More on Radiation

Energy density

$$u_\nu = \text{energy per unit volume per unit frequency} \quad (J\,\mathrm{m}^{-3}\,\mathrm{Hz}^{-1})$$
$$u_\lambda = \text{energy per unit volume per unit wavelength} \quad (J\,\mathrm{m}^{-3}\,\mathrm{m}^{-1})$$

Intensity

$I_\nu =$ energy per unit time per unit area per unit frequency per unit solid angle $(\mathrm{W\,m^{-2}\,Hz^{-1}\,ster^{-1}})$

isotropic field: $$I_\nu = \frac{c}{4\pi} u_\nu$$

Blackbody spectrum

$$I_\nu = B_\nu = \frac{2h}{c^2} \frac{\nu^3}{e^{h\nu/kT} - 1}$$

More on Radiation

Flux

f_ν = energy per unit time per unit area per unit frequency crossing some surface (e.g. a detector) $(\mathrm{W\,m^{-2}\,Hz^{-1}})$

isotropic field: $f_\nu = \frac{c}{4} u_\nu$

blackbody: $f_\nu = \pi B_\nu$

Integrated quantities:
$$u = \int_0^\infty u_\nu \, d\nu = aT^4 \quad (\mathrm{J\,m^{-3}})$$
$$f = \int_0^\infty f_\nu \, d\nu = \sigma T^4 \quad (\mathrm{W\,m^{-2}})$$

$$a = \frac{8\pi^5 k^4}{15 c^3 h^3} = 7.6 \times 10^{-8}\,\mathrm{J\,m^{-2}\,K^{-4}}$$
$$\sigma = \frac{c}{4} a = \frac{2\pi^5 k^4}{15 c^2 h^3} = 5.7 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$$

Radius-Temperature-Luminosity Relation

At the surface of a star of temperature T, the outward-directed total radiative flux (energy/time/area) is

$$f = \sigma T^4$$

total surface area of the star is

$$A = 4\pi R^2$$

total luminosity of the star (blackbody spectrum) is

$$L = fA = 4\pi R^2 \sigma T^4$$

$\Longrightarrow$ important means of estimating a star's <u>radius</u> if the luminosity and temperature are known:

$$R \approx \sqrt{\frac{L}{4\pi\sigma T^4}}$$

Radius-Temperature-Luminosity Relation

Some examples (calibrated to the Sun, assuming blackbodies)

$$L(L_\odot) = R(R_\odot)^2 \; T(T_\odot)^4$$

Sirius A: $L = 25\, L_\odot, \quad \mathrm{T} = 9900\ \mathrm{K} = 1.7\, T_\odot$

$$\Longrightarrow \quad R = \sqrt{25/1.7^4}\, R_\odot = 3.0\, R_\odot$$

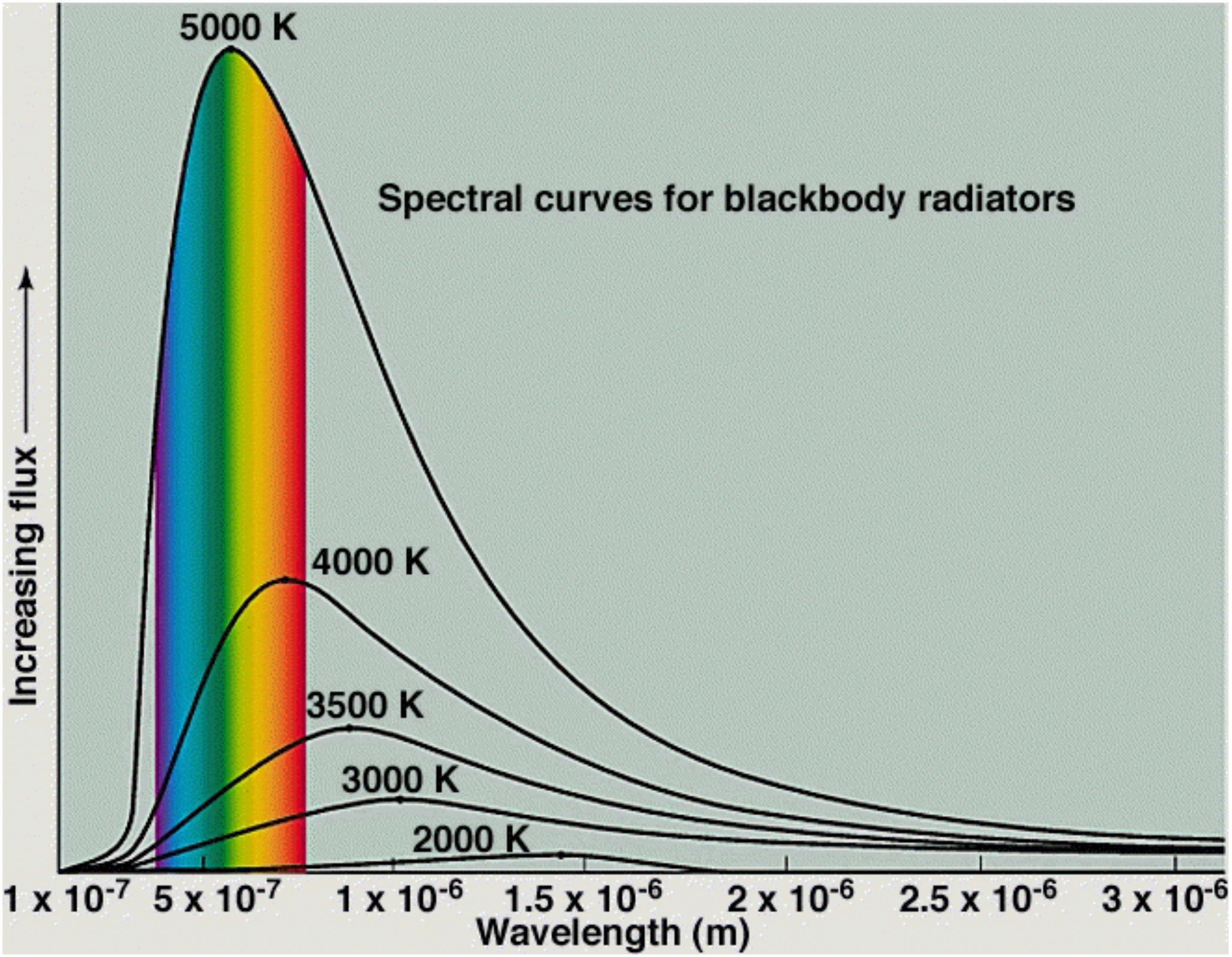
Betelgeuse: $L = 1.4\times10^5\, L_\odot, \quad \mathrm{T} = 3500\ \mathrm{K} = 0.64\, T_\odot$

red giant $\Longrightarrow \quad R = \sqrt{1.4\times10^5/0.64^4}\, R_\odot = 910\, R_\odot = 4.2\ \mathrm{AU}$

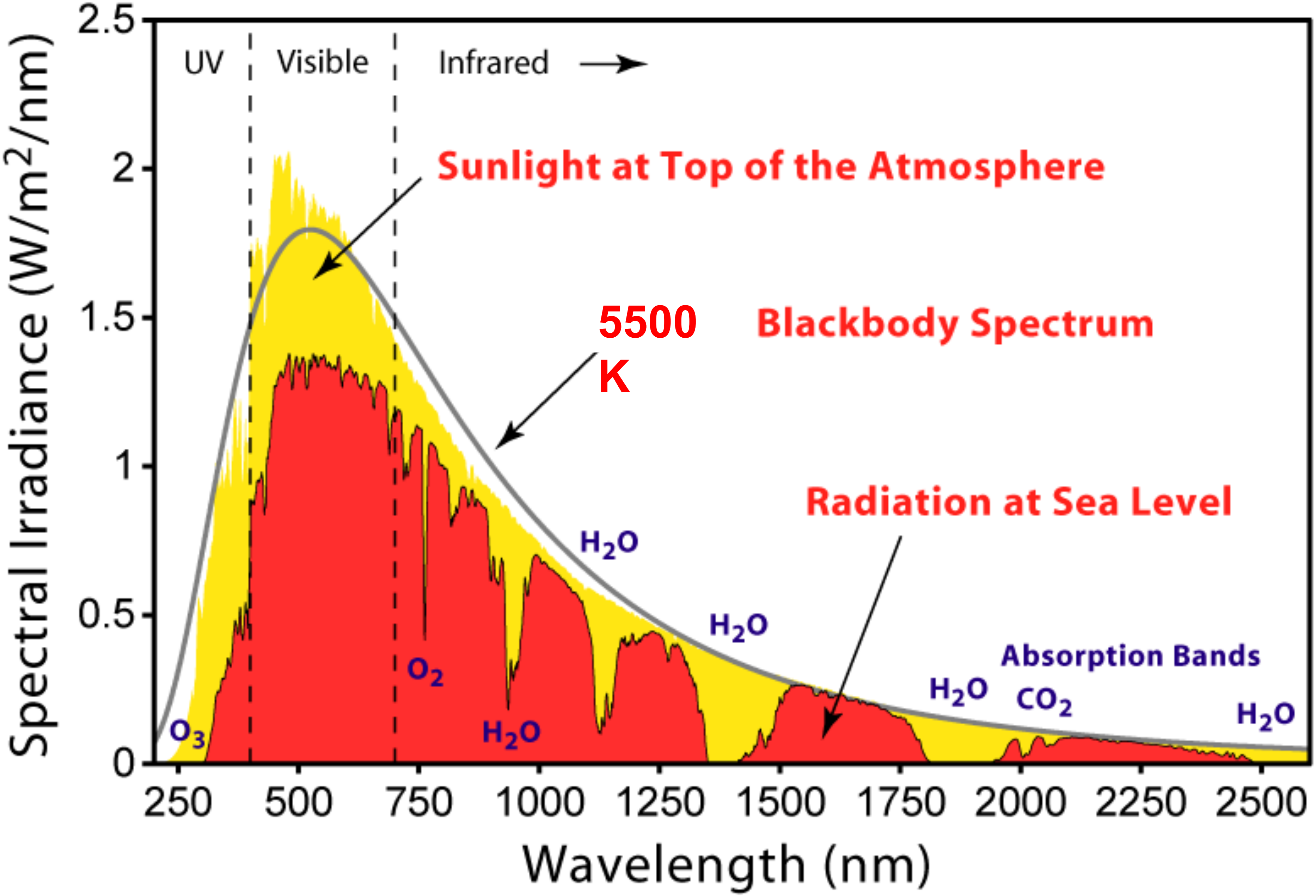
Sirius B: $L = 0.03\, L_\odot, \quad \mathrm{T} = 25{,}000\ \mathrm{K} = 4.3\, T_\odot$

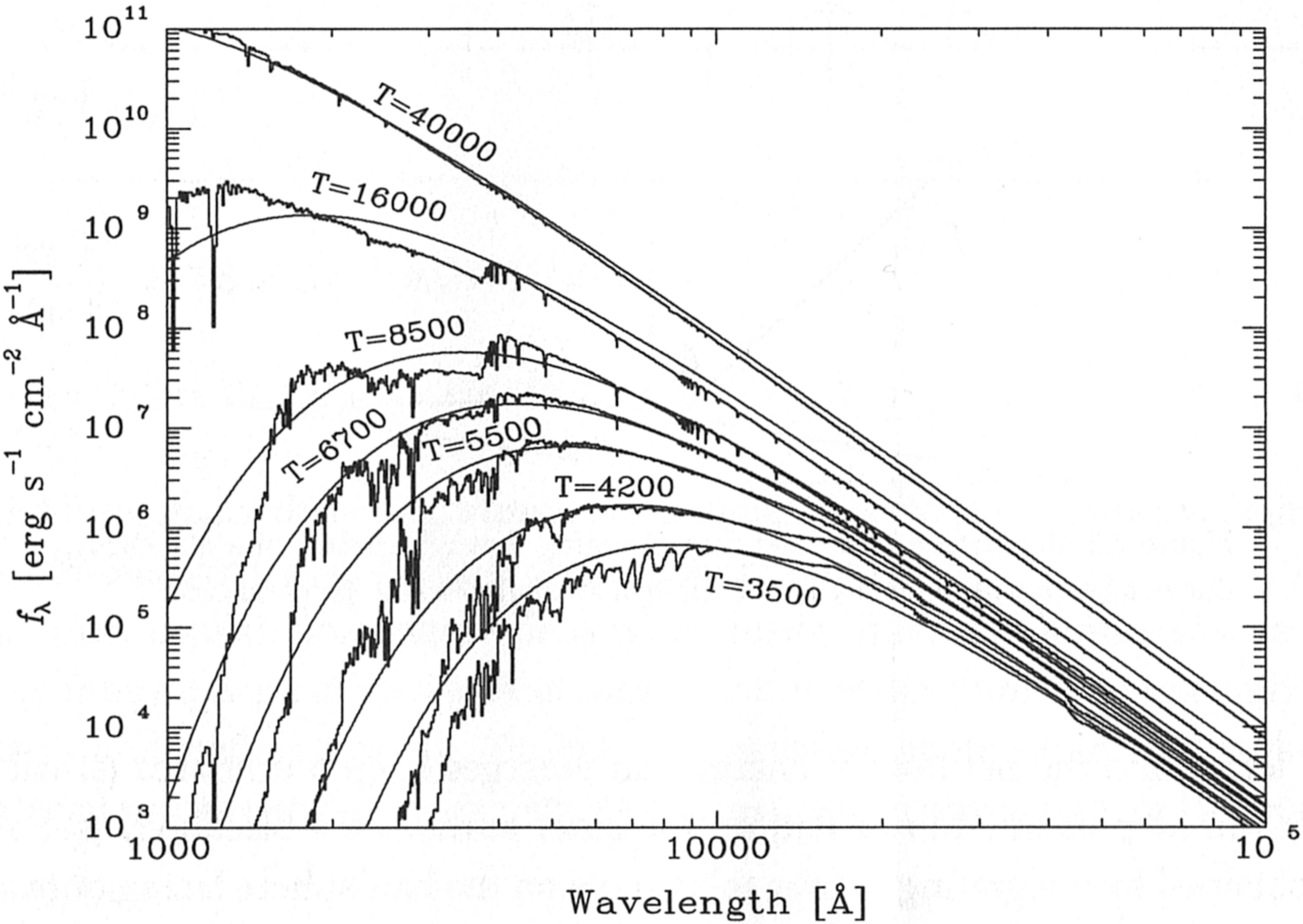
white dwarf $\Longrightarrow \quad R = \sqrt{0.03/4.3^4}\, R_\odot = 0.0094\, R_\odot \approx R_\oplus$

