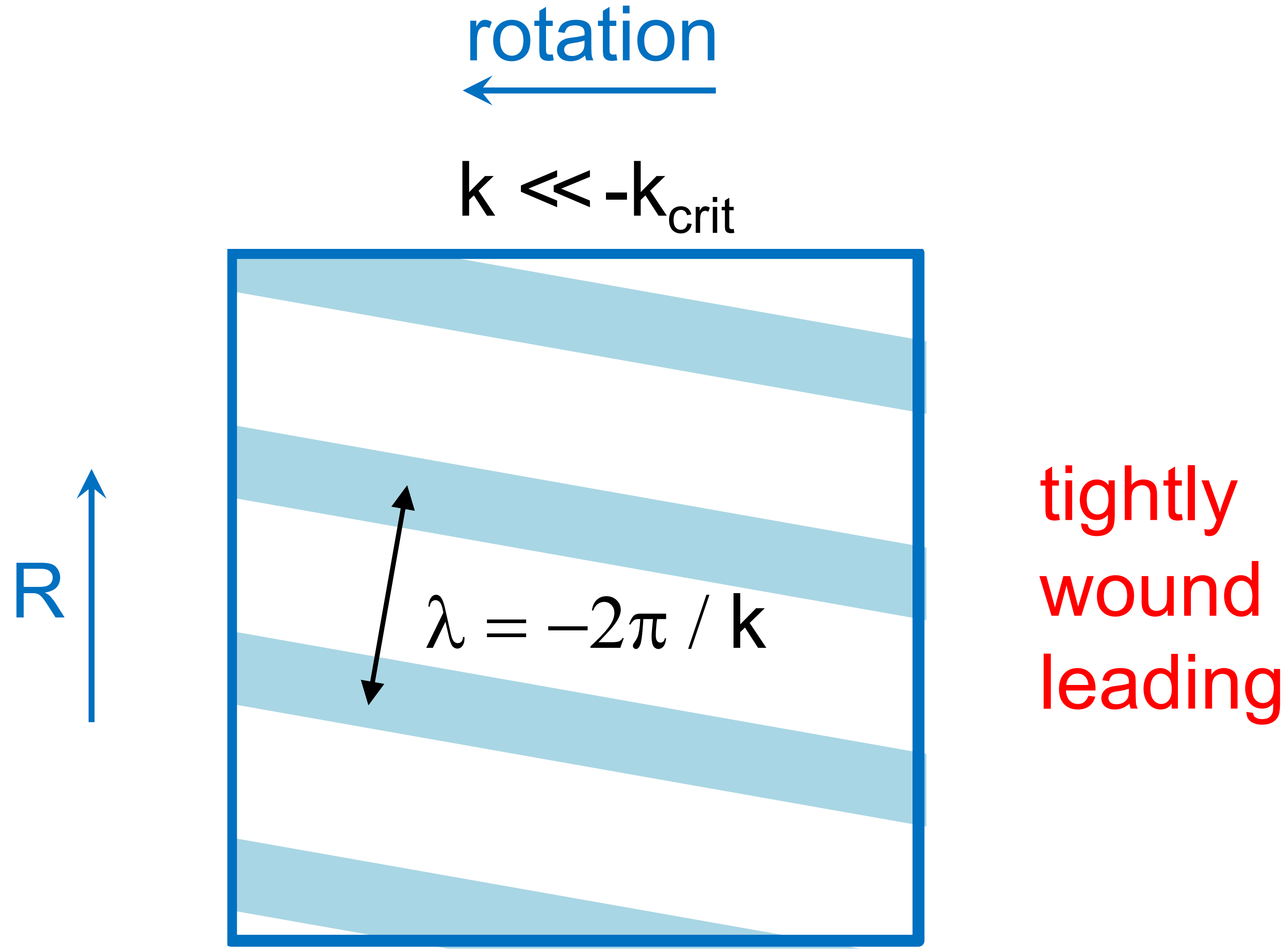
$$s = \frac{\omega - m\Omega}{\kappa}$$

