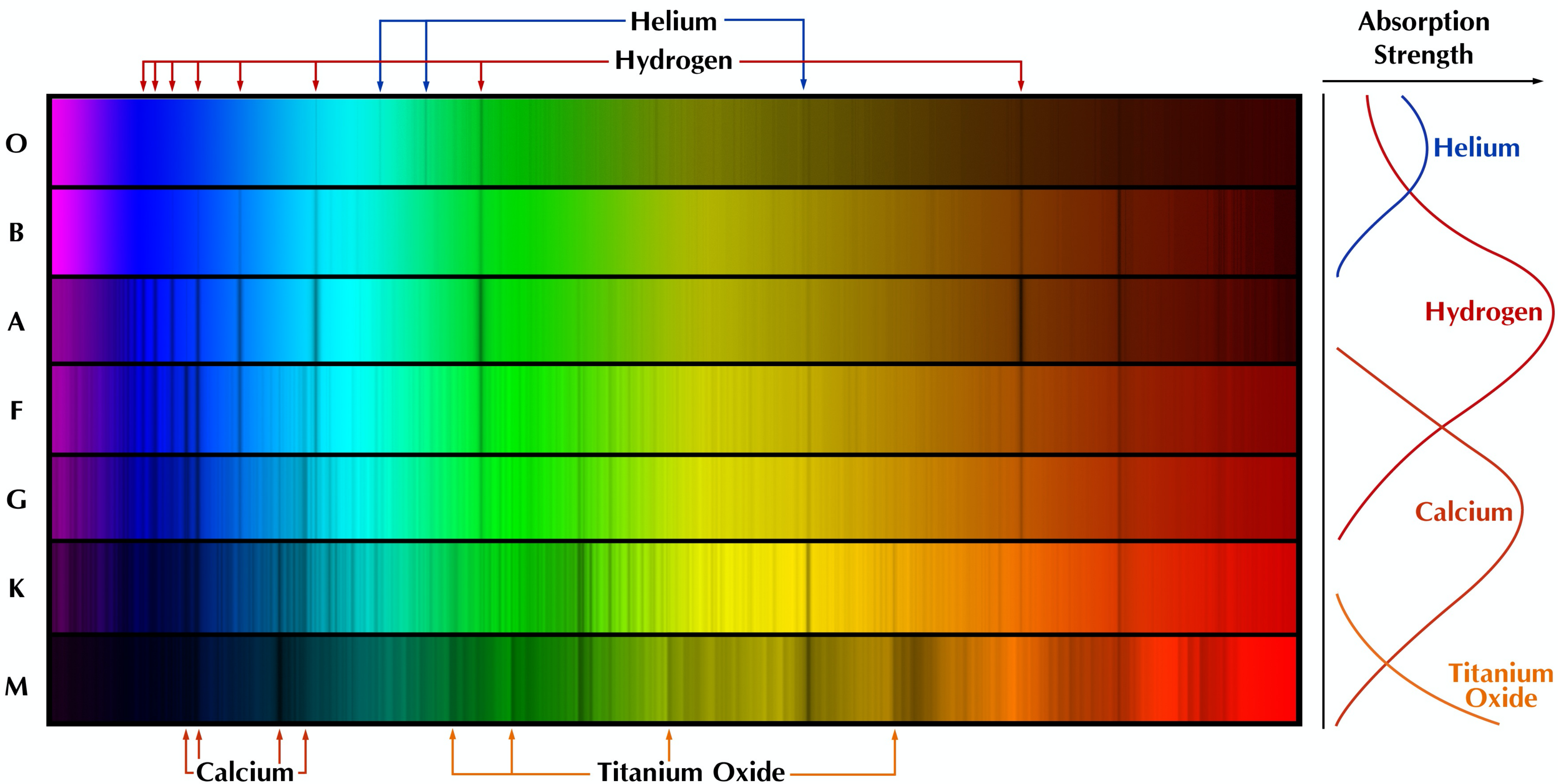

Helium
Hydrogen
O
B
A
F
G
K
M
Calcium
Titanium Oxide
Absorption Strength
Helium
Hydrogen
Calcium
Titanium Oxide

Thermodynamic Equilibrium

Maxwell-Boltzmann

$$p(E) = \left(\frac{m}{2\pi kT}\right)^{1/2} \frac{E}{kT}\, e^{-E/kT} \quad \left(E = \frac{1}{2}mv^2\right)$$