Blackbody Radiation



Solar Radiation Spectrum

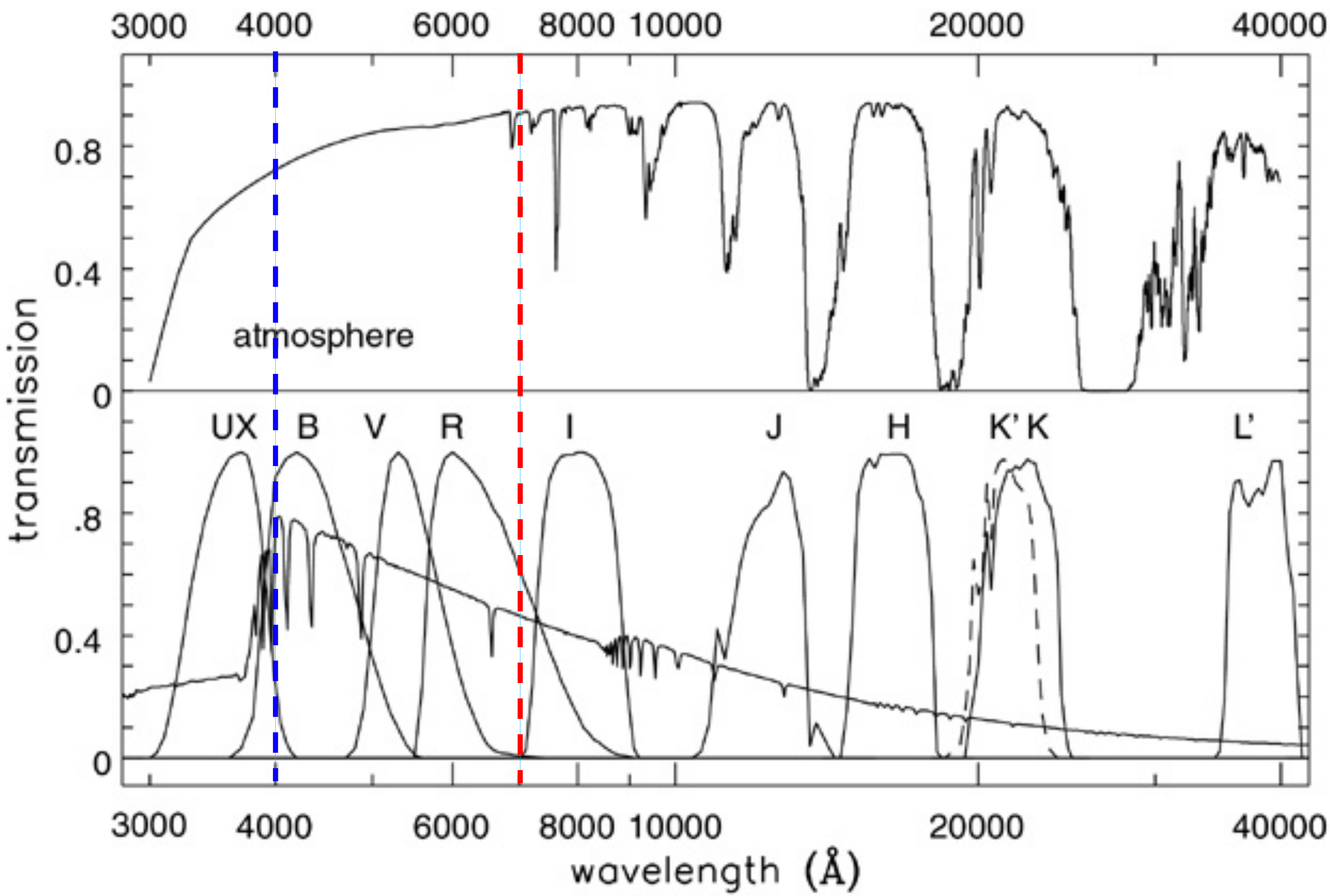


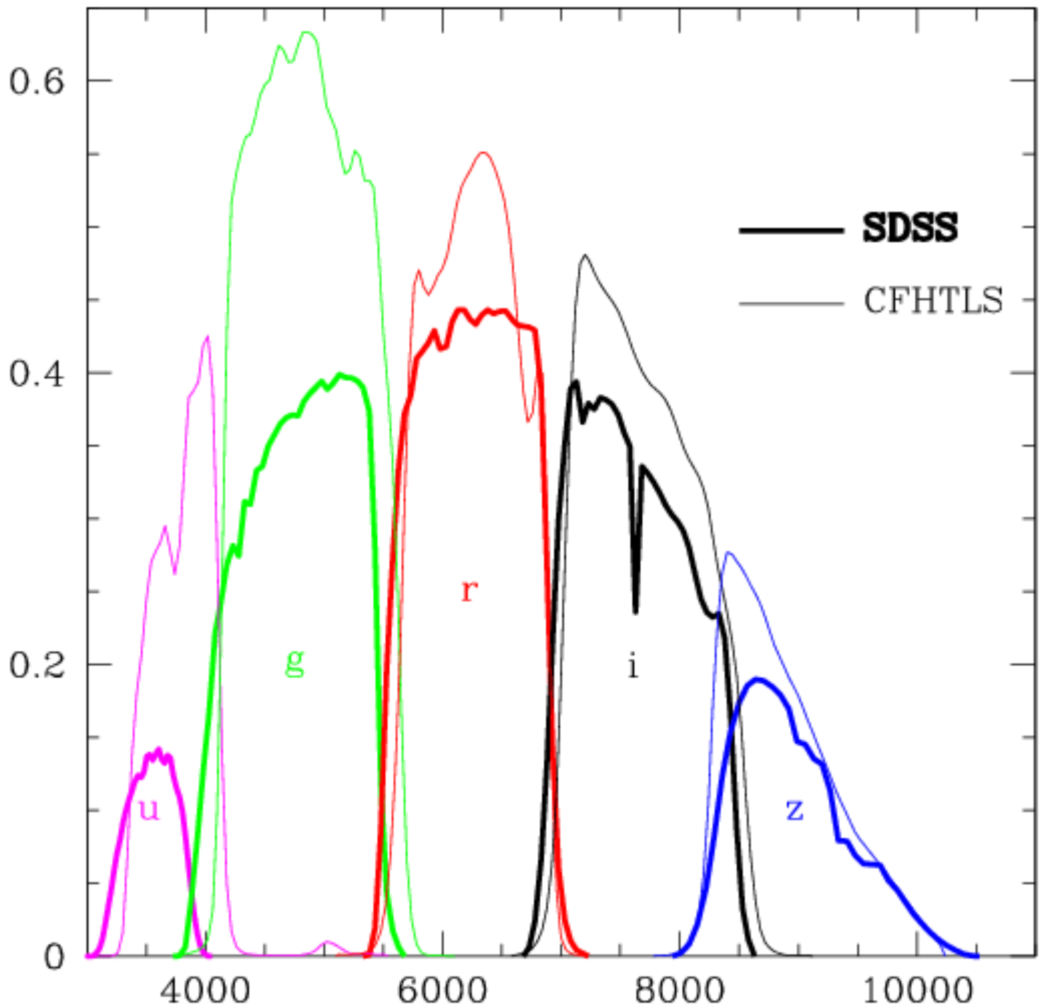


T=40000
T=16000
T=8500
T=6700
T=5500
T=4200
T=3500
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
1000
10000
10^{5}
f_λ [erg s^{-1} cm^{-2} Å^{-1}]
Wavelength [Å]

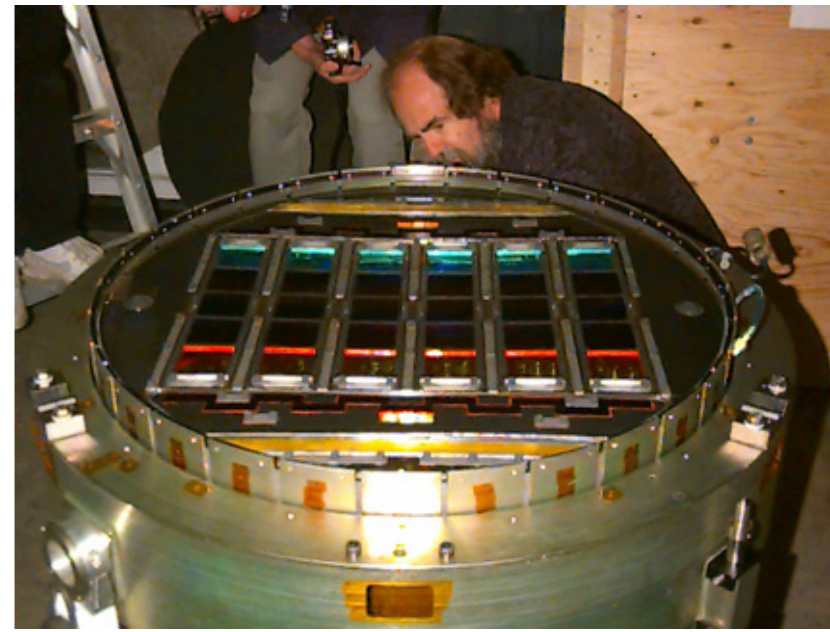
Filters and Colors

- Detailed spectra are expensive
- “bolometric” flux is flux integrated over all wavelengths
  - no detector actually measures this
  - only sensitive to a relatively narrow part of the spectrum
- astronomical instruments generally record light received through a set of standard filters

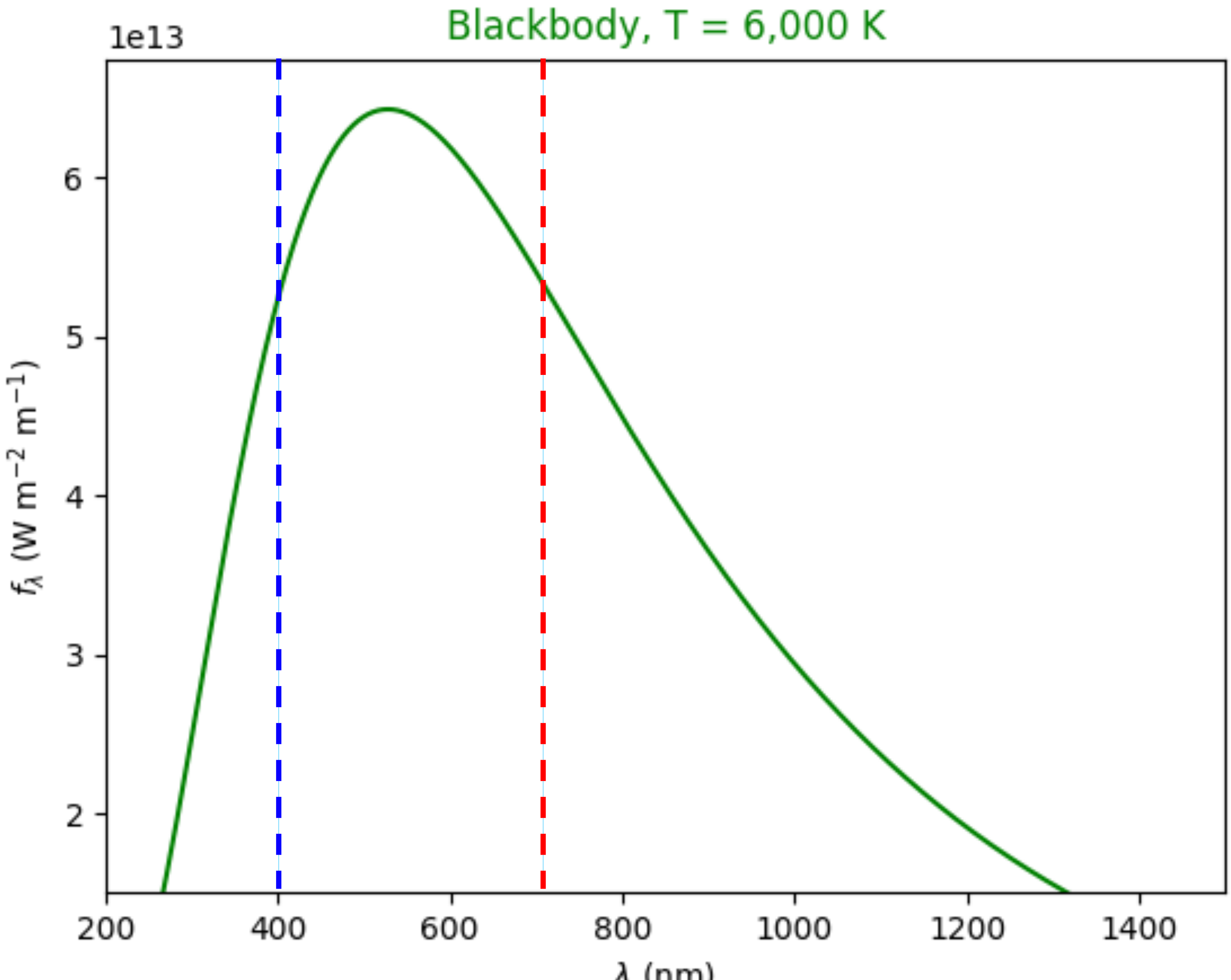

3000
4000
6000
8000
10000
20000
40000
0.8
0.4
0
.8
atmosphere
transmission
UX
B
V
R
I
J
H
K' K
L'
wavelength (Å)



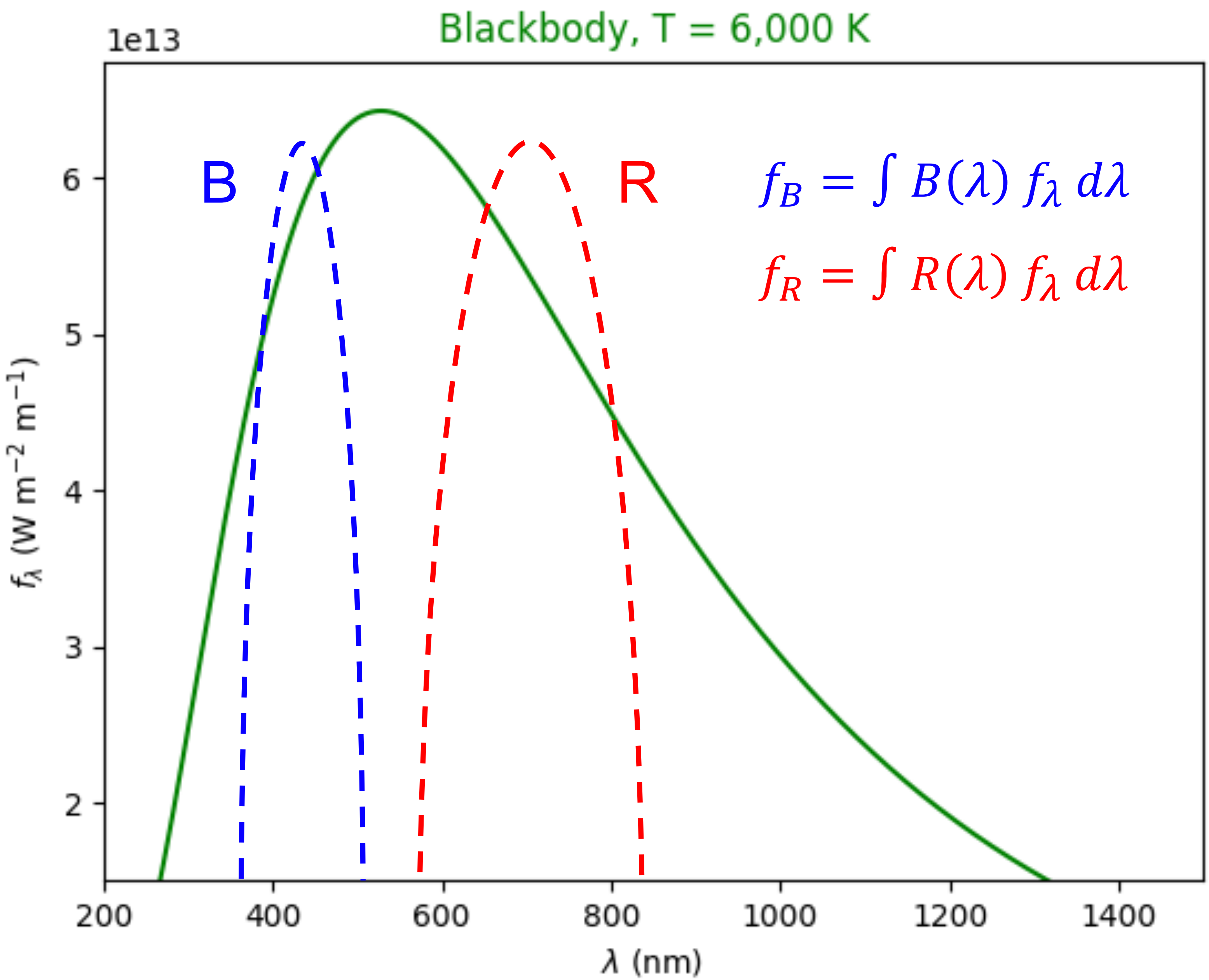
0.6
0.4
0.2
0
4000
6000
8000
10000
SDSS
CFHTLS
u
g
r
i
z



Blackbody Spectrum



Blackbody Spectrum



Filters and Colors

- Detailed spectra are expensive
- "bolometric" flux is flux integrated over all wavelengths
  - no detector actually measures this
  - only sensitive to a relatively narrow part of the spectrum
- astronomical instruments generally record light received through a set of standard filters:

$$f_C = \int C(\lambda)\, f_\lambda\, d\lambda$$

- astronomical "color" is a ratio of fluxes in two different filters,

  e.g. $\frac{f_V}{f_B}, \frac{f_R}{f_I}, \ldots$

  NOTE: <u>independent</u> of distance

- color ~ temperature for blackbody; also approximately true for real spectra

The Magnitude Scale

- Astronomers generally don't work directly with fluxes…
- Greek astronomers ranked visible stars by apparent brightness (~ flux), where brightest = 1, faintest = 6, in roughly equal perceived increments
- modern photometry revealed
    1. the eye's response is logarithmic: an increase of 1 magnitude corresponds to a decrease in flux by a constant factor
    2. first-magnitude stars are about 100 times brighter than sixth-magnitude stars
- modern magnitude scale defines apparent magnitude m by

$$m = -2.5 \log_{10} f + \text{constant}$$

e.g. $$m_V = -2.5 \log_{10} f_V + C_V$$

The Magnitude Scale

- a few stars are brighter than magnitude 1
  - Vega has $m_V \approx 0$ (really 0.03, now)
  - brightest star in the sky is Sirius, with $m_V = -1.47$
  - Sun has $m_V = -26.74$