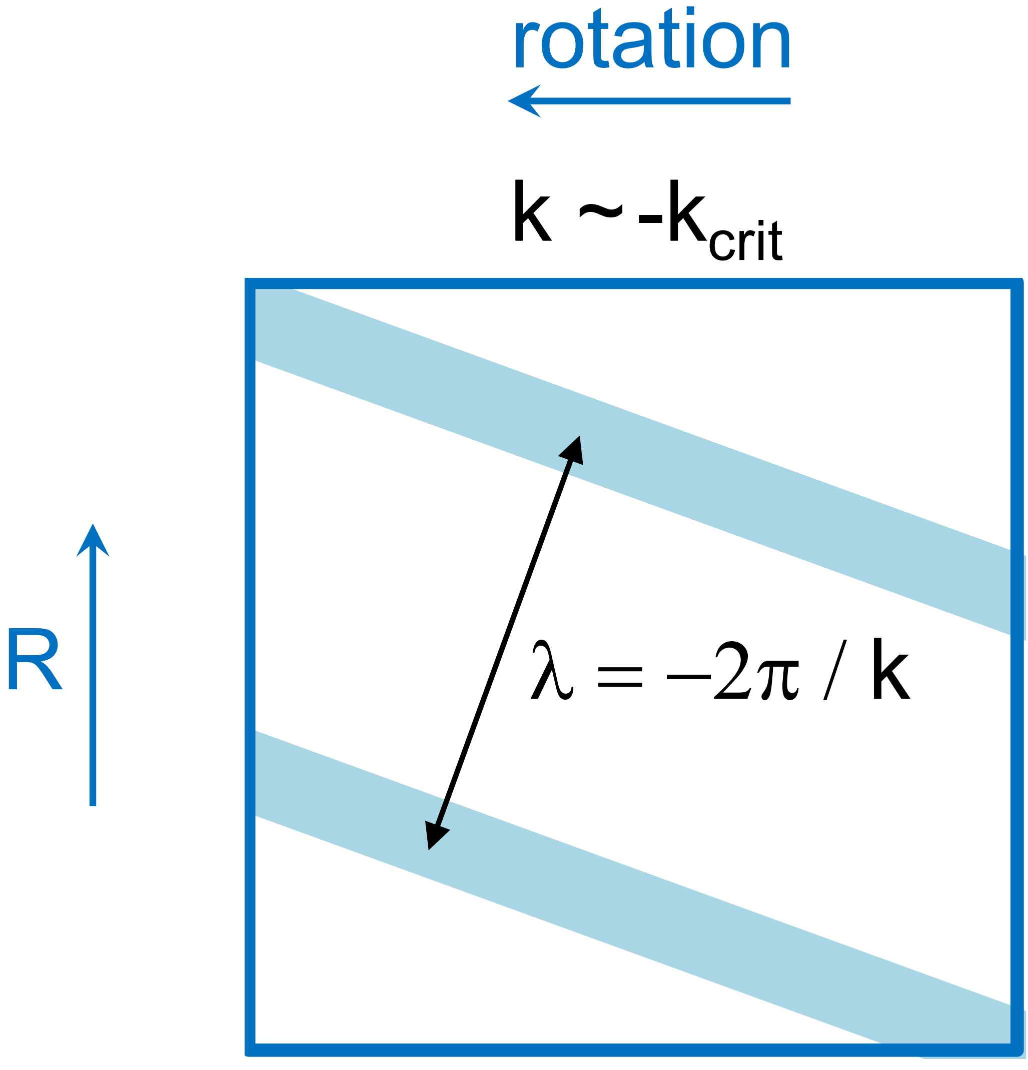
$$k_{\rm crit} = \frac{\kappa^2}{2\pi G\Sigma}$$