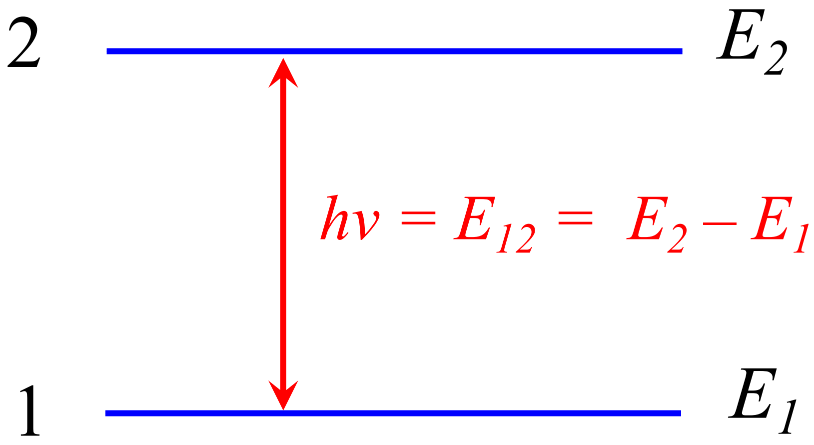
Boltzmann

$$\frac{n_2}{n_1} = \frac{g_2}{g_1}\, e^{-\Delta E/kT}$$

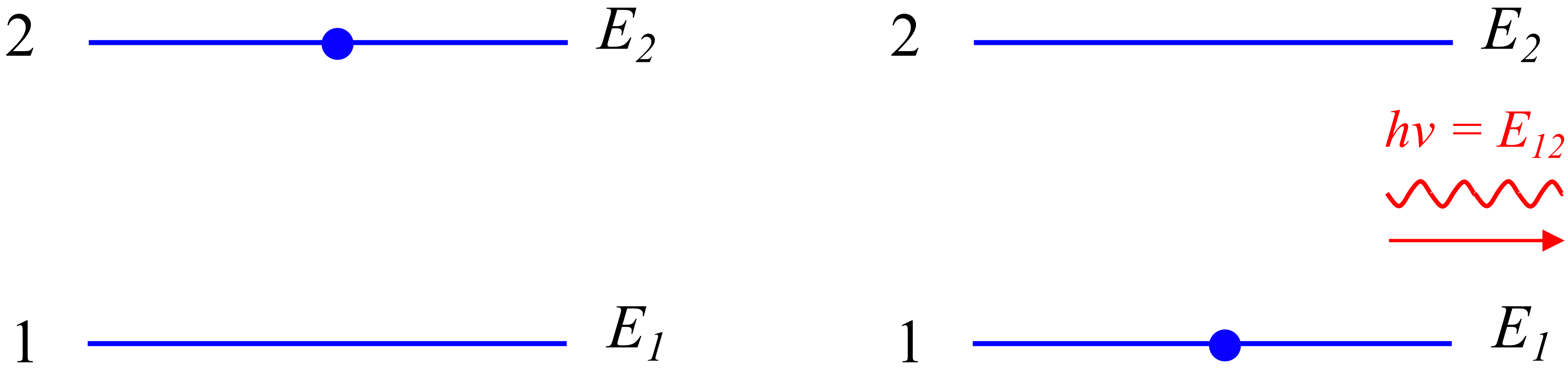
Saha

$$\frac{n_{i+1} n_e}{n_i} = \frac{2(2\pi m_e)^{3/2}}{\mathrm{h}^3} (kT)^{3/2}\, \frac{g_{i+1}}{g_i}\, e^{-\Delta E_i/kT}$$