- vast majority of stars (and galaxies) are fainter than magnitude 6

- ratios of fluxes are differences in magnitudes:

$$m_2 - m_1 = -2.5 \log_{10} f_2 + 2.5 \log_{10} f_1$$

$$= -2.5 \log_{10} \frac{f_2}{f_1}$$

colors: $$m_V - m_B = -2.5 \log_{10} \frac{f_V}{f_B}$$

V − B

Apparent and Absolute Magnitudes

- magnitudes m as just defined (~fluxes) are <u>apparent magnitudes</u>
- flux f depends on both the luminosity L and the distance D to a star:

$$f = \frac{L}{4\pi D^2}$$

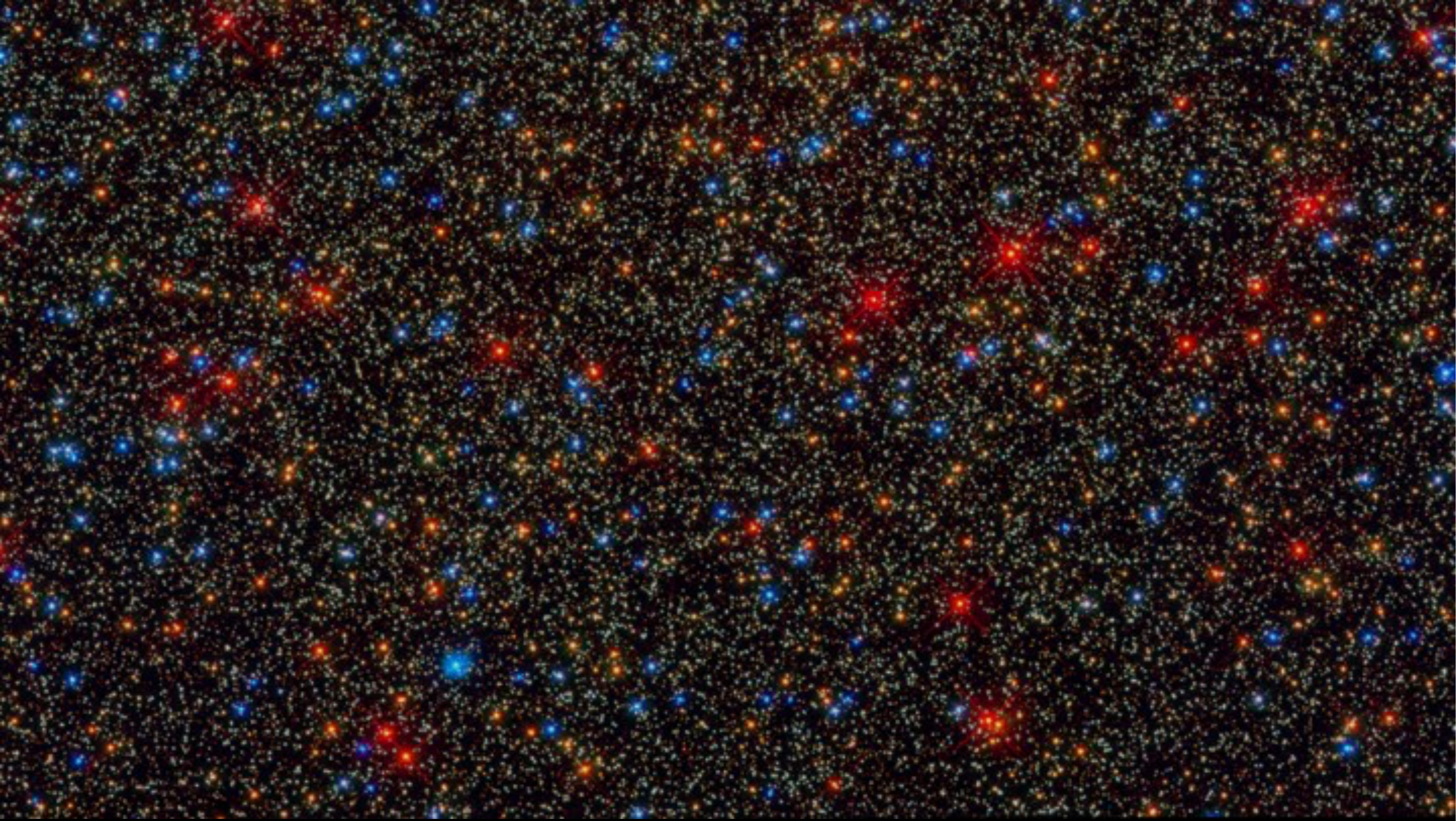
- define <u>absolute magnitude</u> M as the apparent magnitude at a standard distance of $D = 10$ pc
  - equivalent to luminosity since D is fixed (and solar $M_V = 4.8$)
- Then

$$m = -2.5 \log_{10} \frac{L}{4\pi D^2} \qquad M = -2.5 \log_{10} \frac{L}{4\pi (10\, pc)^2}$$

$$\Rightarrow \quad m - M = 5 \log_{10} D(\text{pc}) - 5$$

inverse-square law in magnitudes!

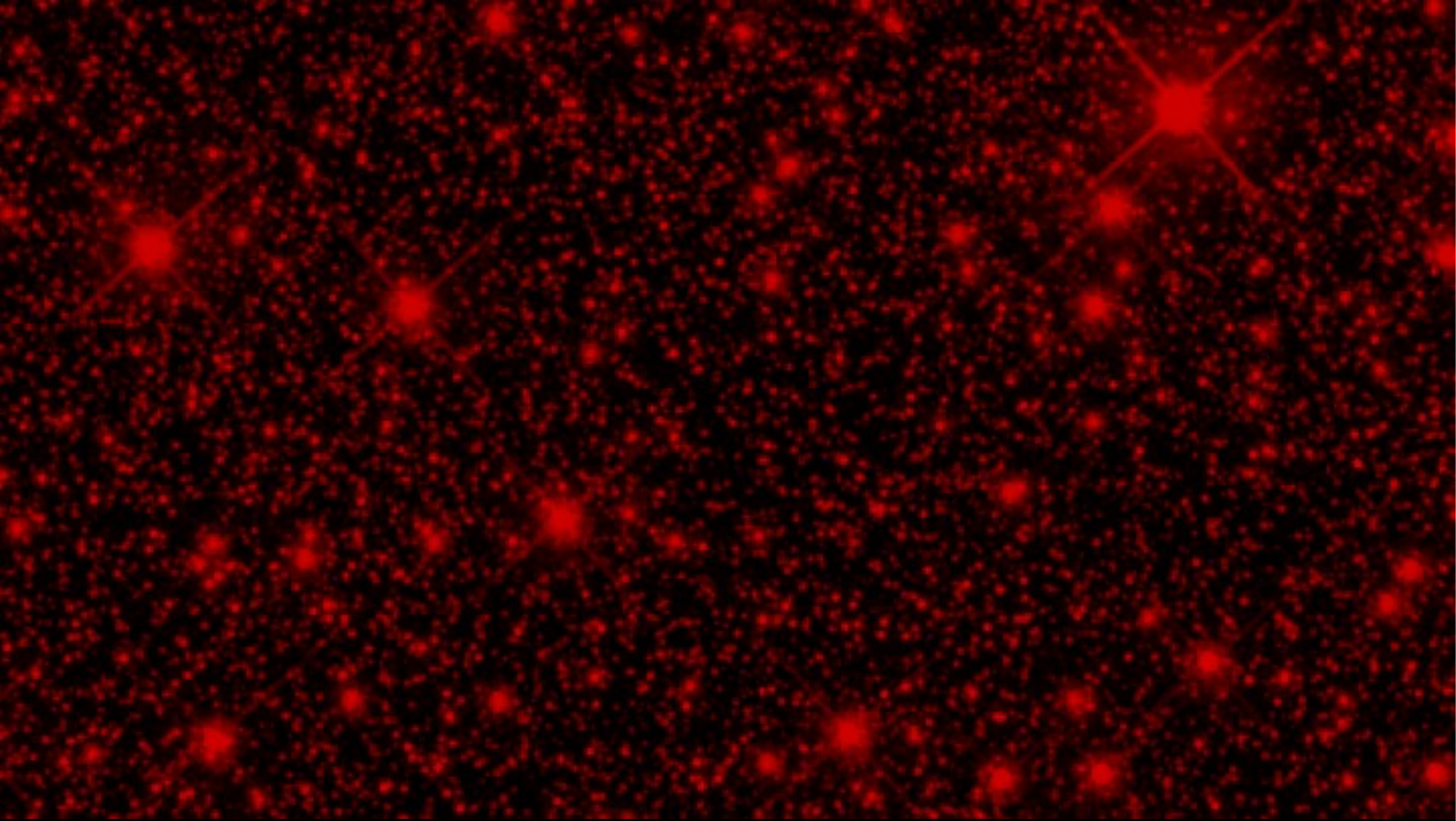
distance modulus



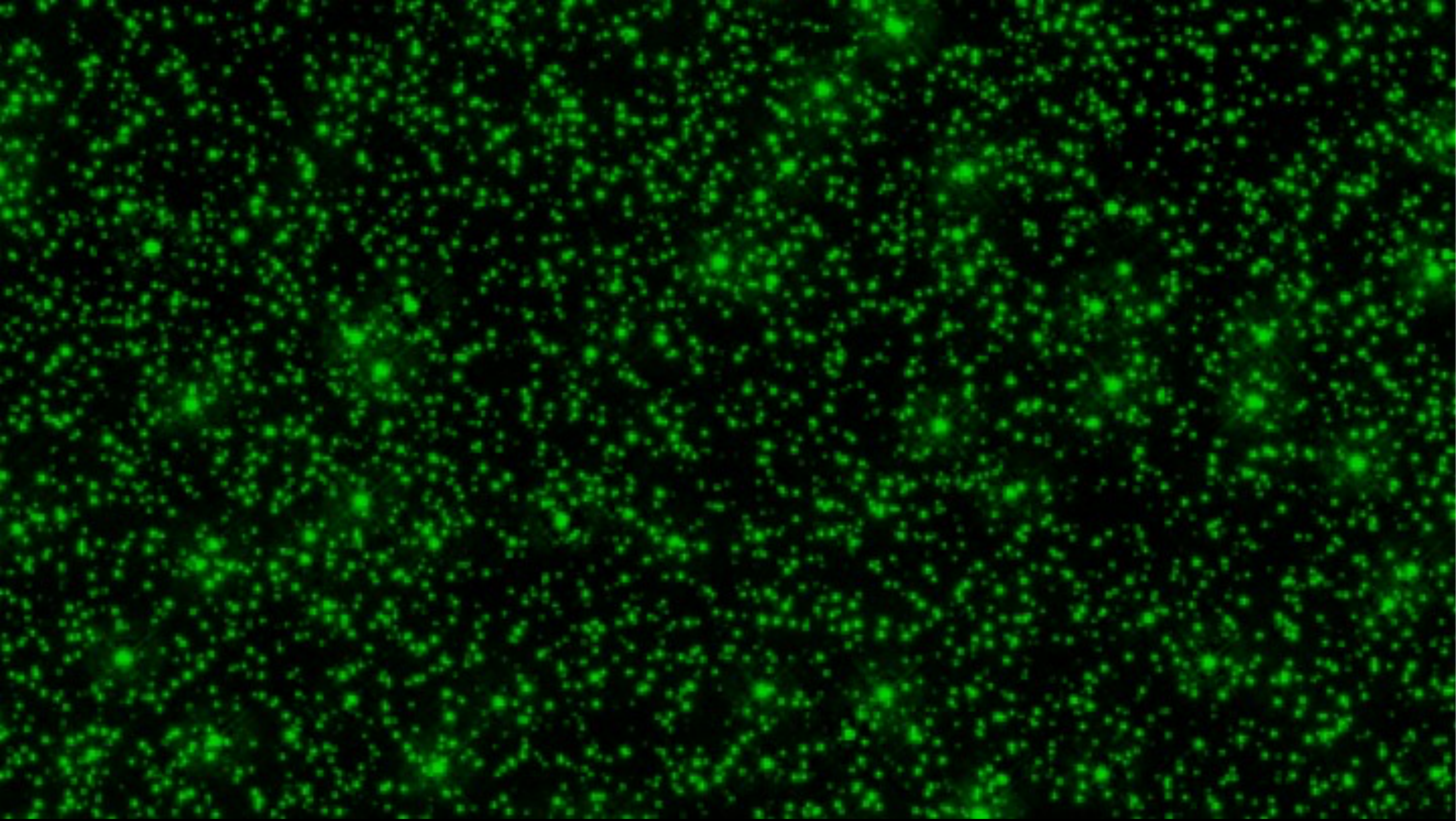
http://hubblesite.org/gallery/wallpaper/pr2009025q/

Image of the inner core of Omega Centauri, taken by WFC3/UVIS on board the Hubble Space Telescope (HST)

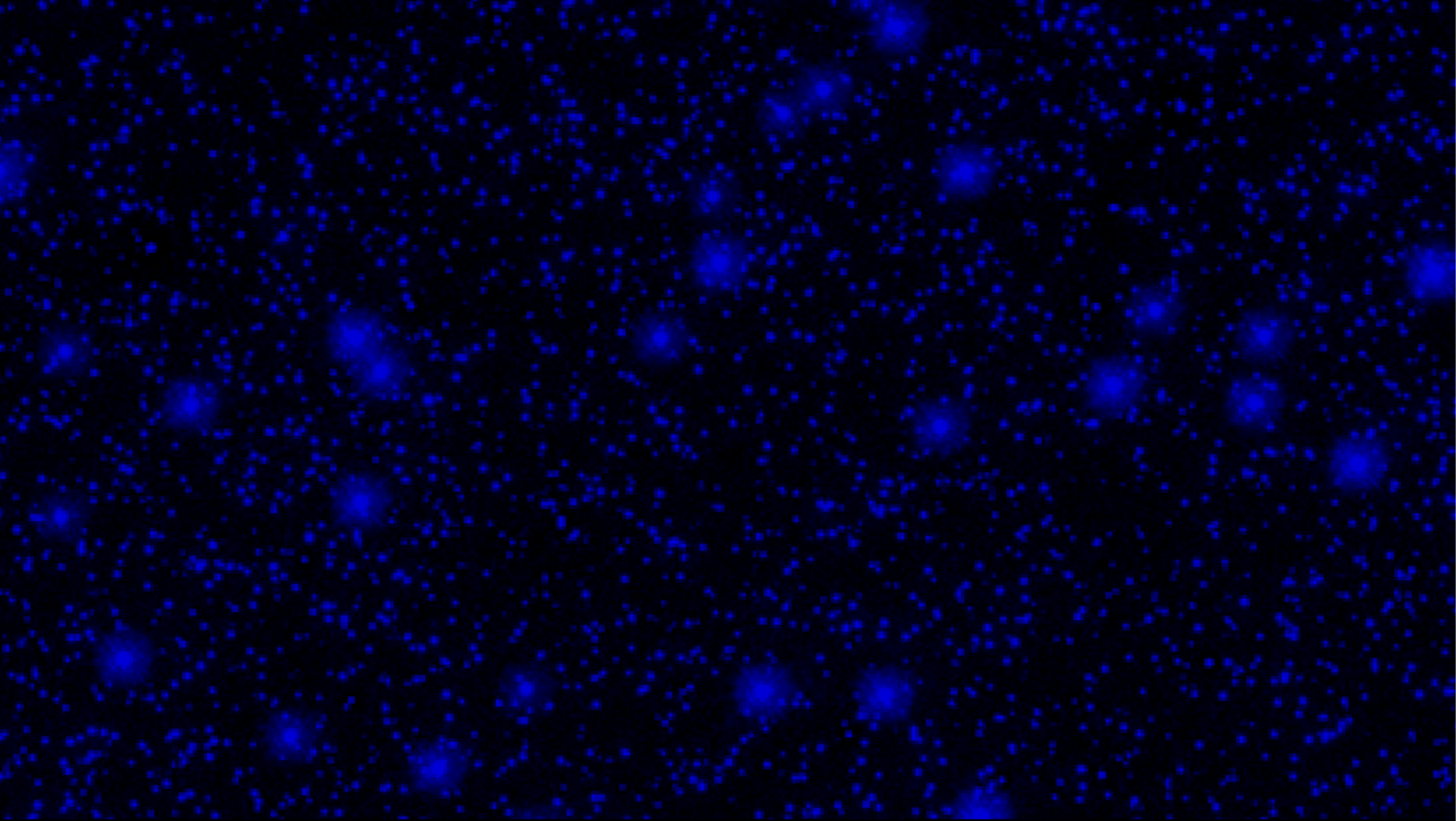
A close-up of the central region. This false-color image was made by combining separate red, green, and blue images



The **red** image is from filter F814W,
which sees only very red light.