Q = 1.2
m = 2

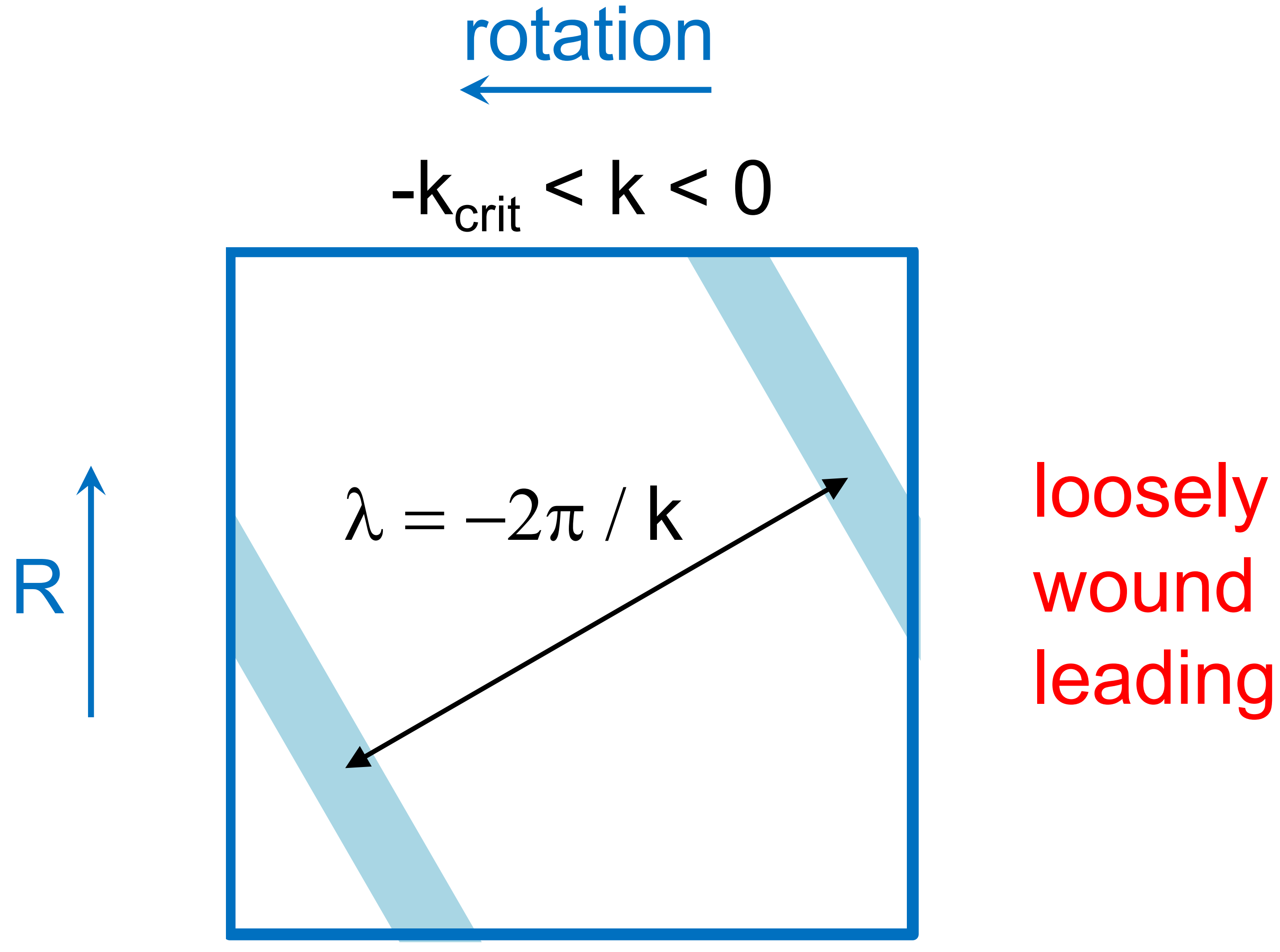


