Fluid Dynamics with Gravity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$ Continuity equation

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\frac{\nabla P}{\rho} - \nabla \phi$$ Euler equation

$$P = P(\rho, S)$$ Equation of state

$$\nabla^2 \phi = 4\pi G \rho$$ Poisson's equation

Small Perturbations

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \boldsymbol{v}_1 = 0$$

Continuity equation

$$\frac{\partial \boldsymbol{v}_1}{\partial t} = -\frac{\nabla P_1}{\rho_0} - \nabla \phi_1$$

Euler equation

$$P_1 = \left.\frac{\partial P}{\partial \rho}\right|_{S,0} \rho_1 = c_s^2 \rho_1$$

Equation of state

$$\frac{\partial^2 \rho_1}{\partial t^2} - c_s^2 \nabla^2 \rho_1 = 4\pi G \rho_0 \rho_1$$

Wave equation?

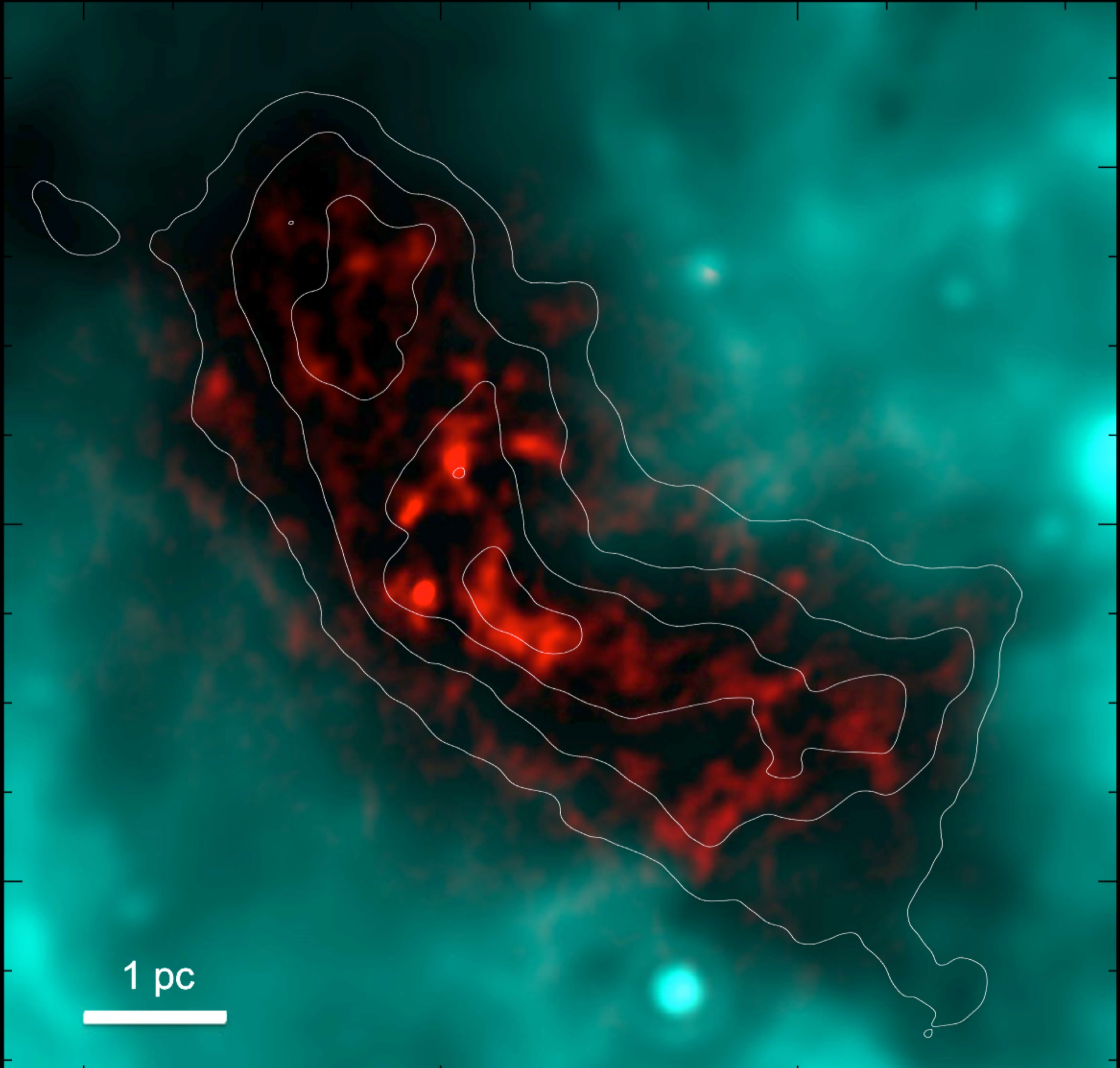
Jeans Instability

$$\frac{\partial^2 \rho_1}{\partial t^2} - c_s^2 \nabla^2 \rho_1 = 4\pi G \rho_0 \rho_1$$

seek solution with $\rho_1 \propto e^{i\boldsymbol{k}\cdot\boldsymbol{x} - i\omega t}$