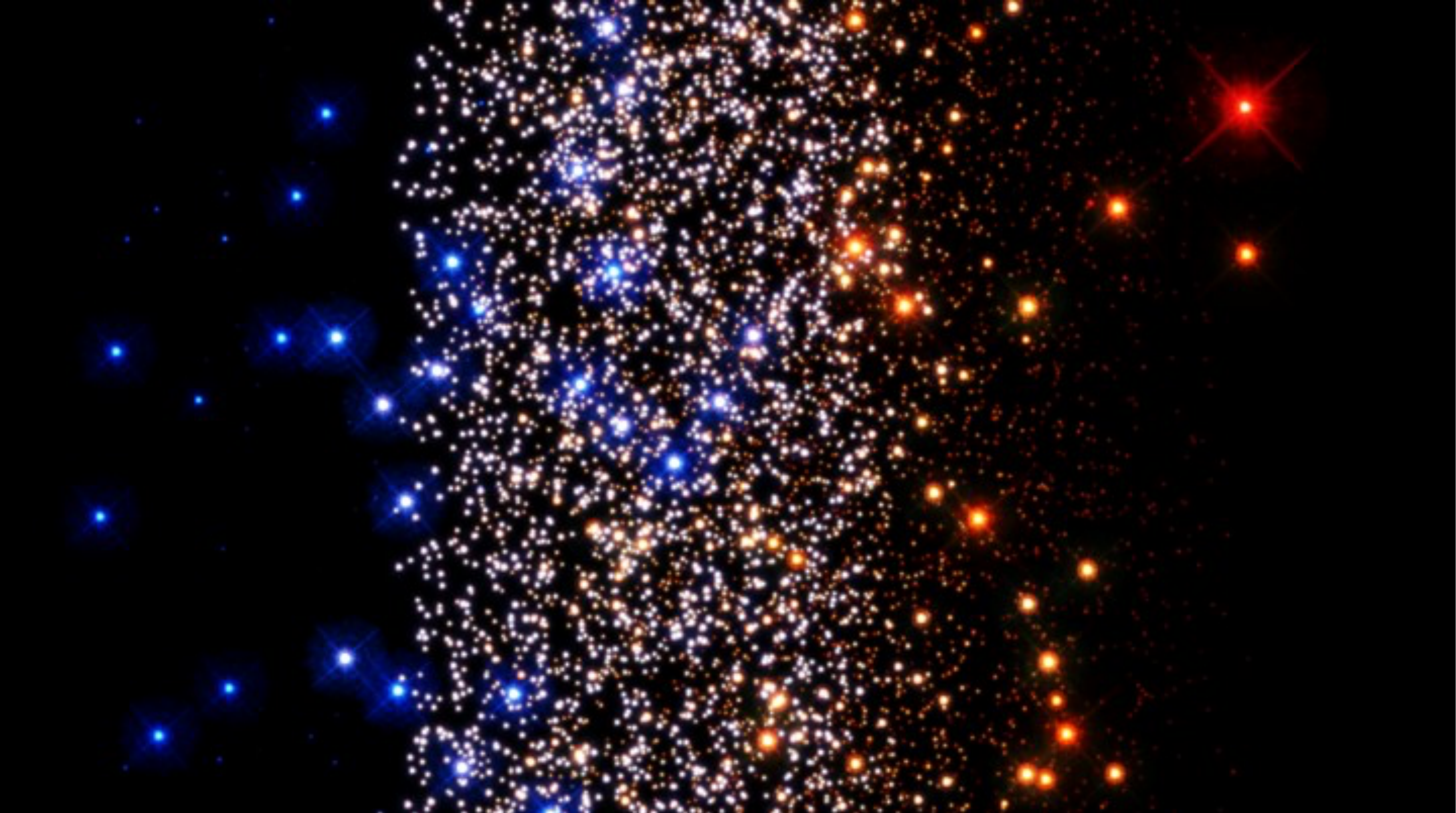


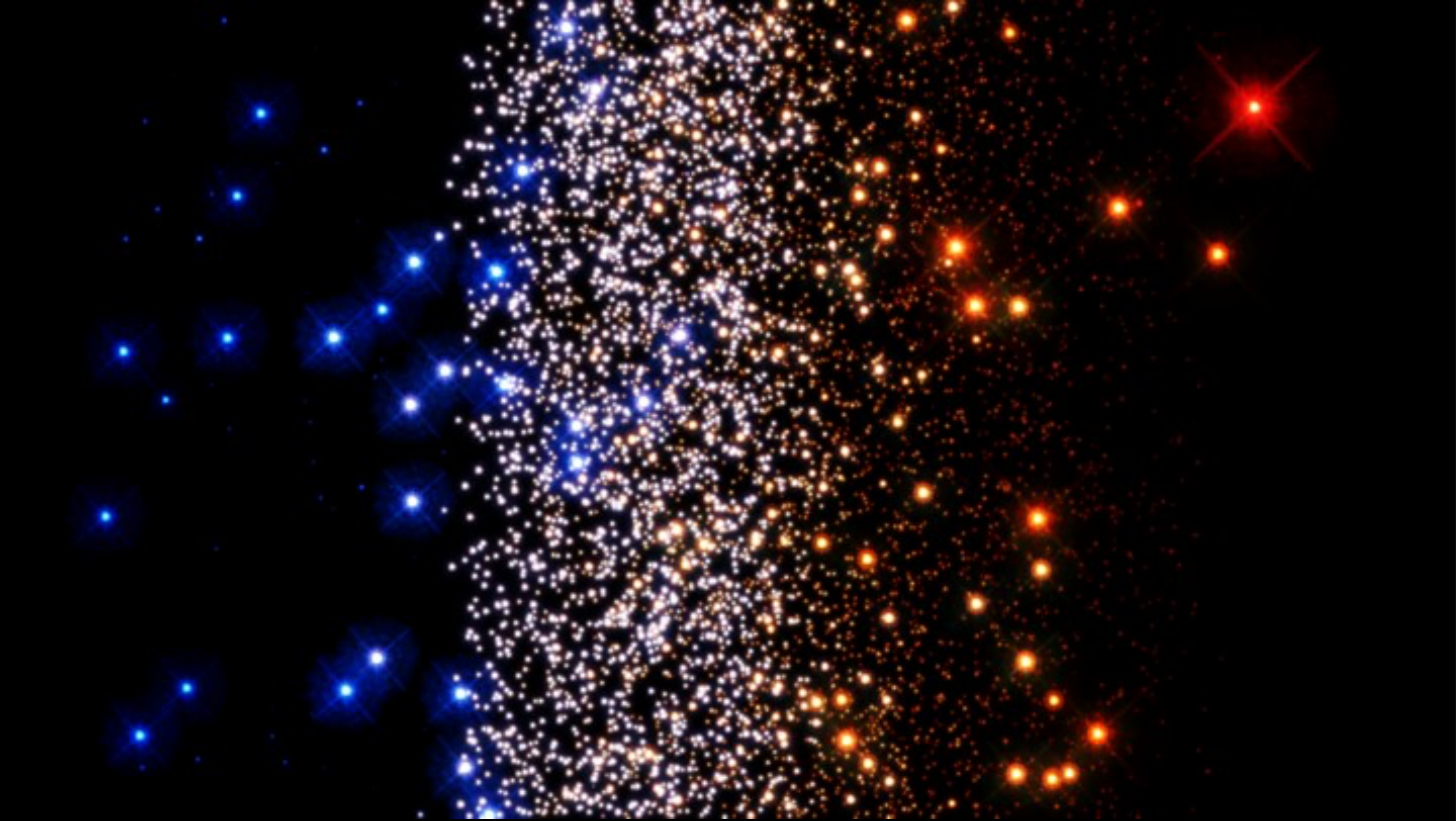
The **green** image is from filter F336W,
which sees only blue light.

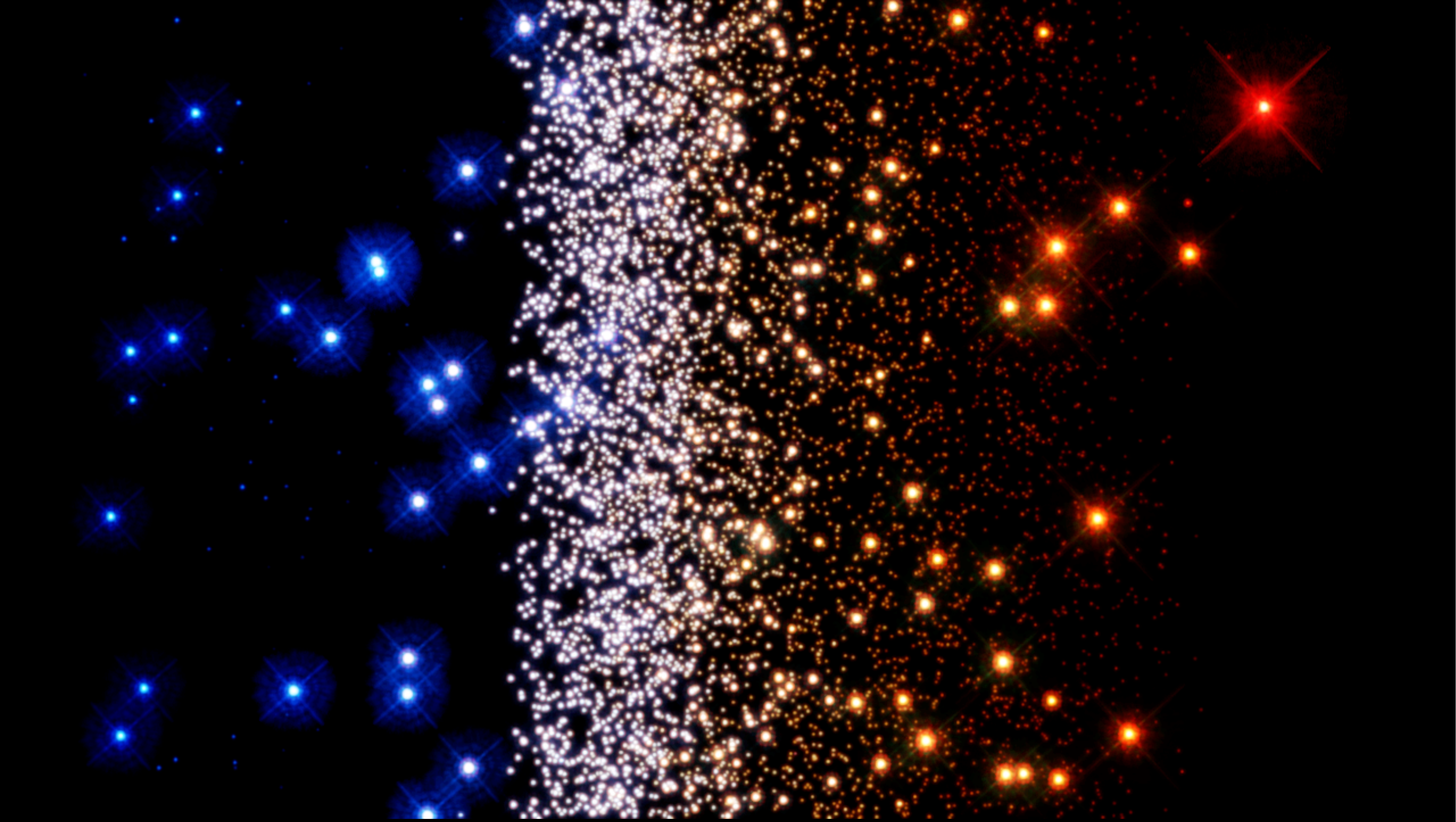


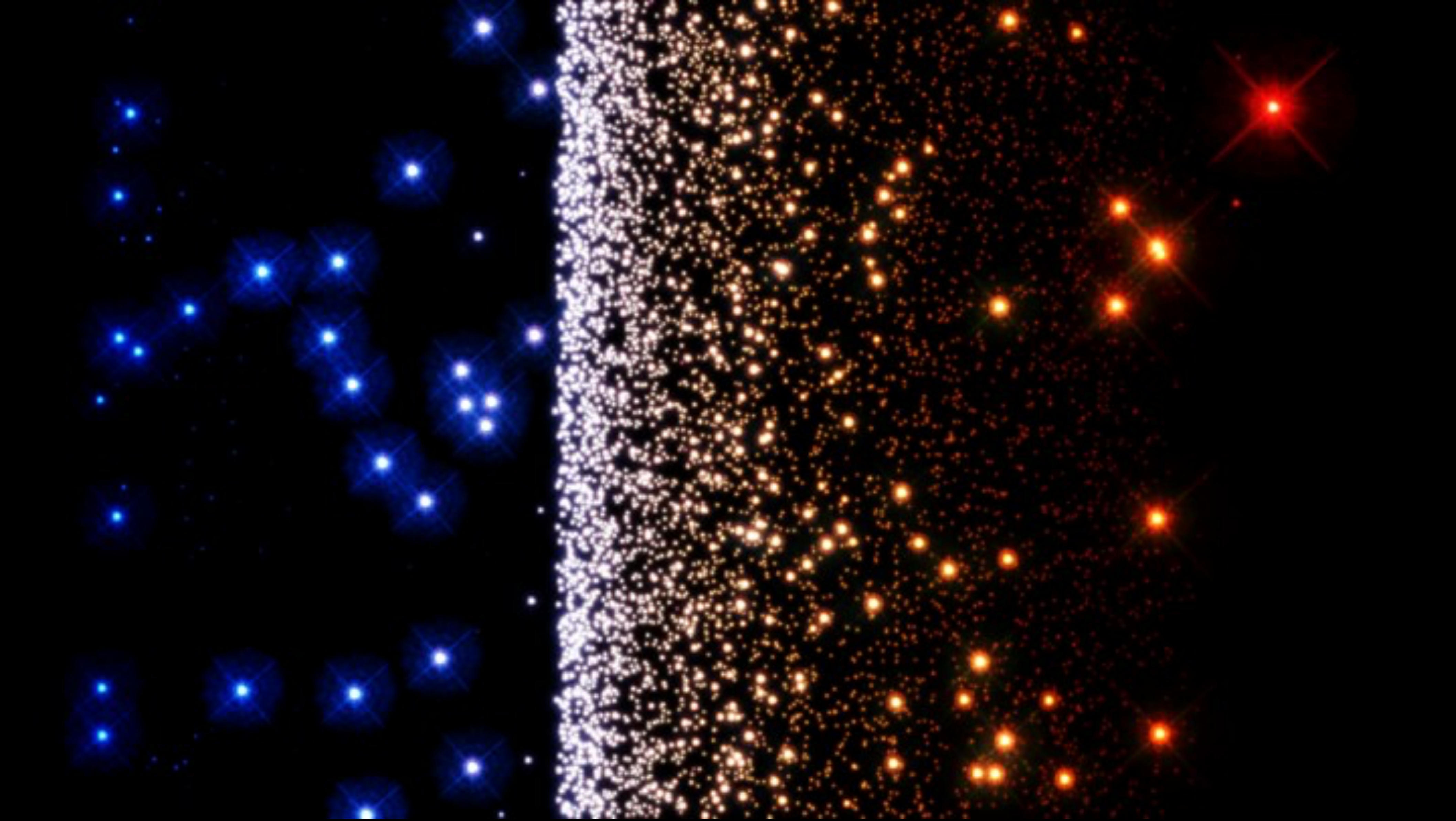
The **blue** image is from filter F225W, which sees only ultraviolet light.