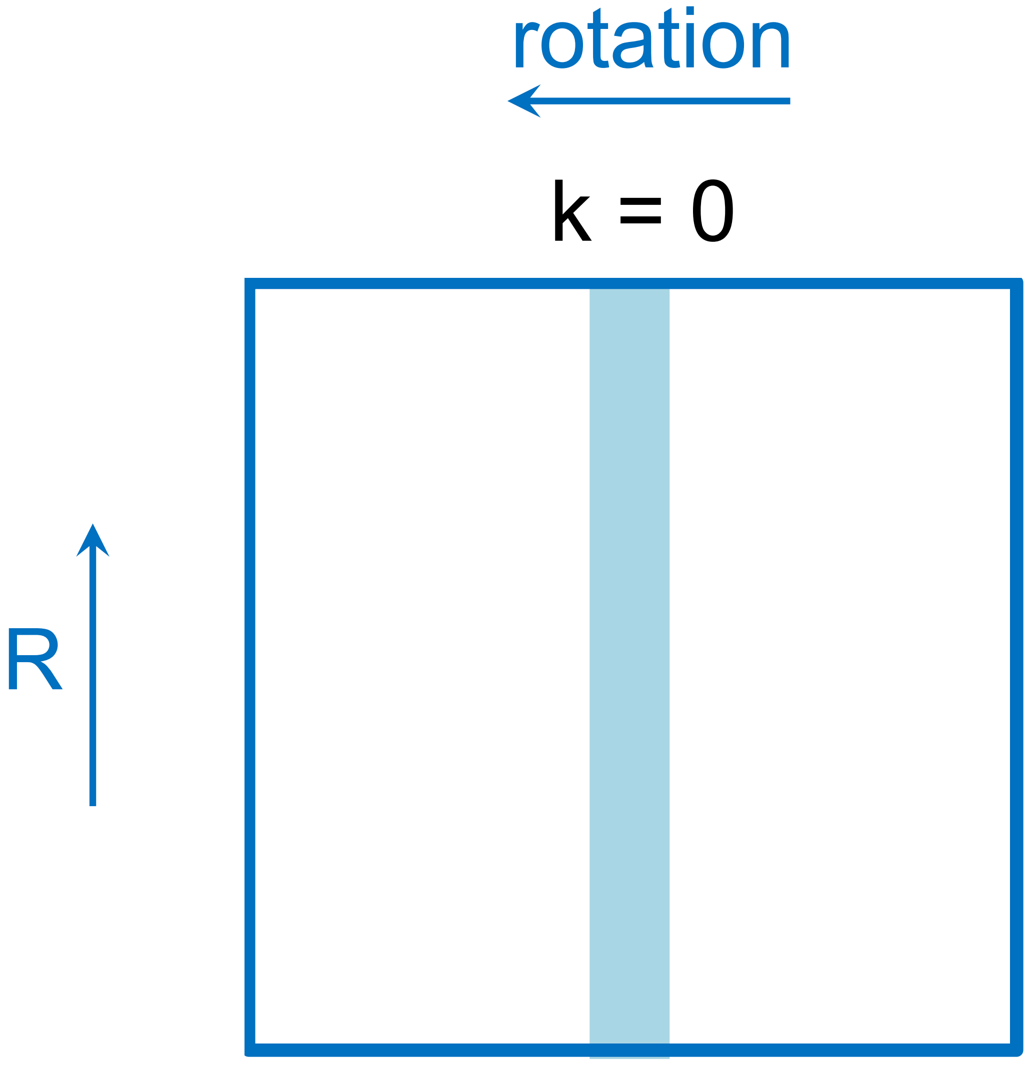
$$v_g = \frac{\partial \omega}{\partial k}$$

