90° deflection distance: $$b_{90} = \frac{G\langle m\rangle}{\langle v^2\rangle}$$

cross section $$\sigma_{90} = \pi b_{90}^2$$

define strong encounter time scale by

$$t_s = \frac{1}{r_s} = \frac{1}{n\sigma v} = \frac{1}{n\sigma_{90}\langle v^2\rangle^{1/2}}$$

$$= \frac{v^3}{\pi G^2 m^2 n}$$

$$= 1.7\times10^{13}\ \mathrm{yr}\ \left(\frac{v}{10\ \mathrm{km/s}}\right)^3 \left(\frac{m}{M_\odot}\right)^{-2} \left(\frac{n}{1\ \mathrm{pc}^{-3}}\right)^{-1}$$

small-angle scattering $(b \gg b_{90})$

$$\Delta v_\perp = 2\frac{bv_\infty^3}{GM}\left(1+\frac{b^2v_\infty^4}{G^2M^2}\right)^{-1} \approx 2\frac{GM}{bv_\infty}$$

expect $\langle \Delta v_\perp \rangle = 0$, $\left\langle \Delta v_\perp^2 \right\rangle = (\Delta v_\perp)^2 \times \#\,\text{of encounters}$

# of encounters in time δt in $[b, b+db)$ is $2\pi b\; db\; n\; v\delta t$, so integrate over b to find

$$\frac{d}{dt}\left\langle \Delta v_\perp^2 \right\rangle = 8\pi\,\frac{G^2m^2n}{v}\ln\frac{b_{max}}{b_{90}} = 8\pi\,\frac{G^2m^2n}{v}\ln\Lambda$$

define <u>relaxation</u> time scale by

$$t_r = \frac{v^2}{\frac{d}{dt}\langle \Delta v_\perp^2 \rangle} = \frac{v^3}{8\pi G^2 m^2 n \ln \Lambda}$$

$$= 2\times10^{11}\,\mathrm{yr}\left(\frac{v}{10\,\mathrm{km/s}}\right)^3 \left(\frac{m}{M_\odot}\right)^{-2} \left(\frac{n}{1\,\mathrm{pc}^{-3}}\right)^{-1}$$