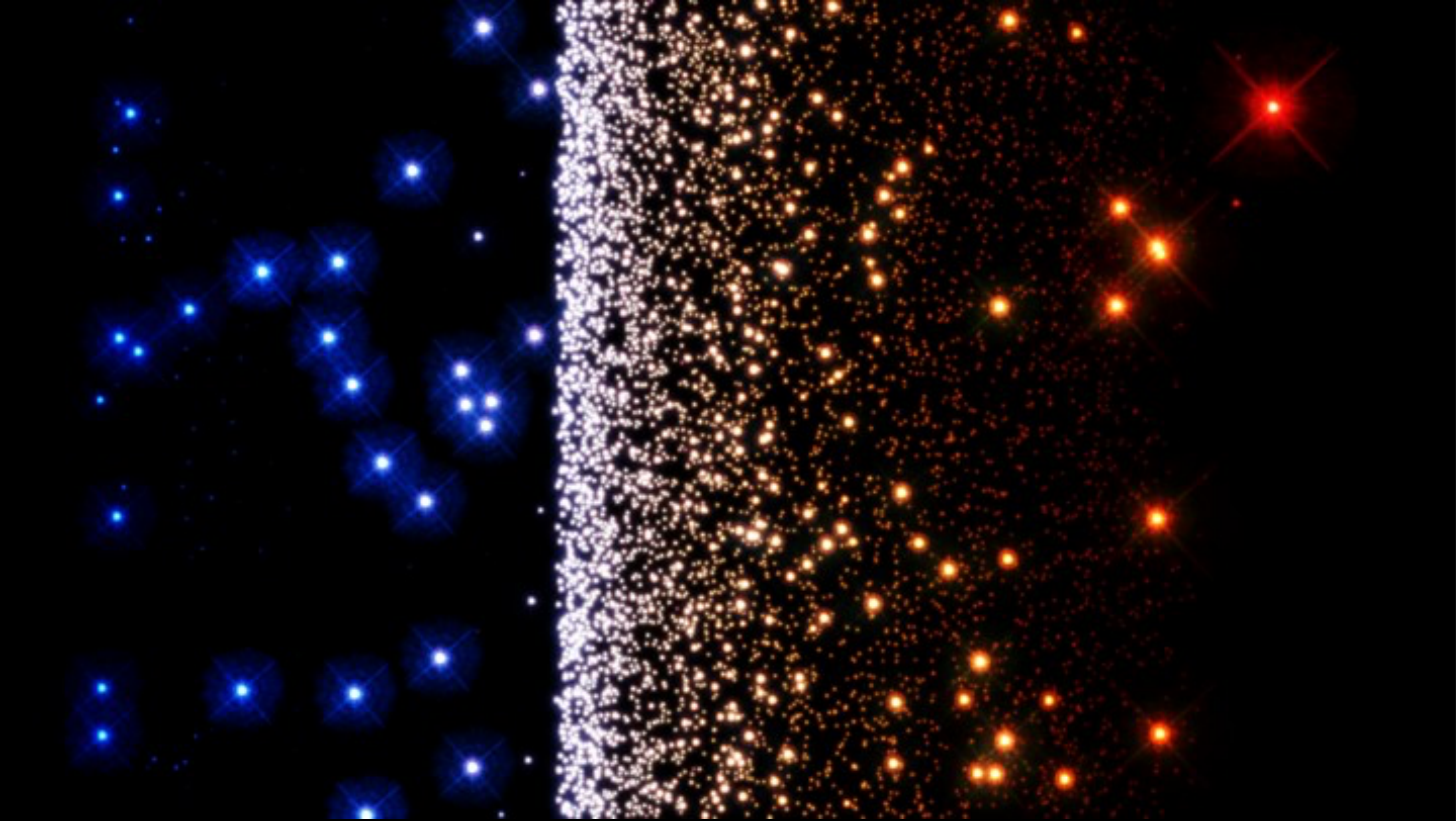
Let’s sort the stars by color, putting the **blue** stars on the left and the **red** stars on the right.



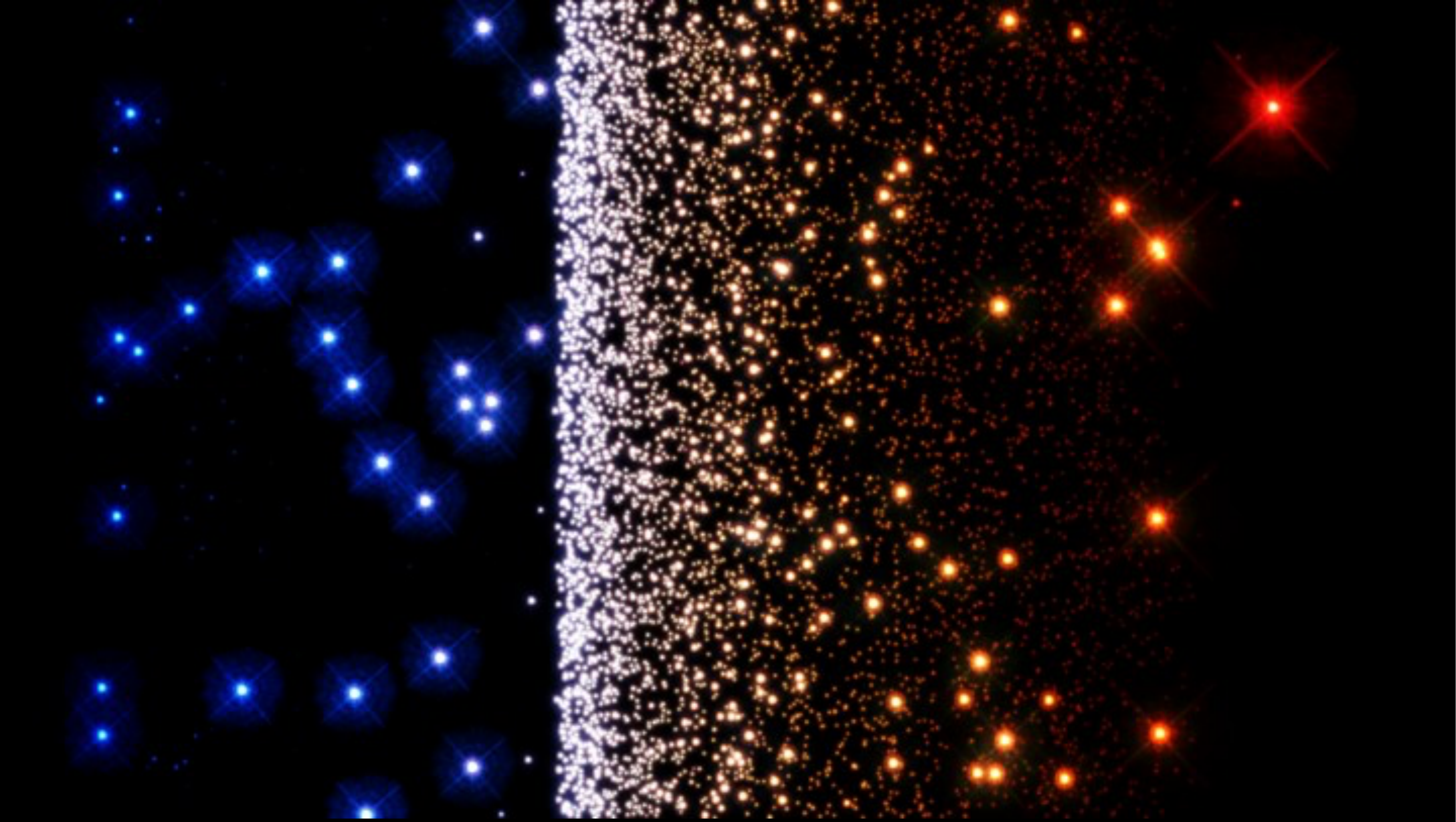




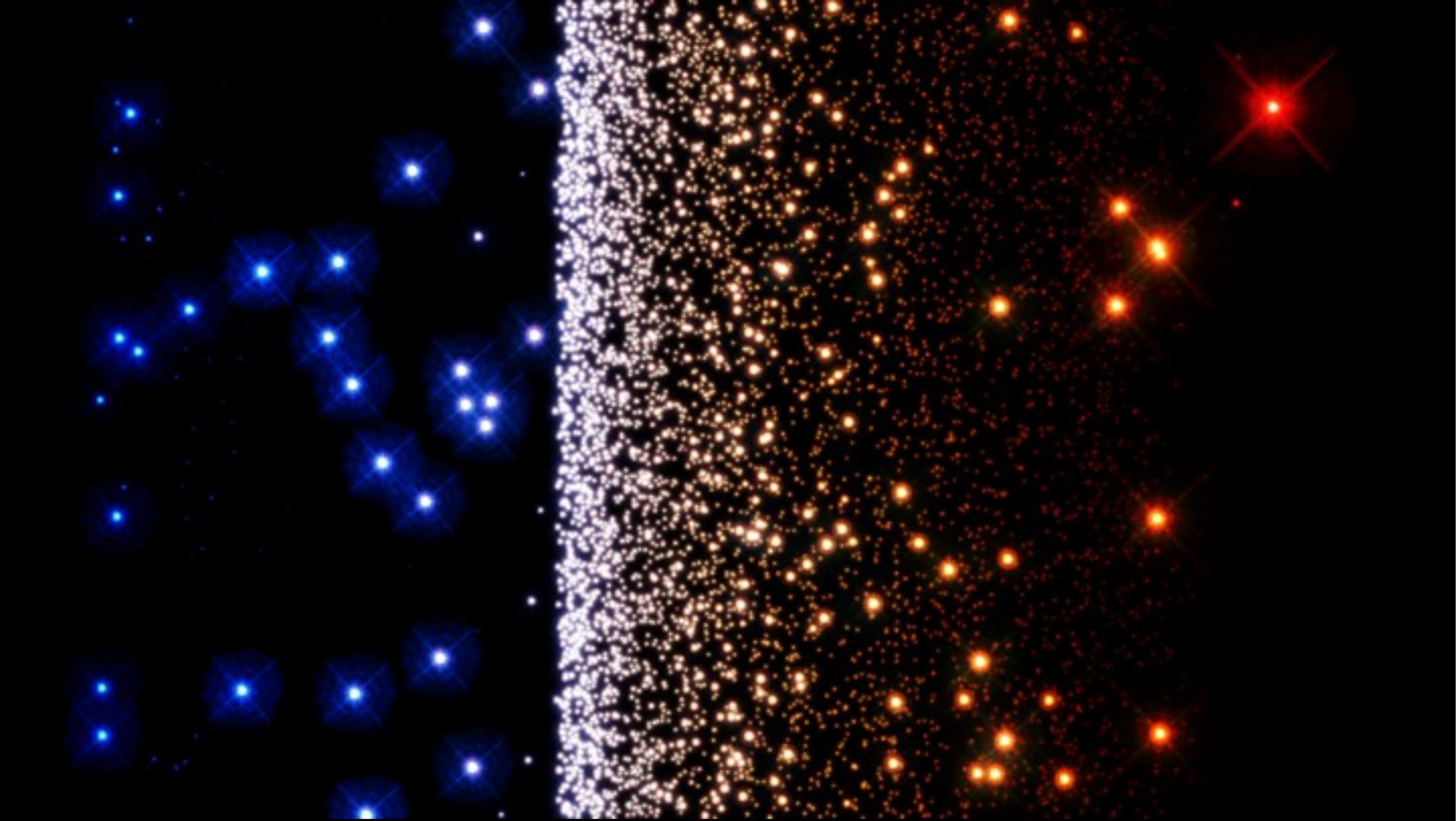




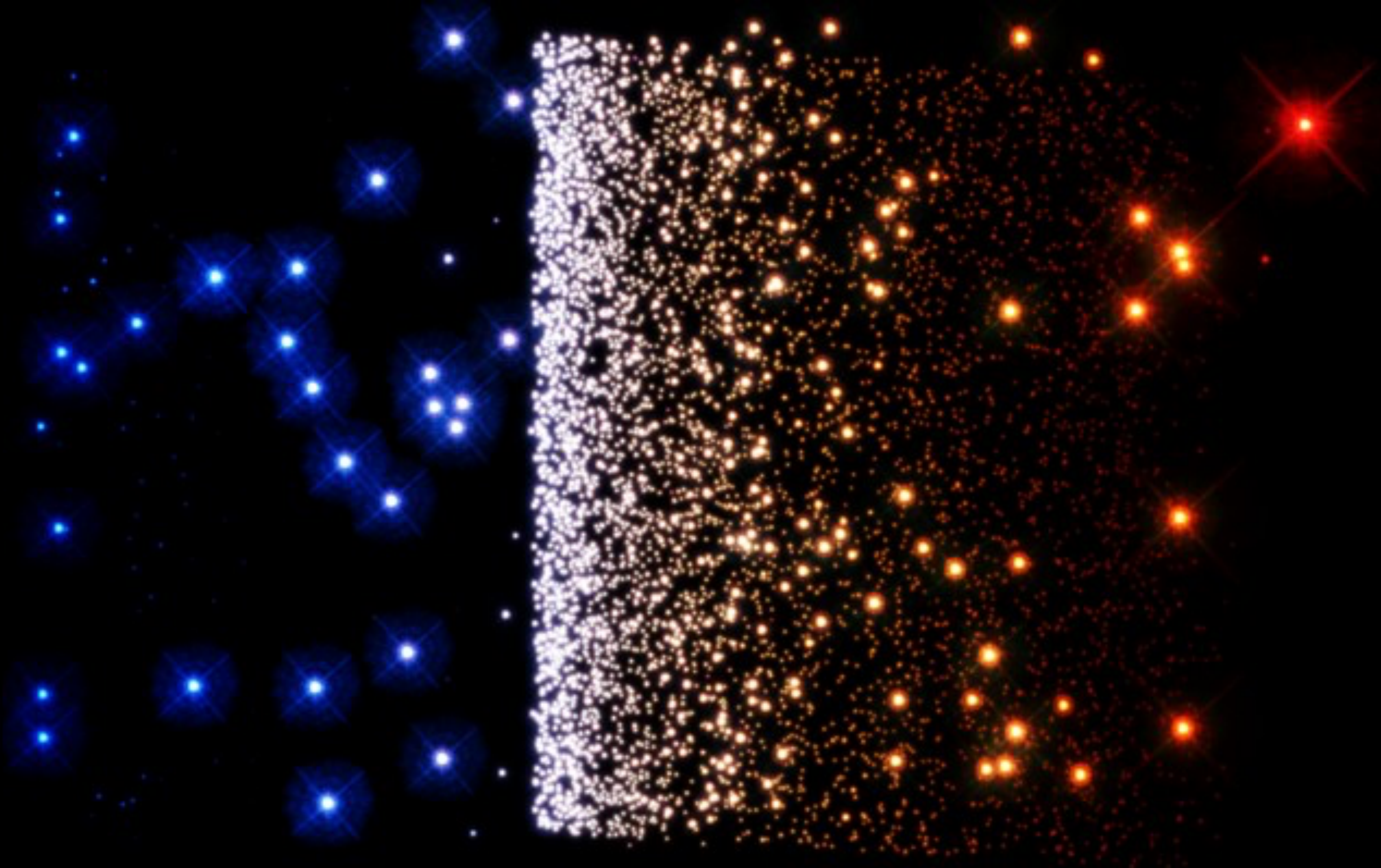
Note that most stars are nearly **white**.