Two-level Atom



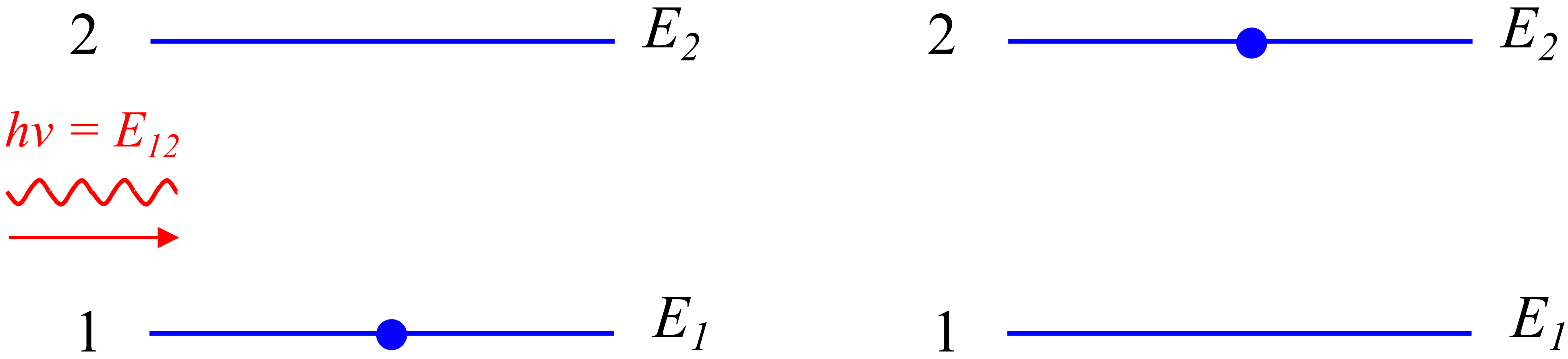
Two-level Atom



spontaneous emission

probability = A_{21} s^{-1}

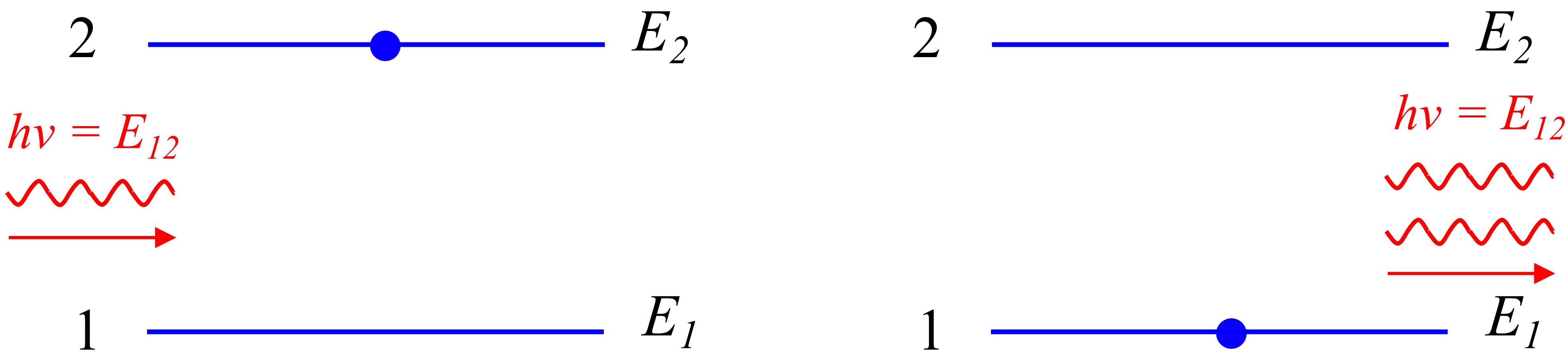
Two-level Atom



absorption

probability = $B_{12}\, u_\nu$ s^{-1}

Two-level Atom



stimulated emission

probability = $B_{21}\, u_{\nu}\ \mathrm{s}^{-1}$

Thermodynamic Equilibrium

Maxwell-Boltzmann

$$p(E) \ = \ \left(\frac{m}{2\pi kT}\right)^{1/2} \frac{E}{kT} \ e^{-E/kT} \qquad \left(E = \frac{1}{2} m v^2\right)$$