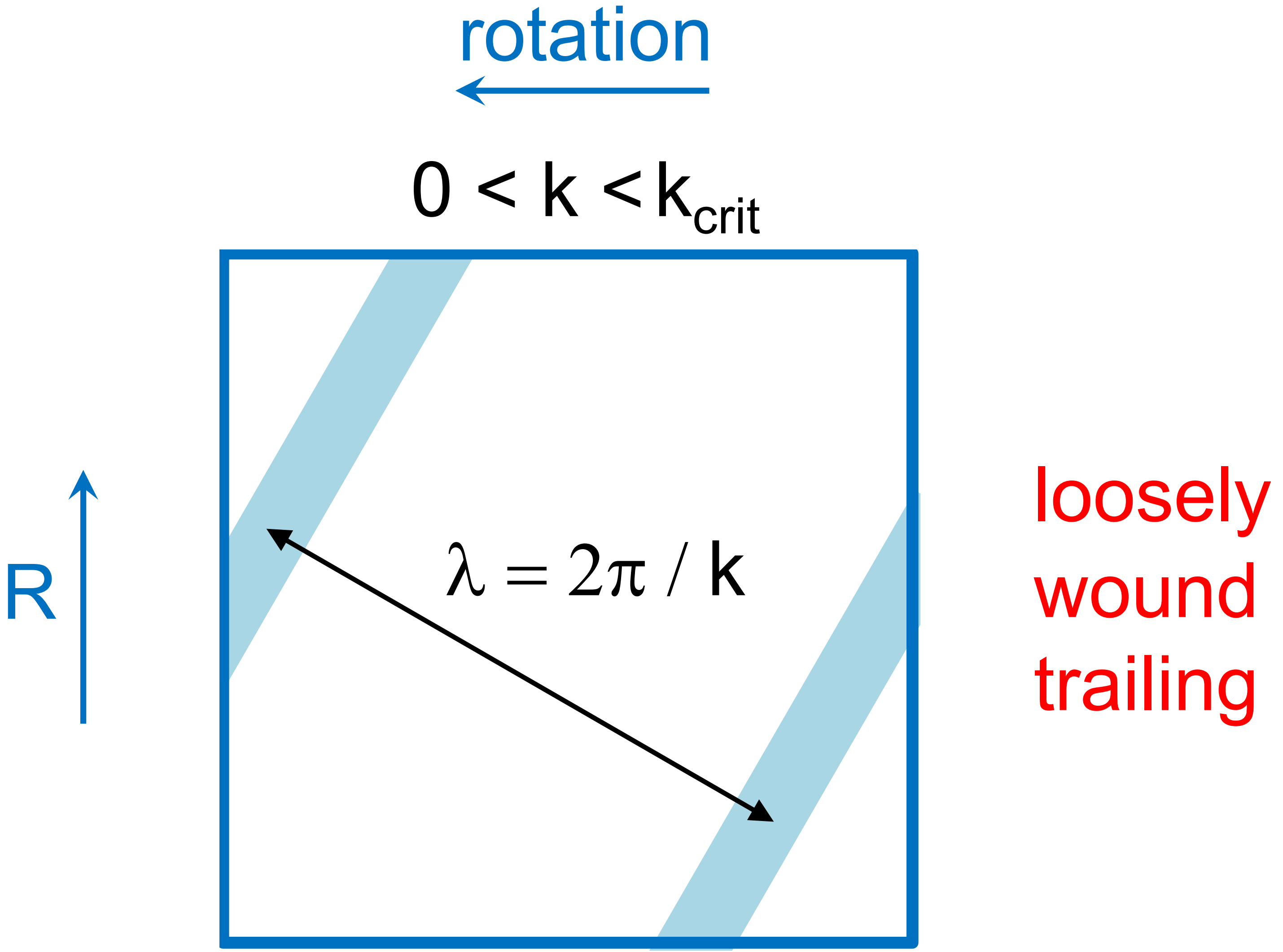

rotation
k ≪ -k_crit
R
λ = −2π / k
tightly
wound
leading



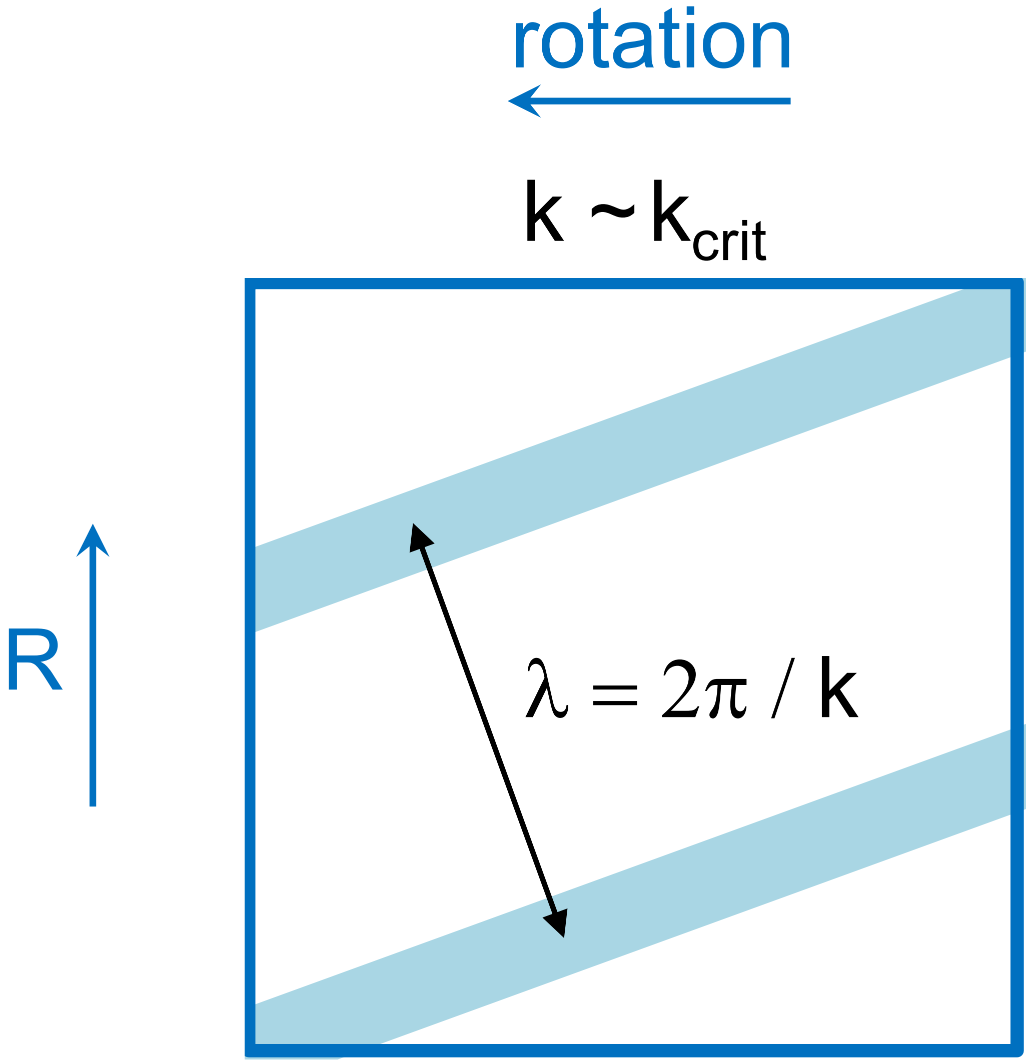
rotation
k ~ -k_crit