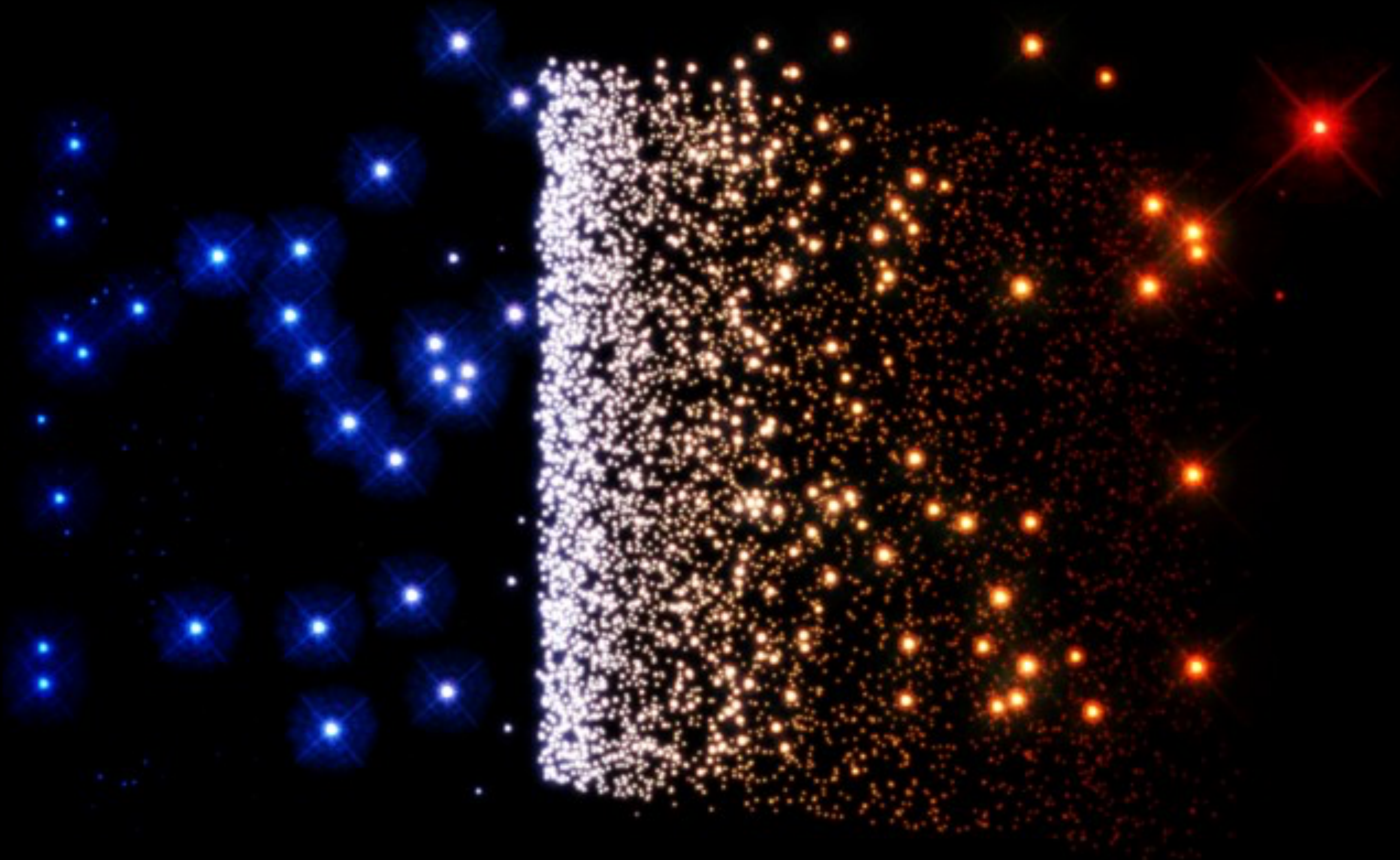


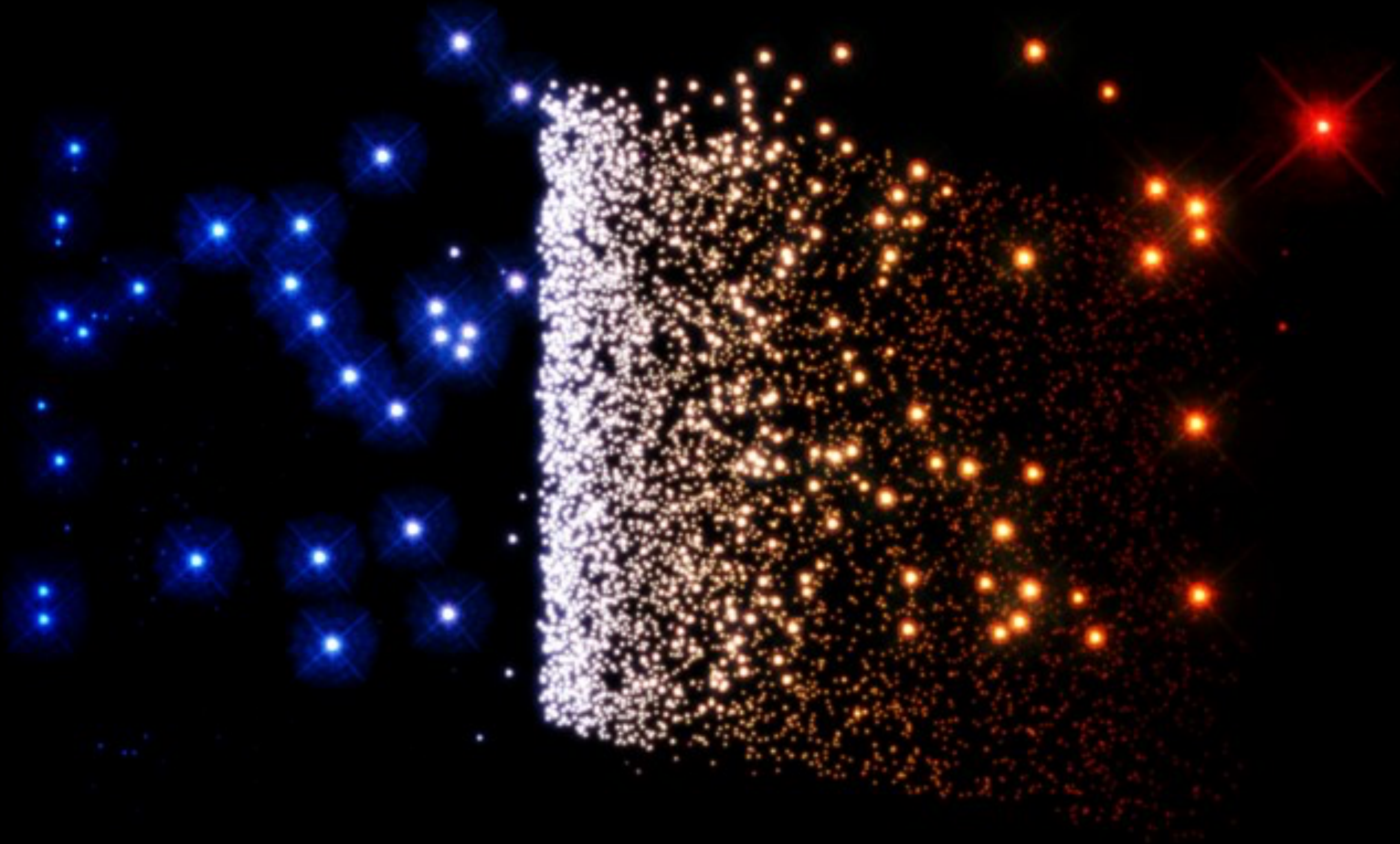
Astronomers also characterize stars in terms of brightness.

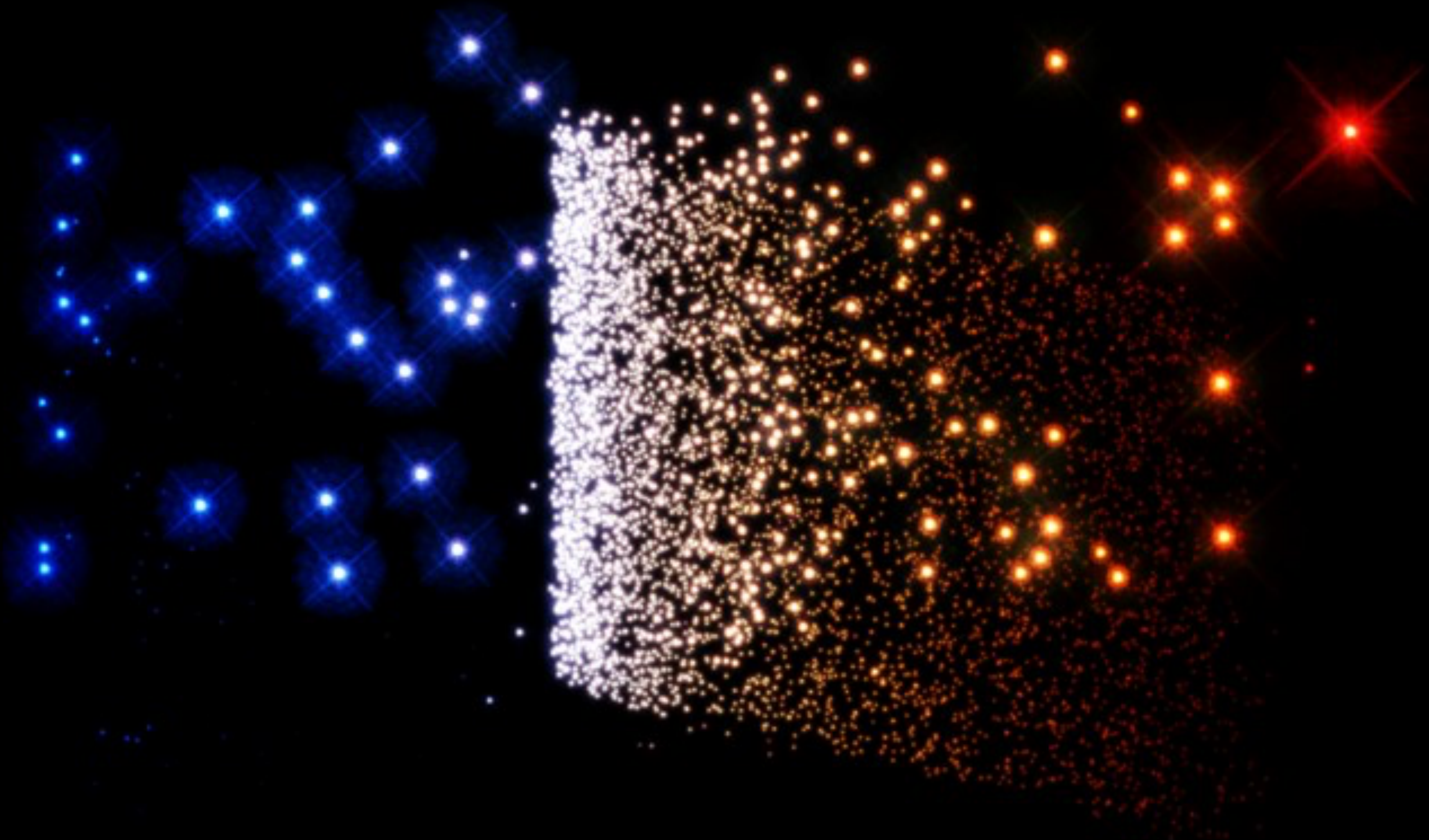


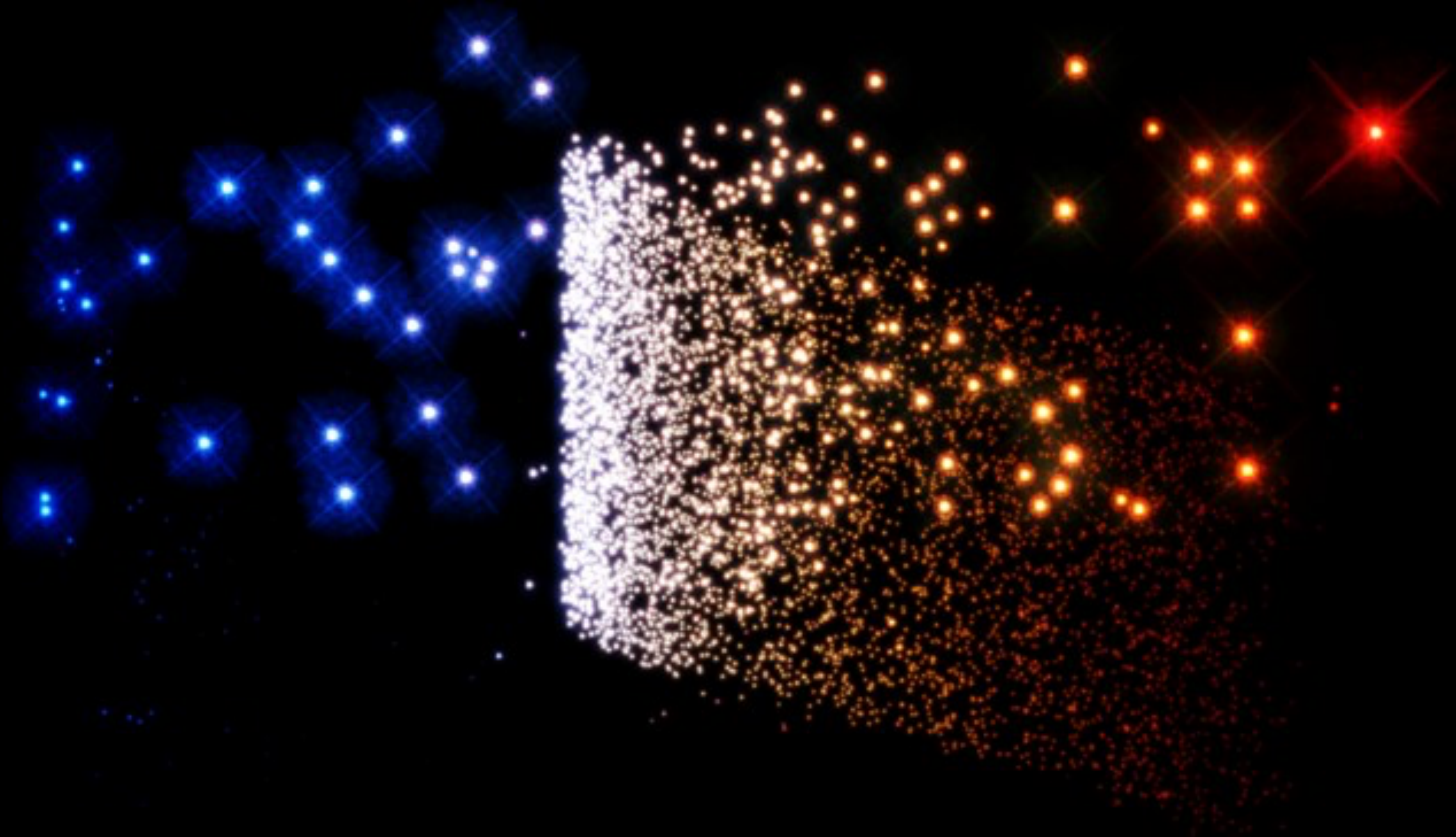
Let’s sort the stars, putting the **bright** stars on top and the **faint** stars on the bottom.