Boltzmann

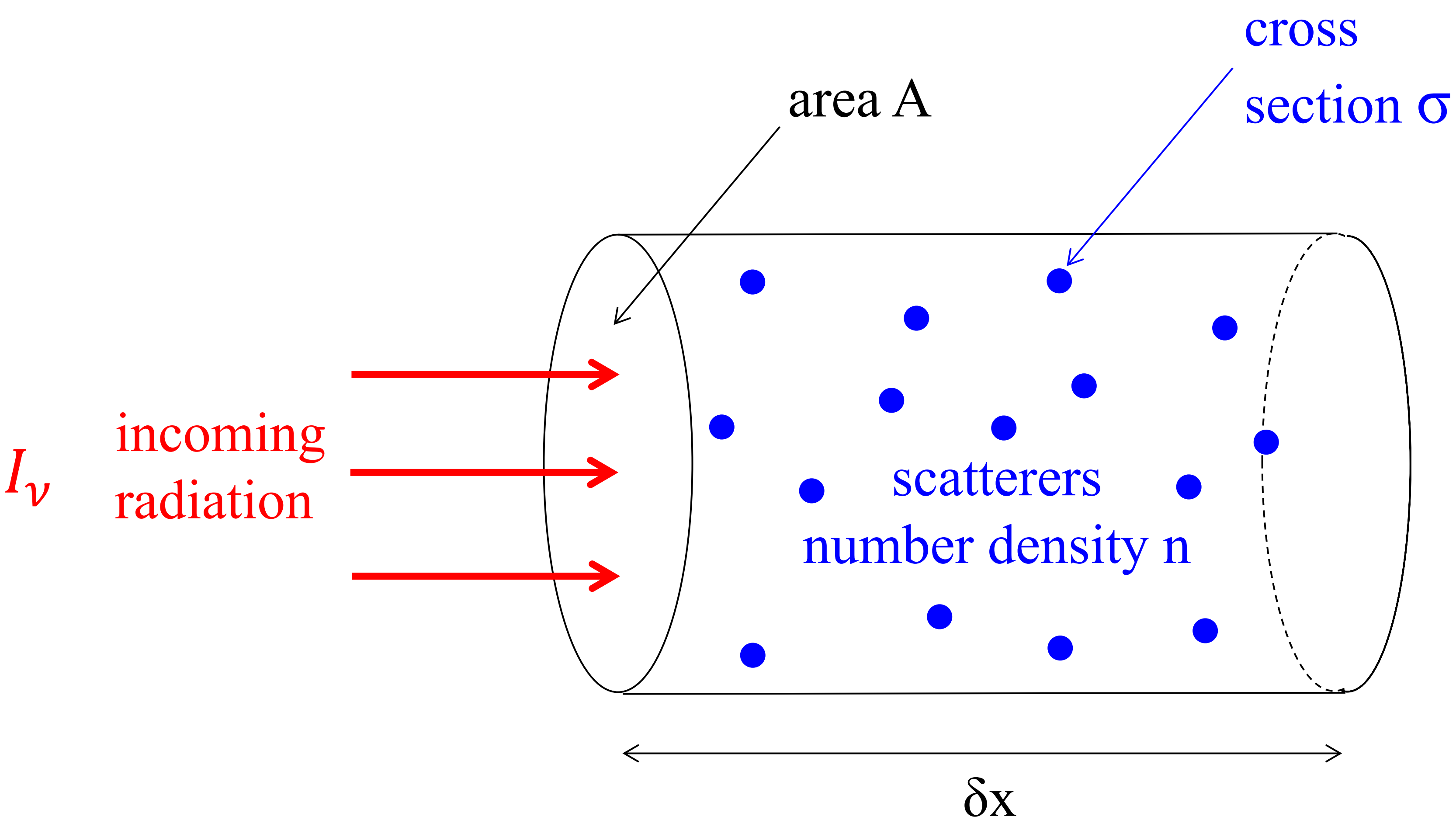
$$\frac{n_2}{n_1} \ = \ \frac{g_2}{g_1} \ e^{-\Delta E/kT}$$

Saha

$$\frac{n_{i+1} n_e}{n_i} \ = \ \frac{2(2\pi m_e)^{3/2}}{\mathrm{h}^3} (kT)^{3/2} \ \frac{g_{i+1}}{g_i} \ e^{-\Delta E_i/kT}$$

Blackbody

$$\mathrm{u}_\nu \ = \ \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/kT} - 1}$$



$$N_{scatterers} = A\delta x\, n$$

$$\text{area fraction} = \frac{A\delta x\, n\sigma}{A} = n\sigma\delta x$$

$$\frac{\delta I_\nu}{I_\nu} = -n\sigma\delta x$$

Radiation Transfer Equation

$$\frac{dI_\nu}{dx} = -\alpha_\nu I_\nu + j_\nu$$

emission processes e.g. $n_2 A_{21} E_{12}/4\pi$

absorption processes

$\alpha_\nu = n\sigma_\nu$ (σ = cross section)

$\alpha_\nu = \rho\kappa_\nu$ (κ = opacity)

Radiation Transfer Equation

$$\frac{dI_\nu}{dx} = -\alpha_\nu I_\nu + j_\nu$$

Absorption only:

$$\frac{dI_\nu}{dx} = -\alpha_\nu I_\nu$$

$\tau = 1$

$\Rightarrow \quad x =$ mean free path

$L \sim 1/\alpha$

$$\Rightarrow \quad I_\nu = I_{\nu,0}\, e^{-\tau_\nu}$$

where $\tau_\nu = \int \alpha_\nu \, dx$ = optical depth

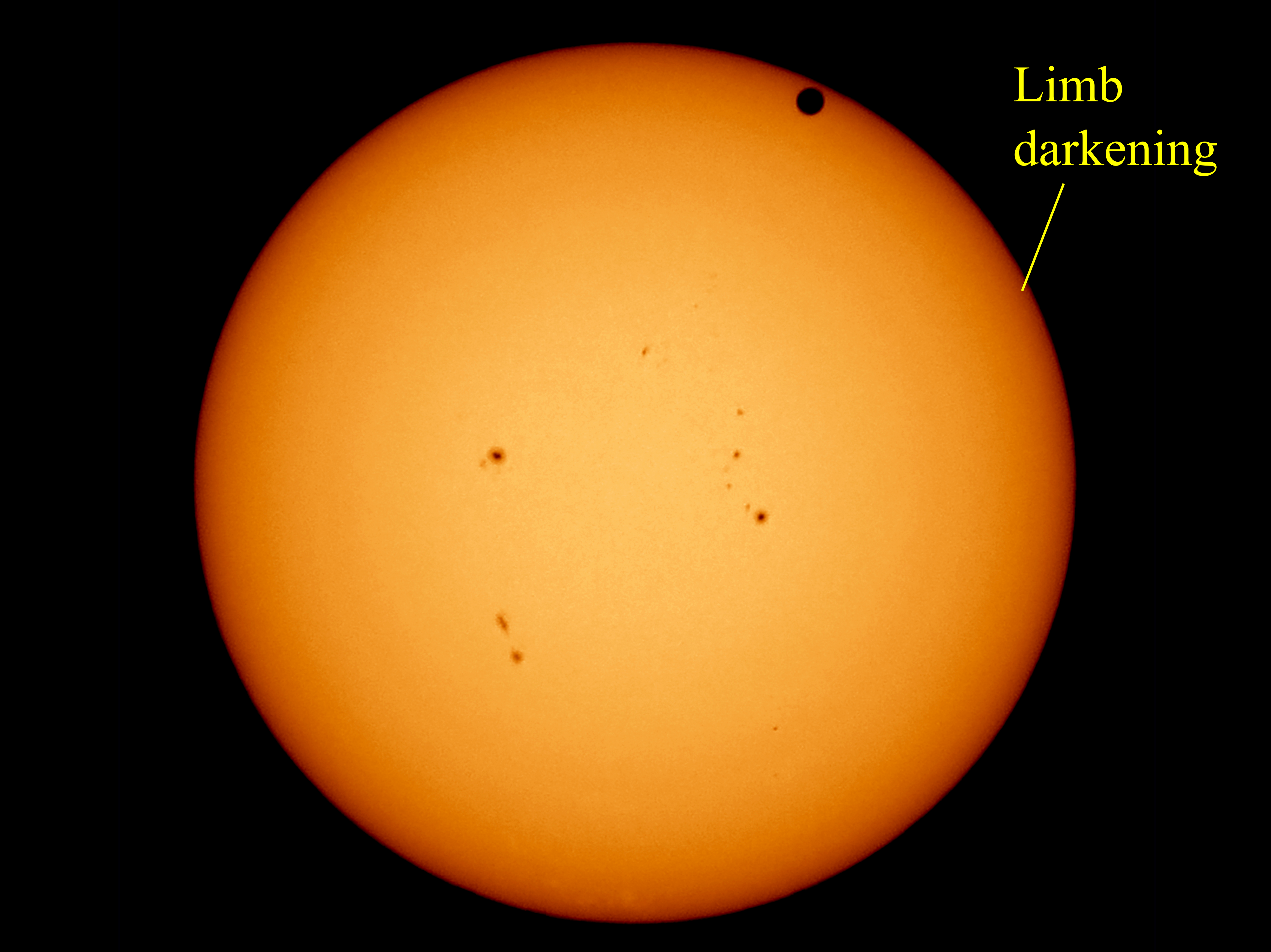
Radiation Transfer Equation

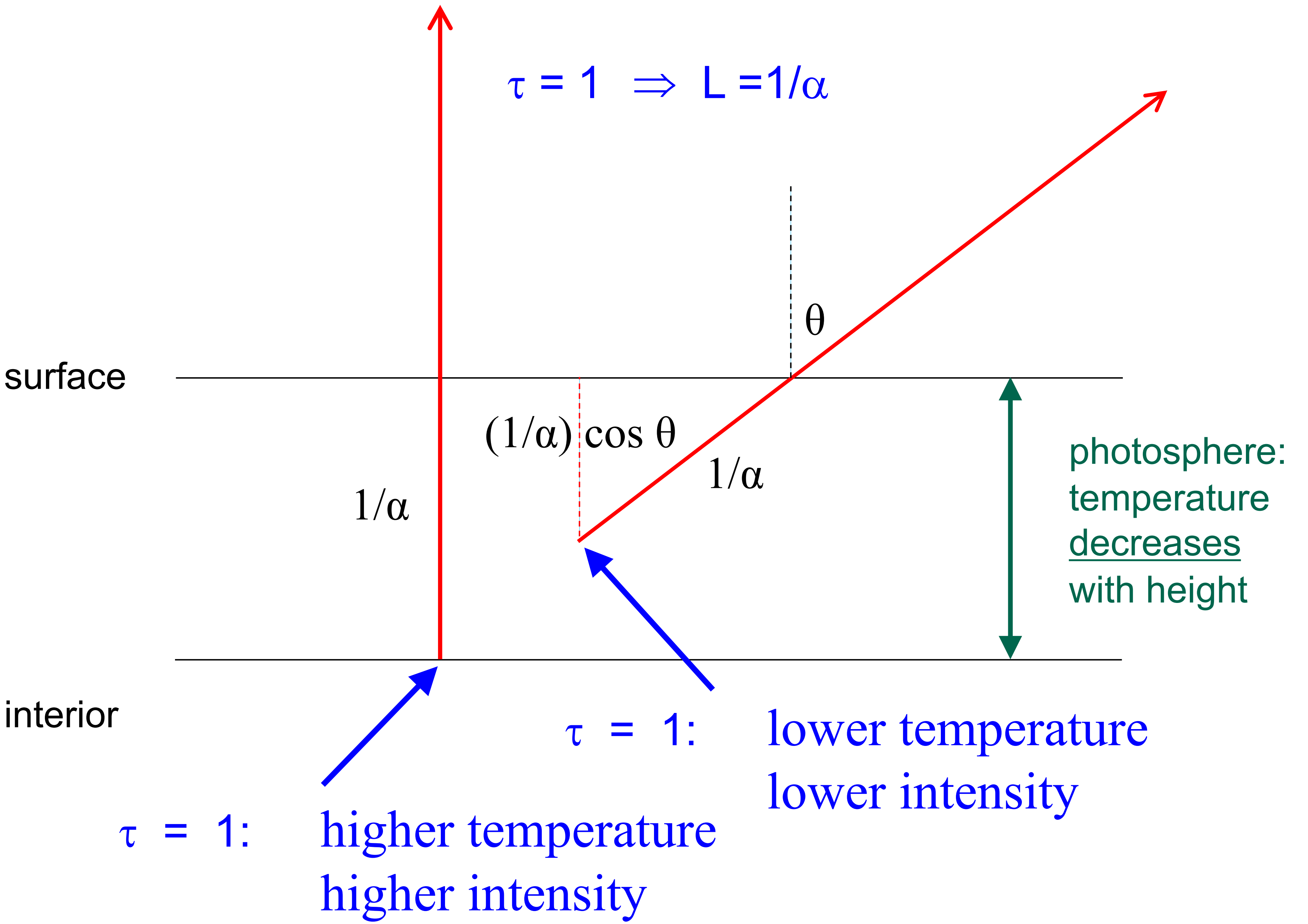
$$\frac{dI_\nu}{dx} = -\alpha_\nu I_\nu + j_\nu$$

Emission only:

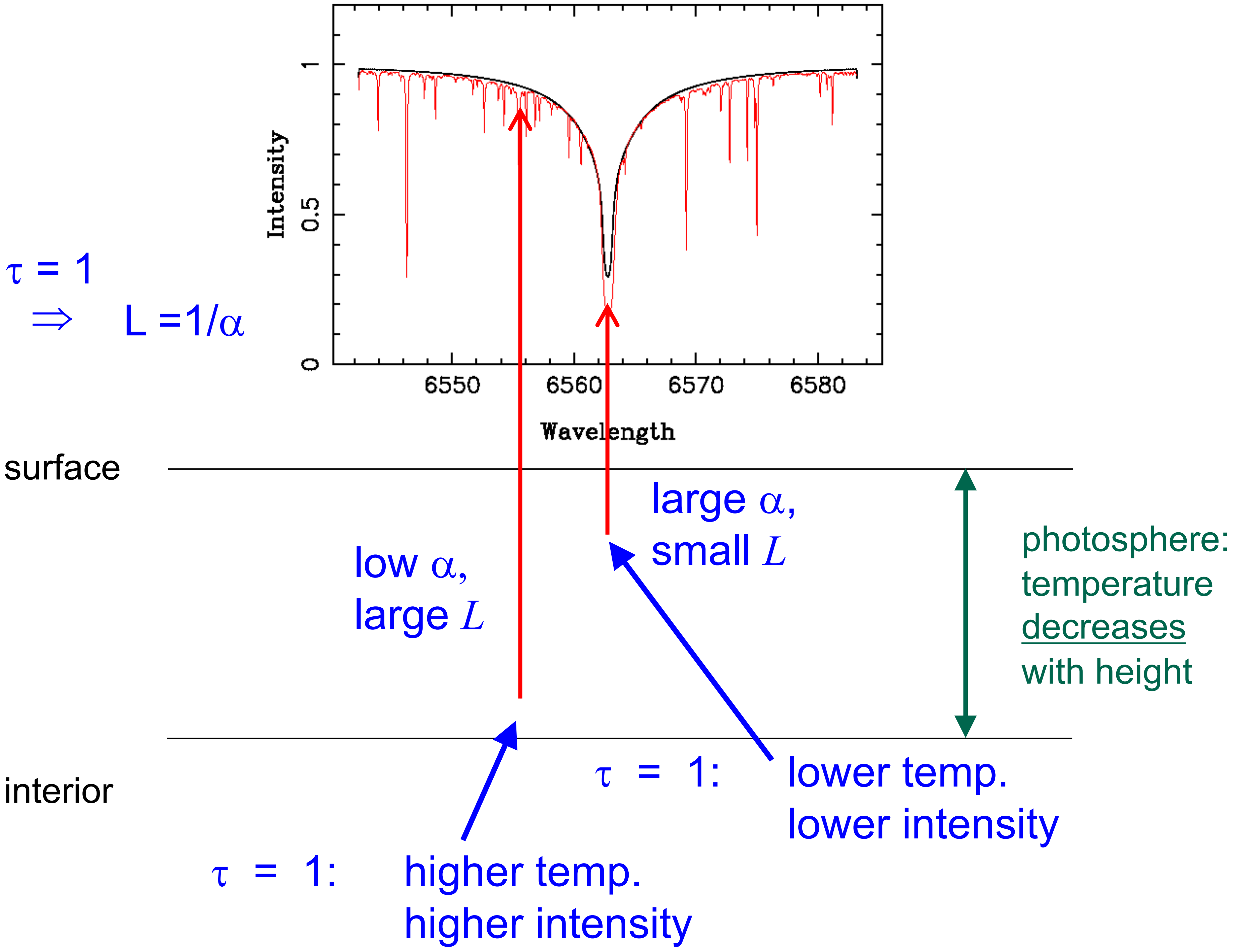
$$I_\nu \approx \int j_\nu \, dx$$

$$\sim j_\nu L$$


Limb
darkening



τ = 1 ⇒ L =1/α
θ
surface
(1/α) cos θ
1/α
1/α
photosphere:
temperature
decreases
with height
interior
τ = 1: lower temperature
lower intensity
τ = 1: higher temperature
higher intensity



τ = 1
⇒ L =1/α
Intensity
1
0.5
0
6550
6560
6570
6580
Wavelength
surface
interior
low α,
large L
large α,
small L
photosphere:
temperature
decreases
with height
τ = 1:
lower temp.
lower intensity
τ = 1:
higher temp.
higher intensity

Solar Properties

$$D_{\odot} = 1\,\mathrm{AU} = 1.5 \times 10^{8}\ \mathrm{km}$$

$$M_{\odot} = 2.0 \times 10^{30}\ \mathrm{kg}$$

$$R_{\odot} = 7.0 \times 10^{5}\ \mathrm{km}$$

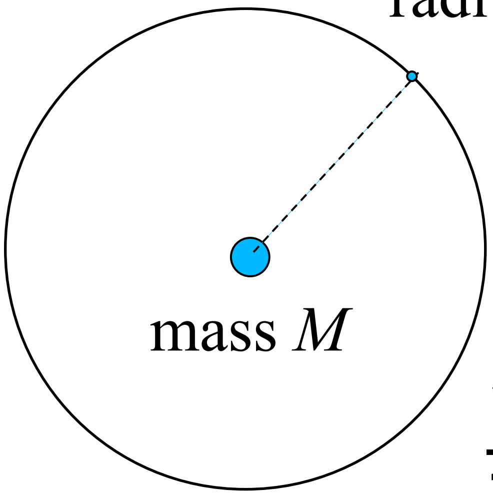
$$\rho_{\odot} = 1400\ \mathrm{kg/m^{3}}$$

$$L_{\odot} = 3.8 \times 10^{26}\ \mathrm{W}$$

$$T_{\odot} = 5800\ \mathrm{K}$$

$$t_{\odot} = 4.5 \times 10^{9}\ \mathrm{yr}$$

Free-Fall Time



$$\frac{d^2r}{dt^2} = a_r = -\frac{GM}{r^2}$$

$$E = \frac{1}{2}v_r^2 - \frac{GM}{r} = -\frac{GM}{R}$$

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{GM}{r} - \frac{GM}{R}$$

$$\frac{dr}{dt} = -\left[2GM\left(\frac{1}{r} - \frac{1}{R}\right)\right]^{\frac{1}{2}}$$

