PHYS 431/531: Galactic Astrophysics

Fall 2021, Homework #3

(Due October 16, 2021)

1. Neutral hydrogen atoms in the cool interstellar medium have number density $n_H \sim 1\,\mathrm{cm}^{-3}$ and temperature $T \sim 100\,\mathrm{K}$.

   (a) Show that the average speed $\bar{v}$ of these atoms, defined by $\frac{1}{2}m_H\bar{v}^2 = \frac{3}{2}kT$ (where m_H is the mass of a hydrogen atom and k is Boltzmann's constant), is

   $$\bar{v} \approx 2,\mathrm{km\,s}^{-1}\left(\frac{T}{100\,\mathrm{K}}\right)^{1/2}.$$

   (b) Hence show that the typical atomic center-of-mass kinetic energy is much greater than the energy difference between the hyperfine states associated with the 21-cm radio line.

   The mean time between collisions for atoms in this environment is a few thousand years, while the mean time for an excited atom to emit a 21-cm photon is $\sim 1.1 \times 10^7\,\mathrm{yr}$. As a result, the populations of the lower and upper hyperfine states are determined entirely by collisional processes and the states are populated proportional to their statistical weights, so three-quarters of all hydrogen atoms are in the upper state.

   (c) Calculate the total 21-cm luminosity of a galaxy containing a total of $5 \times 10^9\,M_\odot$ of neutral hydrogen.

2. (a) A spherical dust grain lies 0.5 pc from an O star of luminosity $5 \times 10^5\,L_\odot$ and absorbs all electromagnetic radiation striking it. It is spinning, so the absorbed energy is evenly distributed and the grain's temperature T is uniform across the surface. The star's emission is isotropic and the grain radiates (also isotropically) as a blackbody at temperature T. What is its equilibrium temperature, and at what wavelength does its emission peak?

   (b) *[GRADUATE STUDENTS.]* The dust grain is part of a spherical cloud of gas of radius 2 pc and uniform density $n_H = 150\,\mathrm{cm}^{-3}$ centered on the star. Assuming a dust density of $10^{-12}\,n_H$ and taking all dust grains to have radius $r_d = 0.15\mu m$, calculate the optical depth in dust absorption between the star and the edge of the cloud. Hence determine what fraction of the star's light escapes from the cloud.

3. What are the sound speed and Jeans mass

   (a) in a molecular cloud core (pure H_2) of temperature 10 K and number density 10^6 molecules/cm^3?

   (b) in atomic hydrogen gas with temperature 100 K and number density 1 atom/cm^3?

   (c) in hot ionized hydrogen with temperature 10^6 K and number density 10^{-3} protons/cm^3?

   In all cases, assume an adiabatic index $\gamma = \frac{5}{3}$.

4. A particle is dropped (from radius a with zero velocity) into the gravitational potential corresponding to a static homogeneous sphere of radius a and density ρ. Calculate how long the particle takes to reach the center of the sphere. [Hint: the equation of motion is $d^2r/dt^2 = -GM(r)/r^2$.]

5. *[GRADUATE STUDENTS.]* Calculate the time required for a homogeneous sphere of radius a and density ρ with no internal pressure support to collapse under its own gravity. [Apply the previous equation of motion to a particle on the surface.]