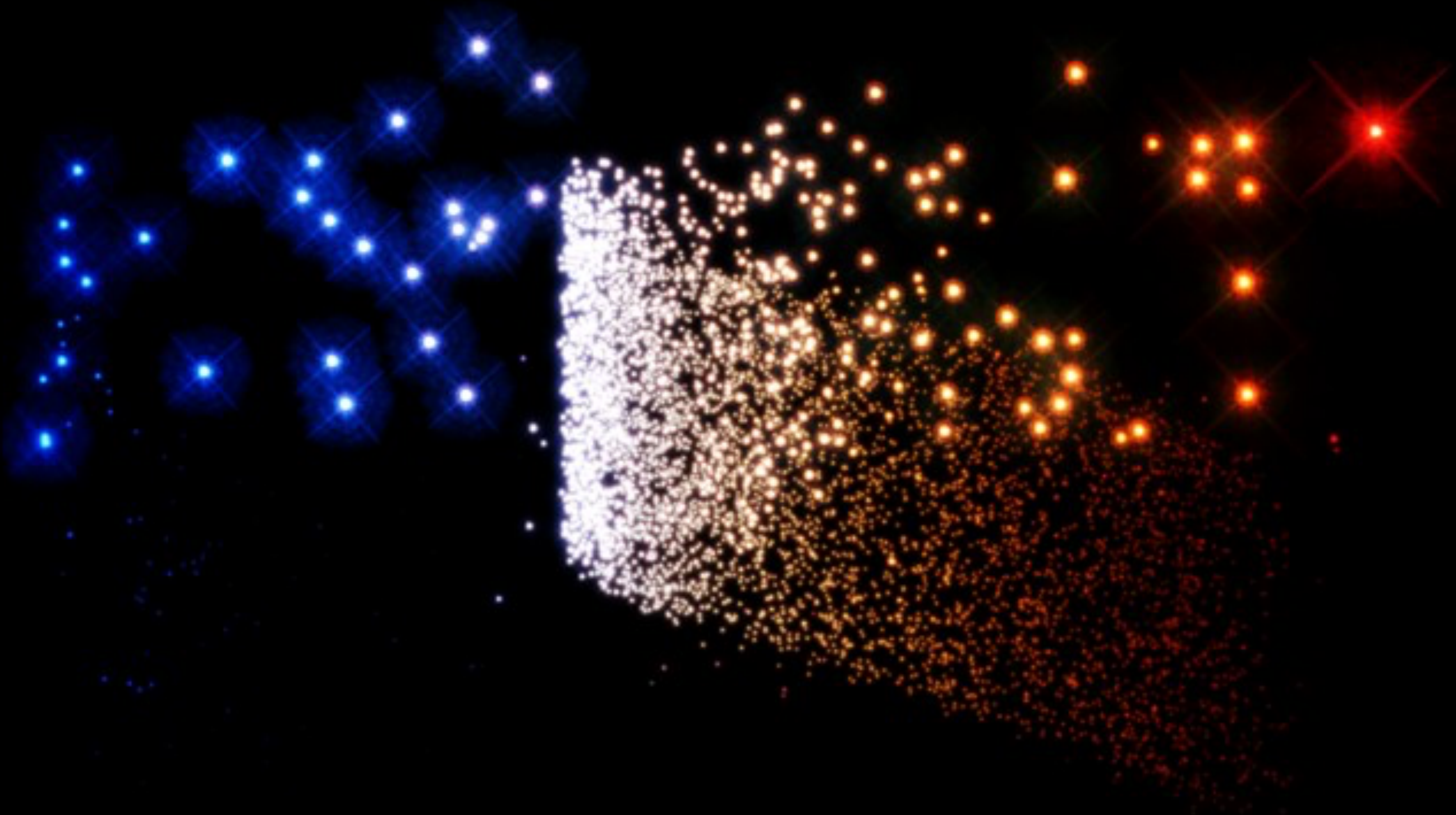


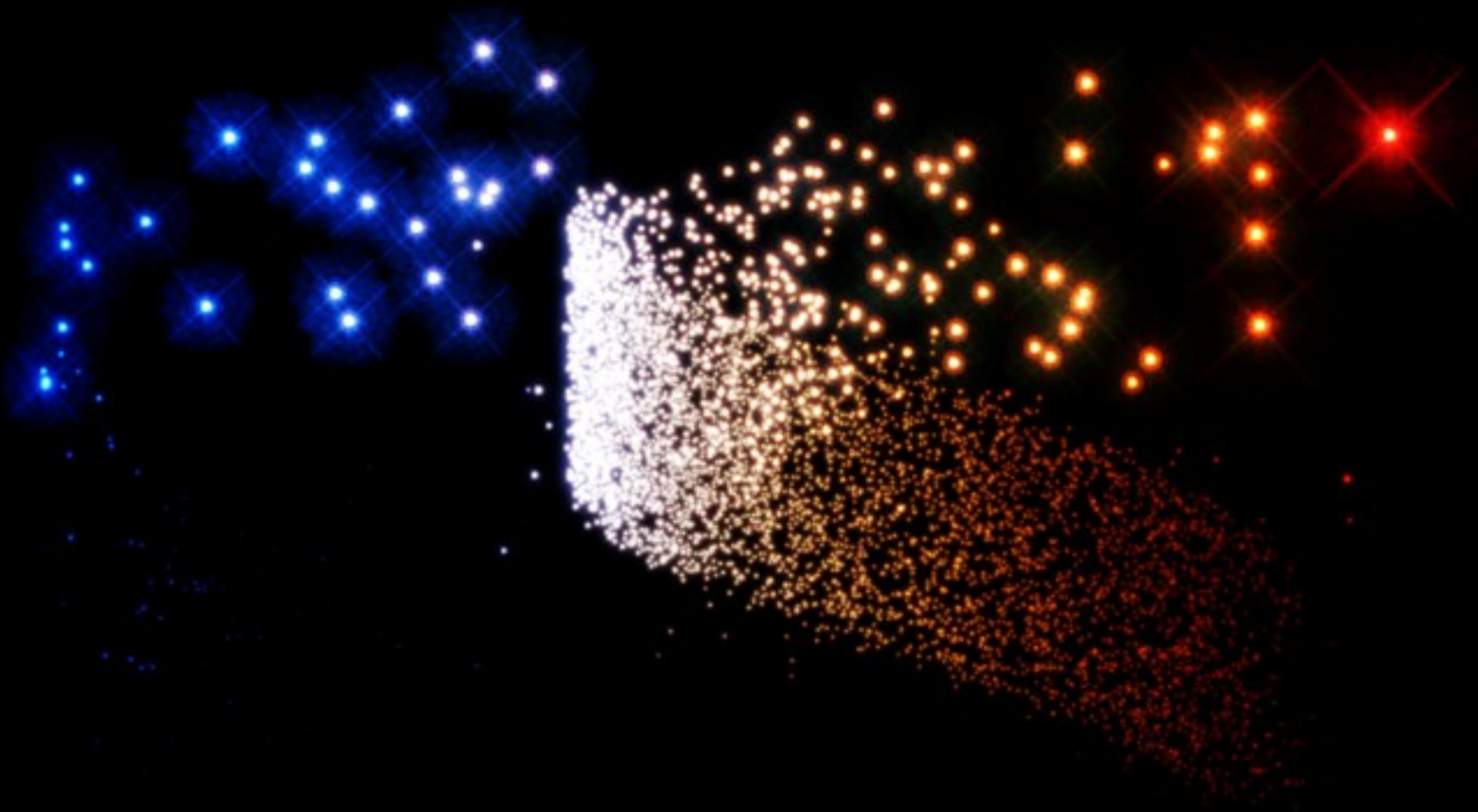


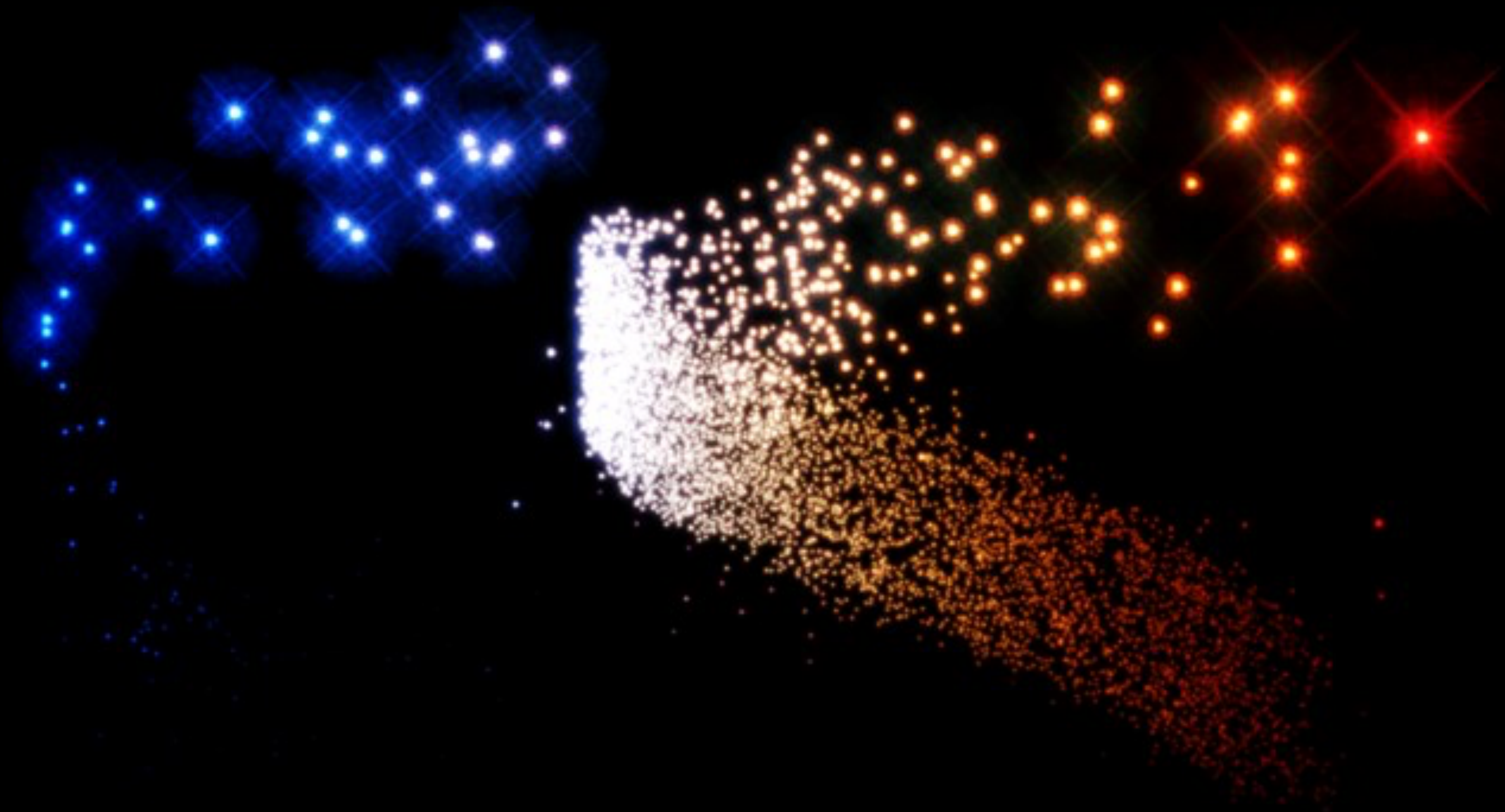


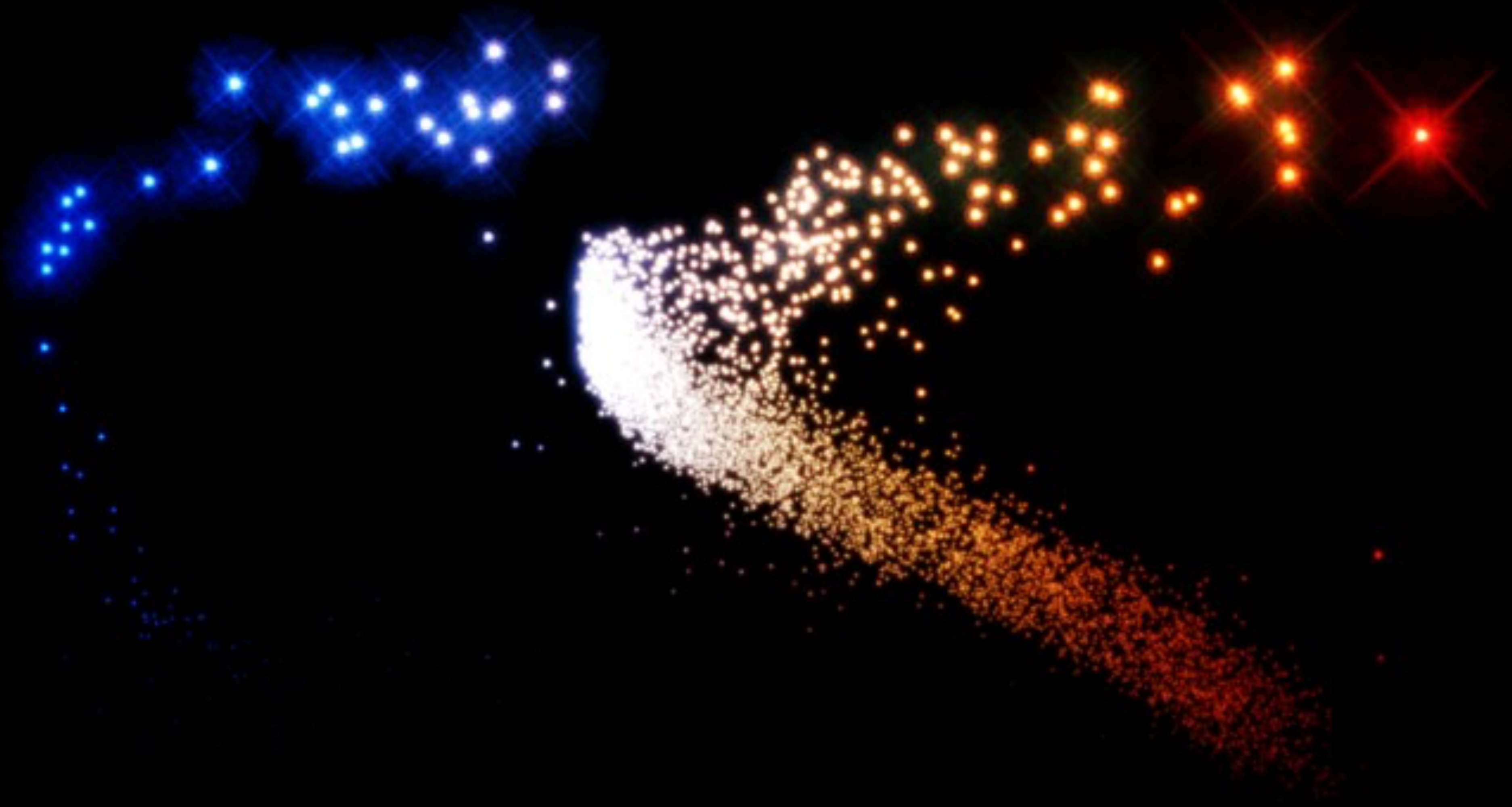


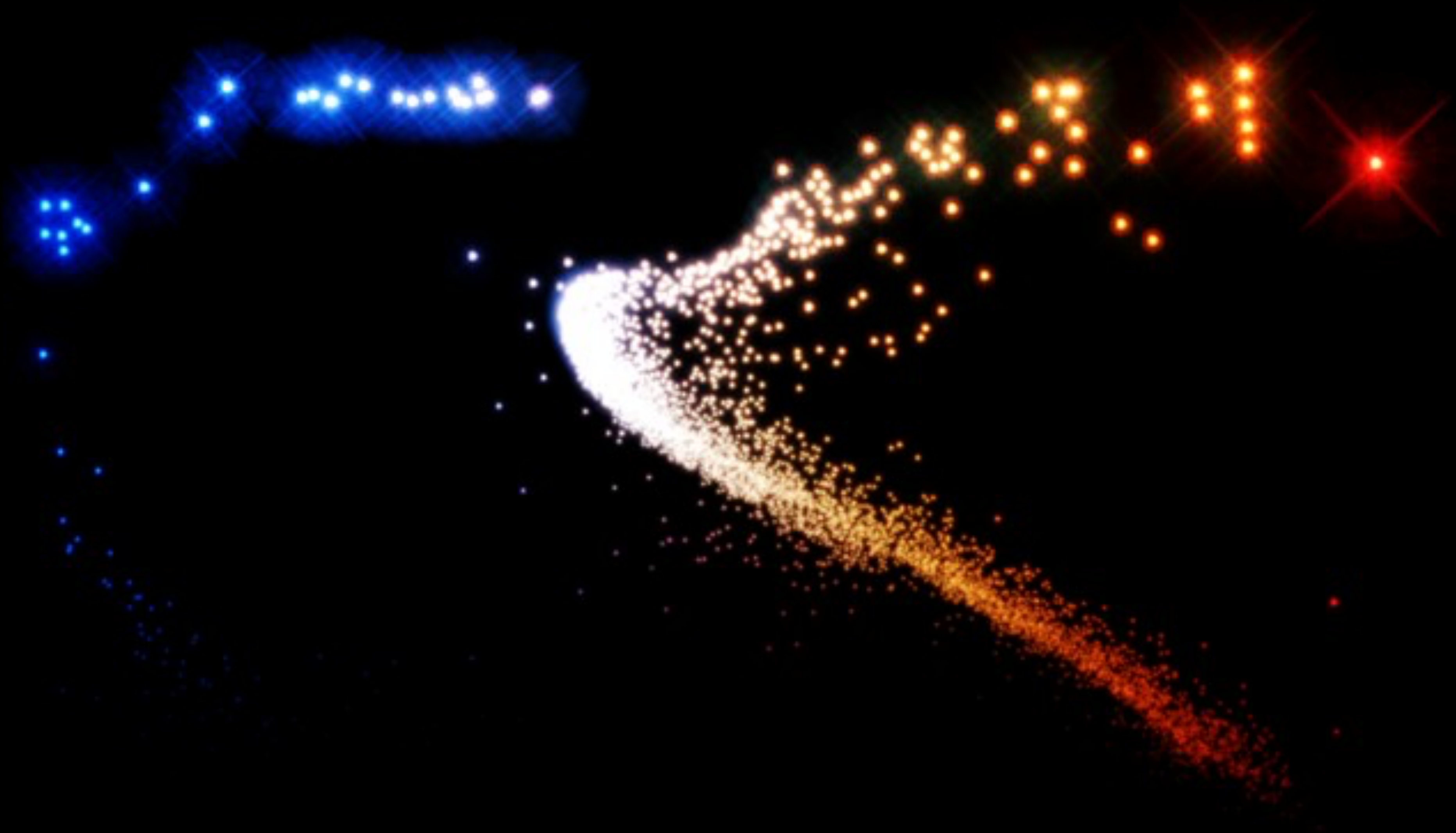




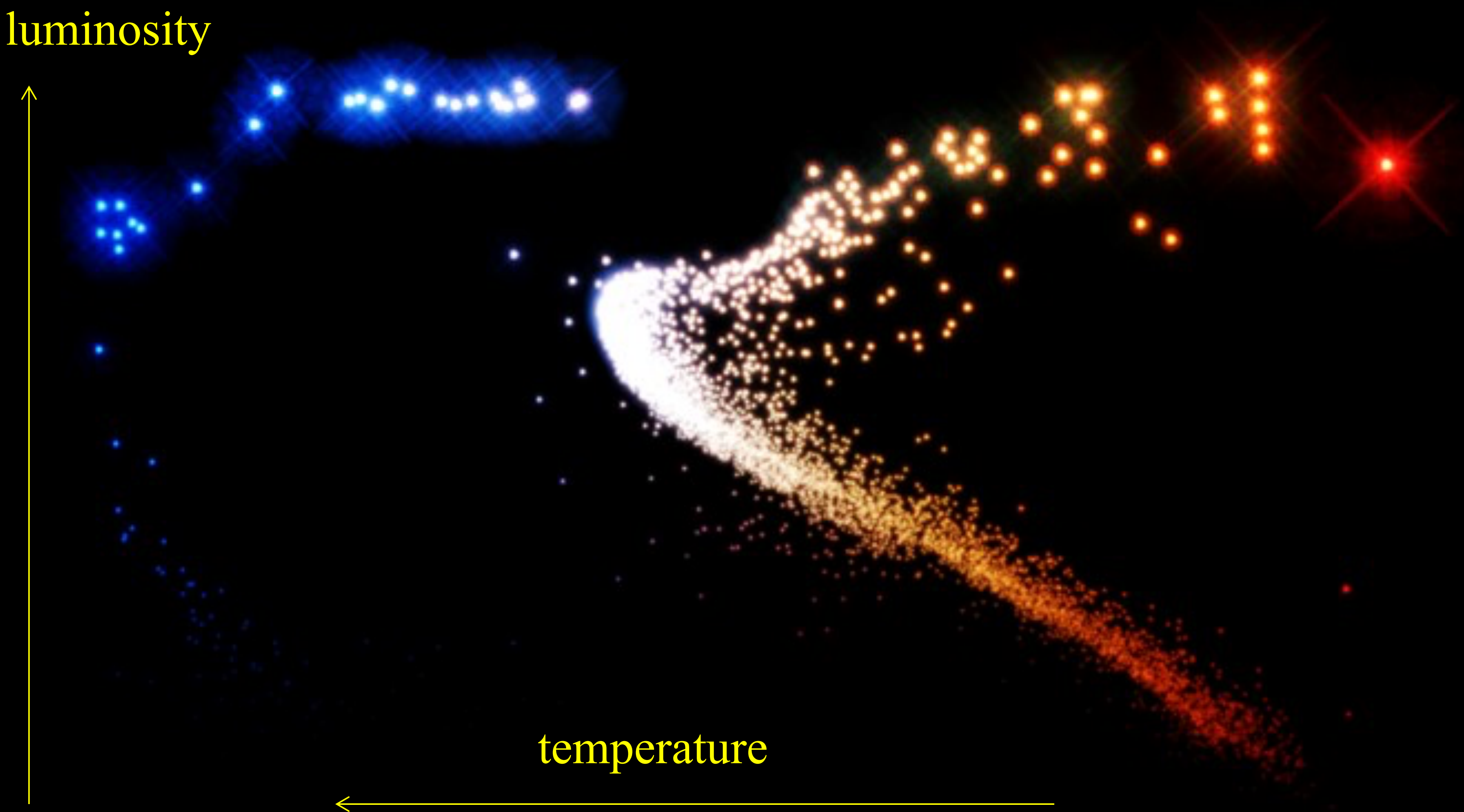




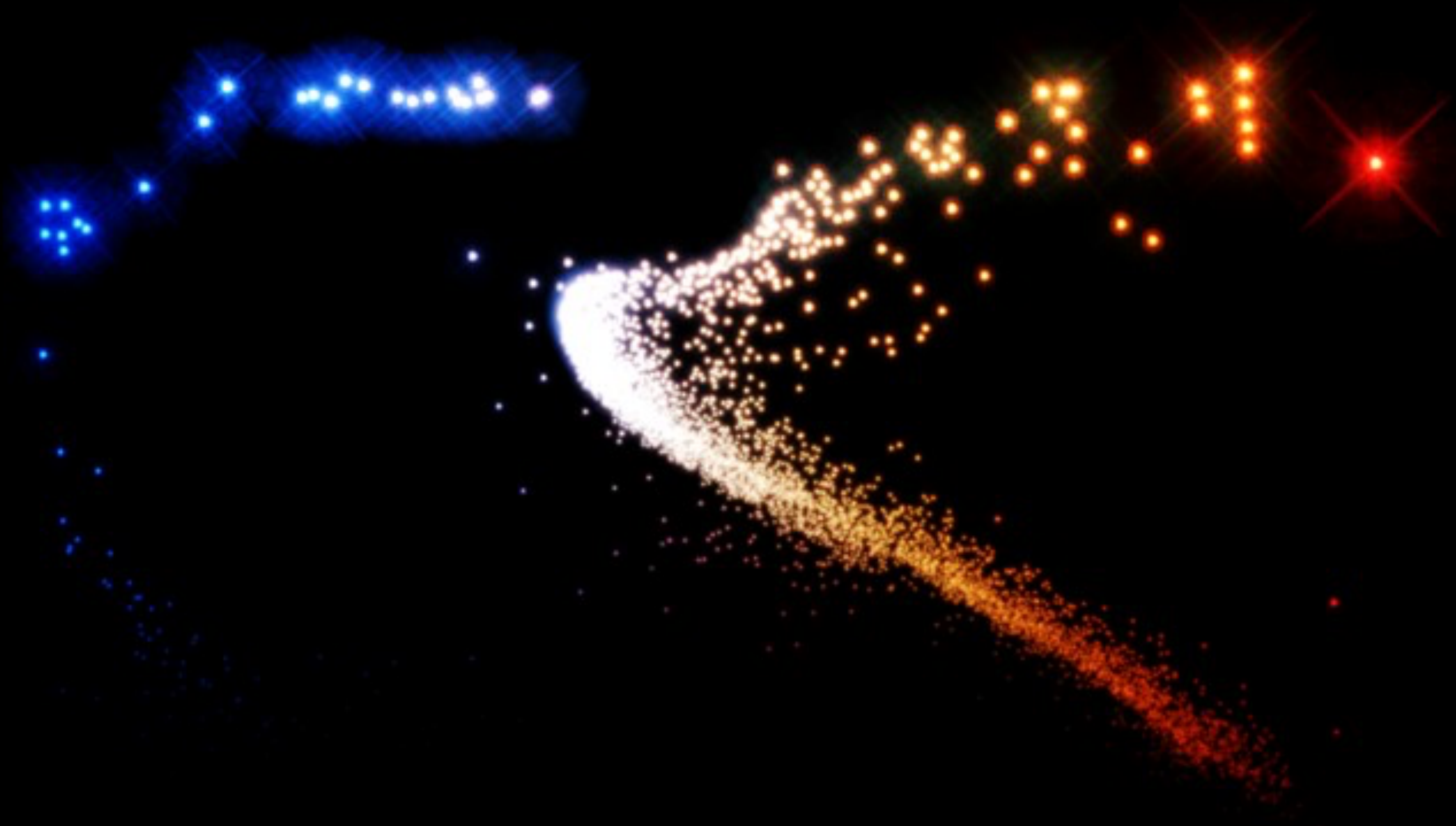