$$\frac{dt}{dr} = -\left[2GM\left(\frac{1}{r} - \frac{1}{R}\right)\right]^{-\frac{1}{2}}$$

$$\Longrightarrow \quad \mathrm{t_{ff}} = -\int_R^0 \left[2GM\left(\frac{1}{r} - \frac{1}{R}\right)\right]^{\frac{1}{2}} dr = \frac{\pi}{2\sqrt{2}}\left(\frac{GM}{R^3}\right)^{-\frac{1}{2}}$$

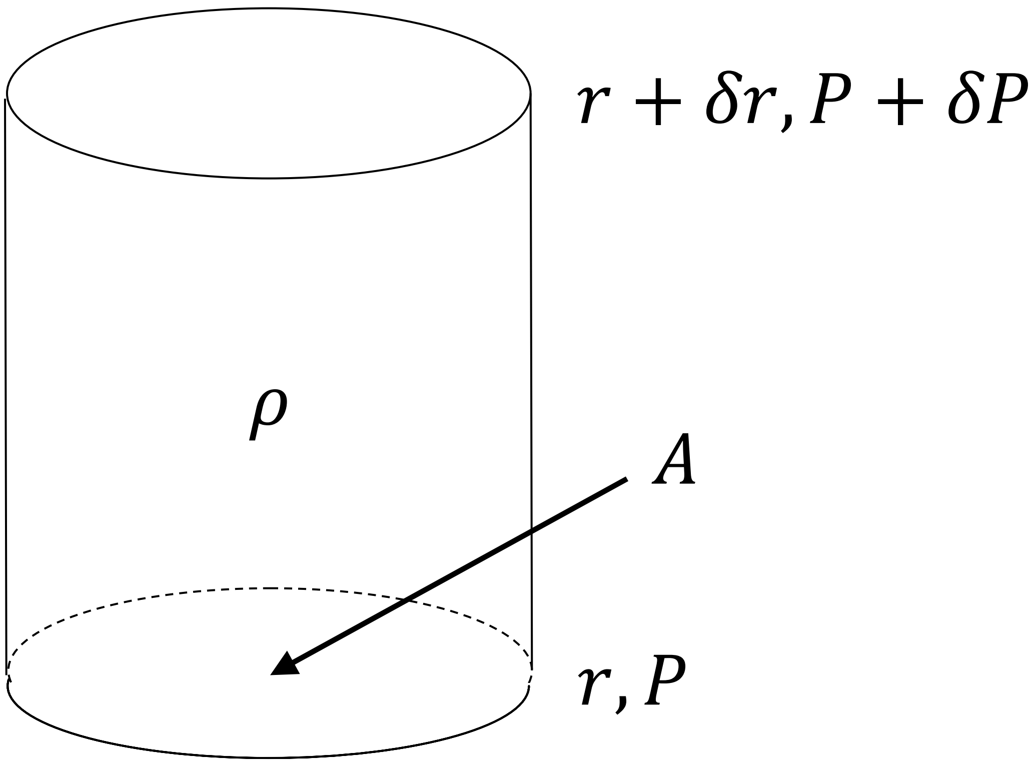
Free-Fall Time

$$t_{ff} = \frac{\pi}{2\sqrt{2}} \left(\frac{GM}{R^3}\right)^{-1/2} = 29.5 \text{ min for the Sun}$$

$$t_{orb} = 2\pi \left(\frac{GM}{R^3}\right)^{-1/2}$$

$$t_{ff} = \sqrt{\frac{3\pi}{32}} \, (G\overline{\rho}\,)^{-1/2}$$

Hydrostatic Equilibrium



$$\delta m = A \delta r \, \rho$$

$$F_p = \; -A \delta P \qquad \uparrow$$

$$F_g = \; \frac{GM(r)\delta m}{r^2} \qquad \downarrow$$

$$\Longrightarrow \frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

$$
\begin{aligned}
\frac{dP}{dr} &= -\frac{GM\rho}{r^2} \\
\frac{dM}{dr} &= 4\pi r^2 \rho \\
\frac{dT}{dr} &= -\frac{3\kappa\rho L}{16\pi a c r^2 T^3} \\
\frac{dL}{dr} &= 4\pi r^2 \rho (\epsilon - \epsilon_\nu)
\end{aligned}
$$