R
λ = −2π / k

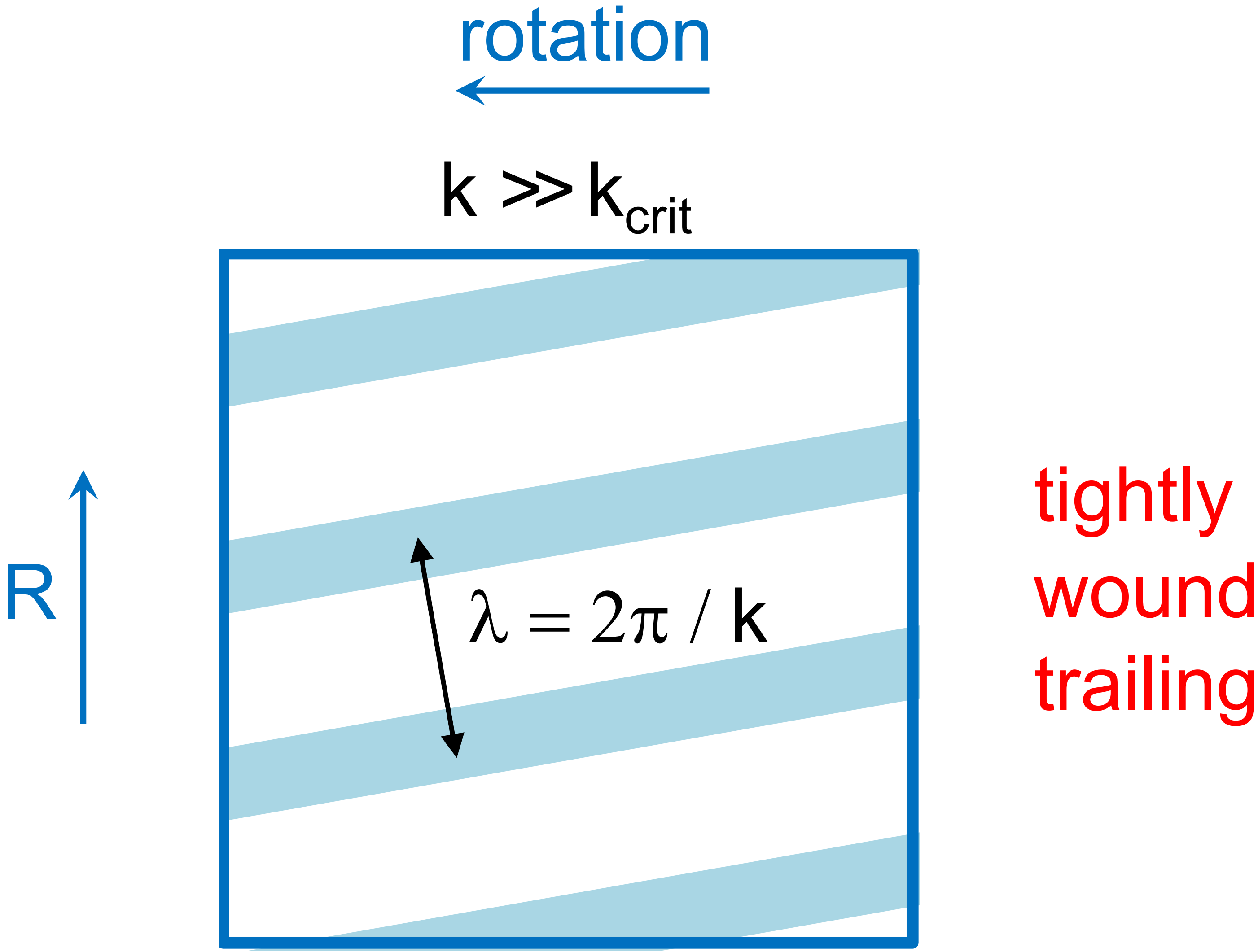

rotation
$-k_{crit} < k < 0$
R
$\lambda = -2\pi / k$
loosely
wound
leading

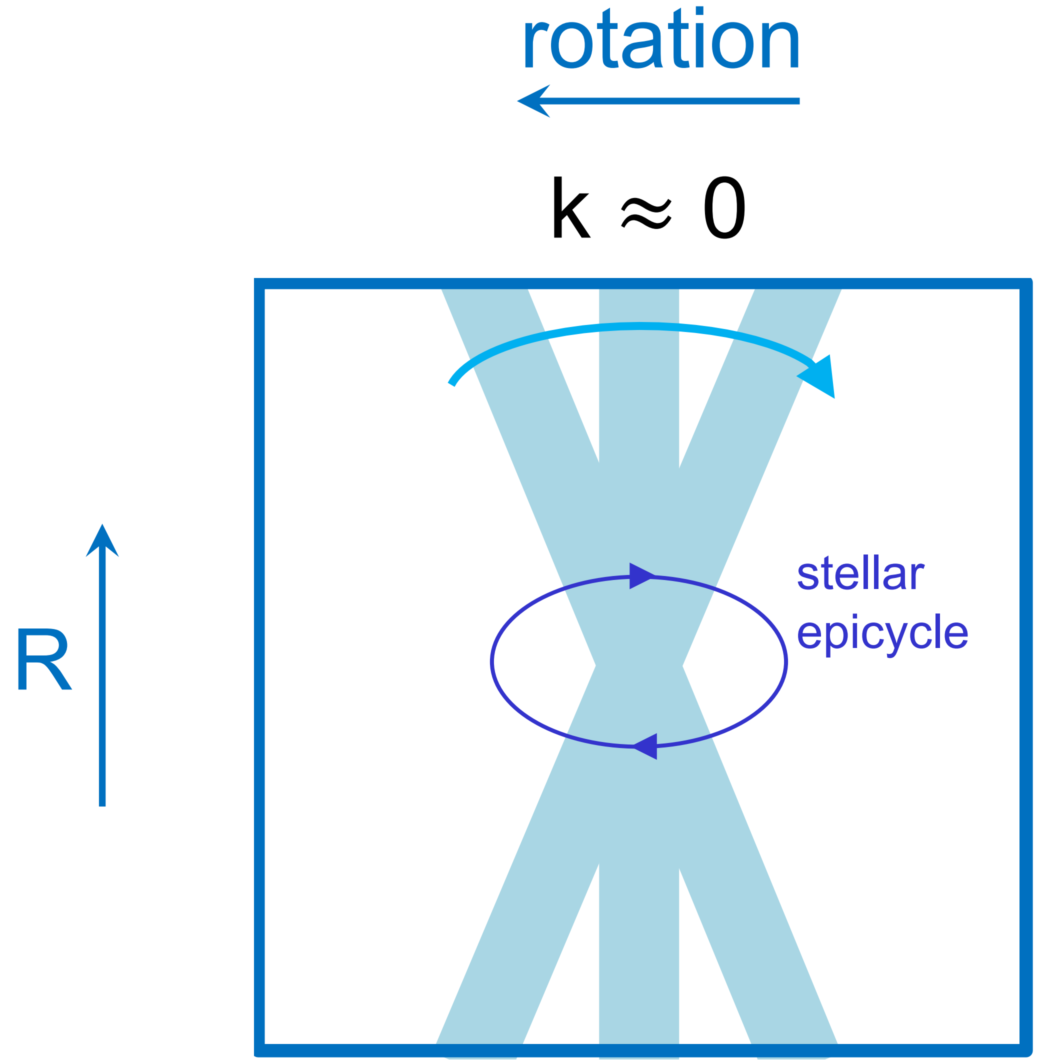