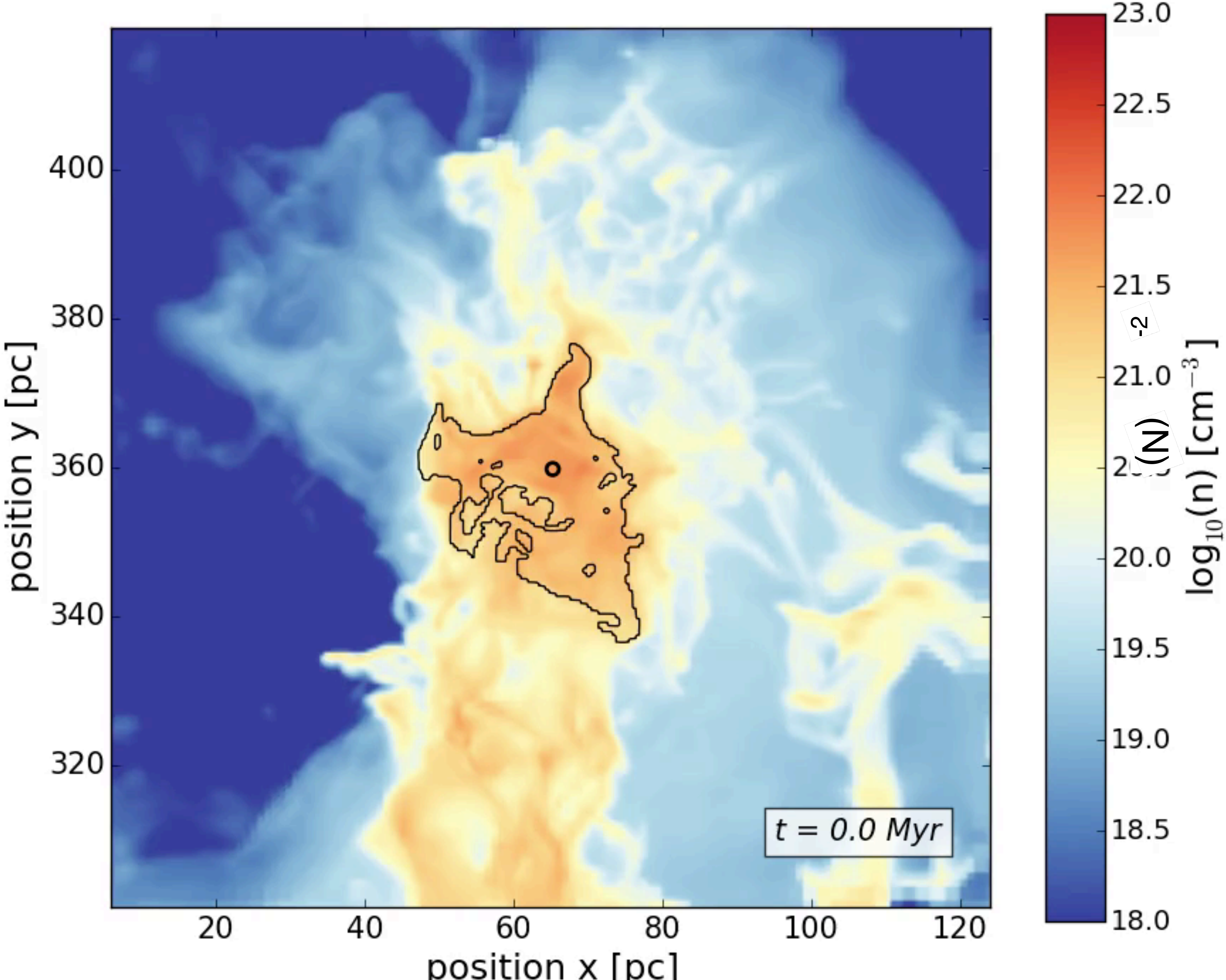
$$\Longrightarrow \quad \omega^2 = c_s^2 k^2 - 4\pi G \rho_0$$

$$\Longrightarrow \quad \omega^2 < 0 \quad \text{for} \quad k^2 < \frac{4\pi G \rho_0}{c_s^2} \equiv k_J^2$$


1 pc

$M \sim 10^4 M_{\odot}$
$\Delta x = 0.12$ pc

Ibáñez-Mejía et al. 2017



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Visible • WFPC2

Infrared • NICMOS

Trapezium Cluster • Orion Nebula

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