rotation
k = 0
R



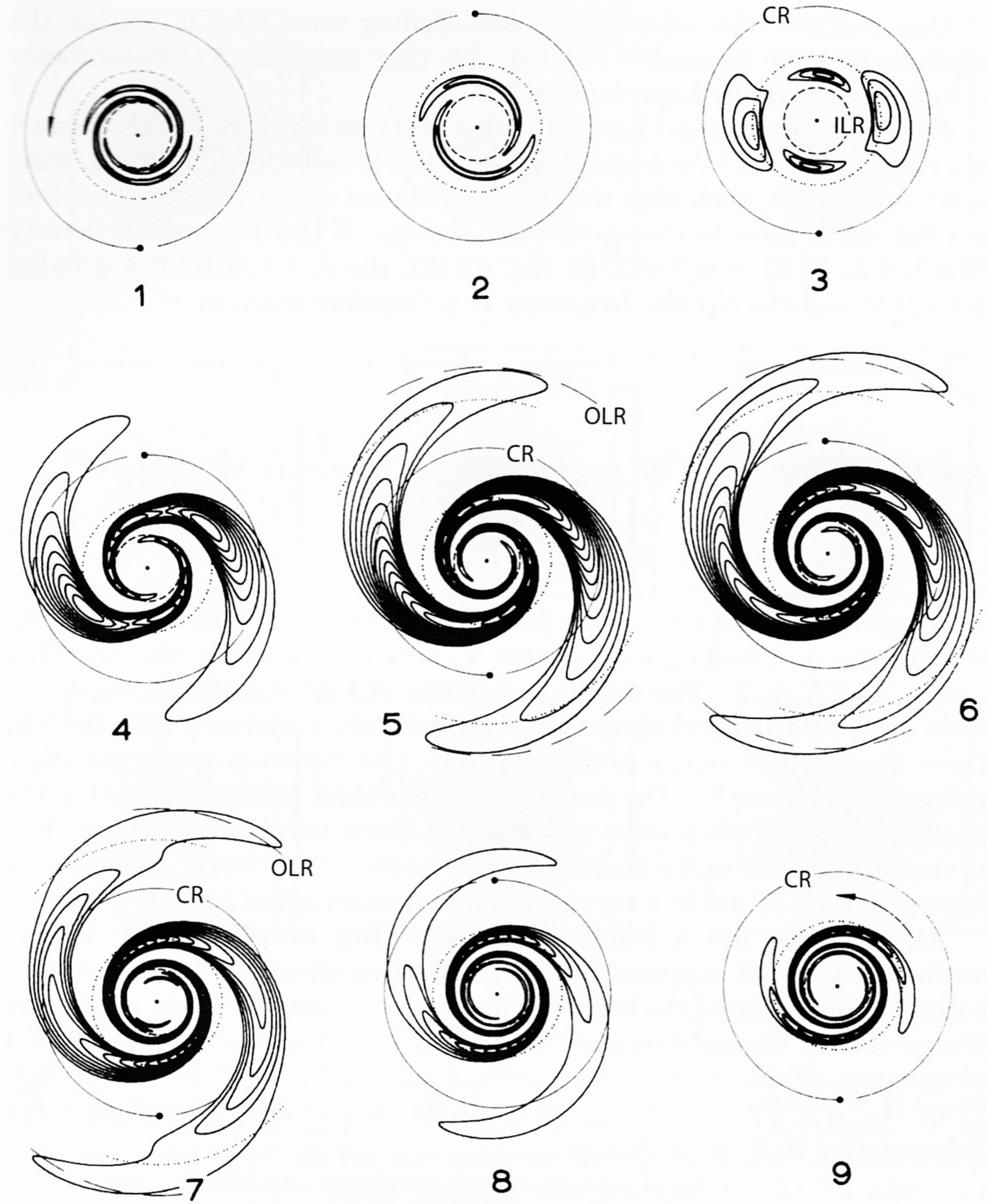
rotation
$0 < k < k_{crit}$
R
$\lambda = 2\pi / k$
loosely
wound
trailing


rotation
k ~ k$_{crit}$
R
$\lambda = 2\pi / k$


rotation
k >> k_crit
R
λ = 2π / k
tightly
wound
trailing



Swing
amplification


CR
ILR
OLR
CR
OLR
CR
CR
1
2
3
4
5
6
7
8
9