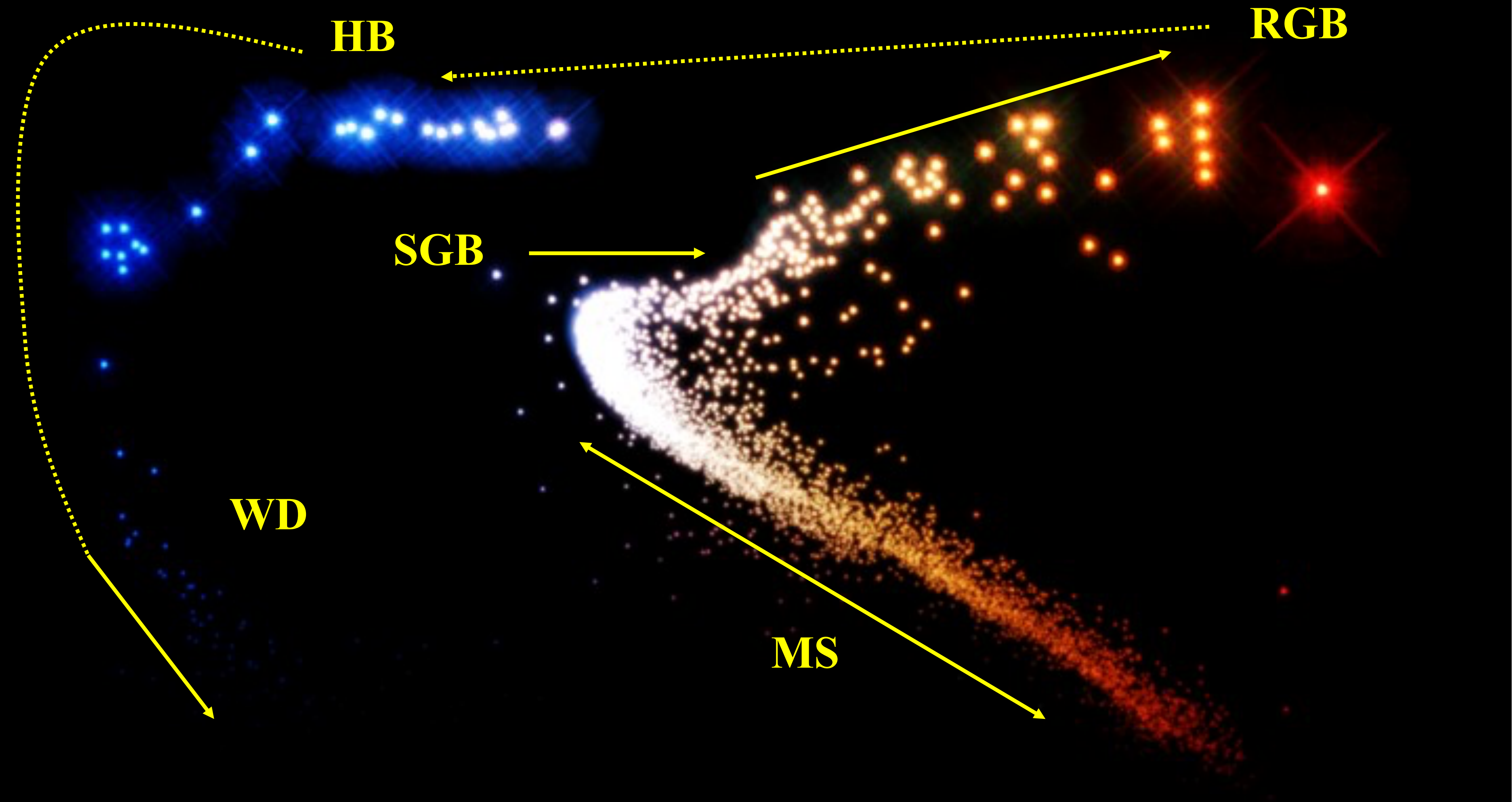
This is called a Hertzsprung-Russel (H-R) Diagram.



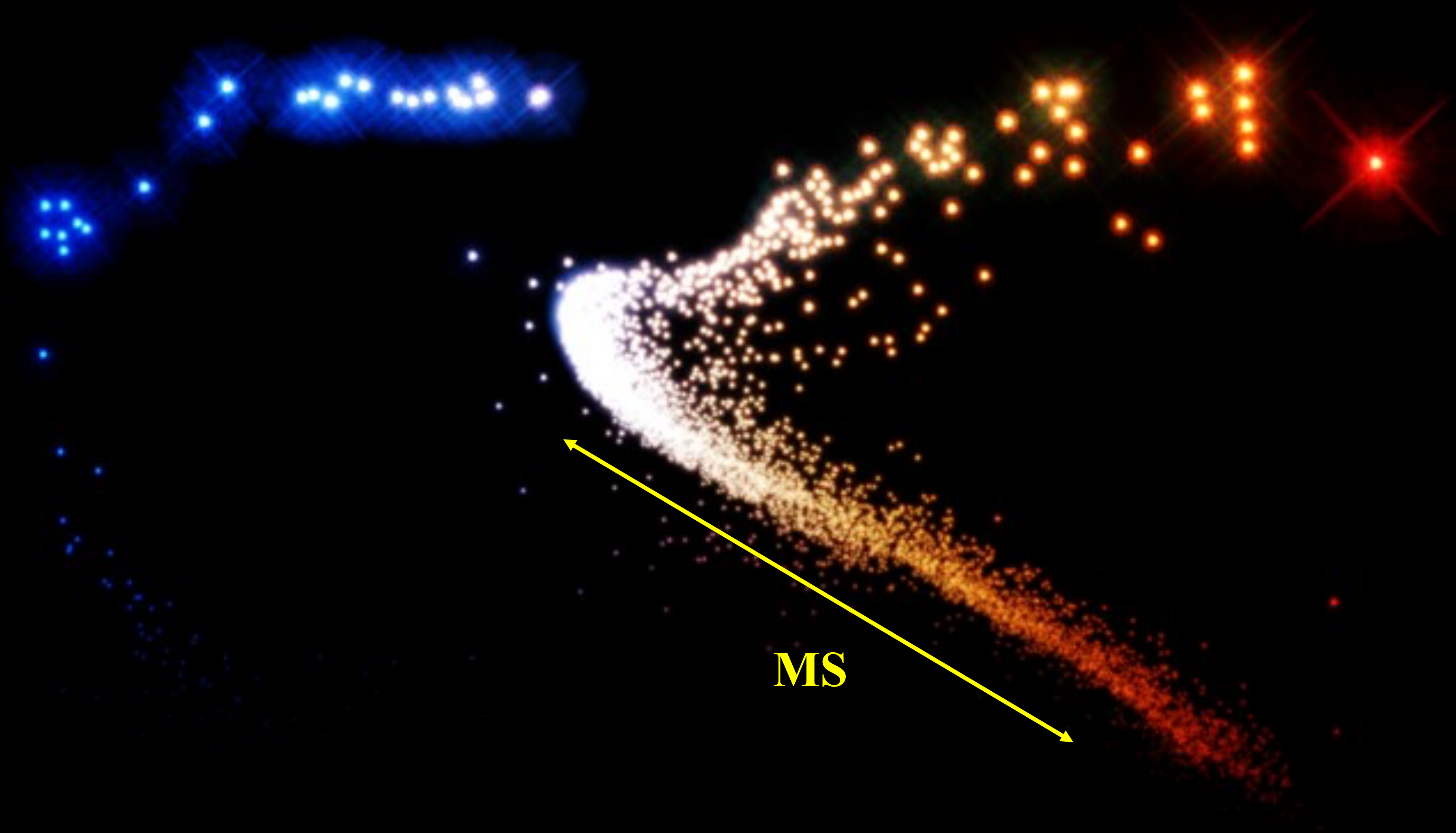
The unmistakable order in diagrams like this led astronomers to develop theories to explain stellar evolution.



Stars don’t fall just anywhere in the H-R diagram.

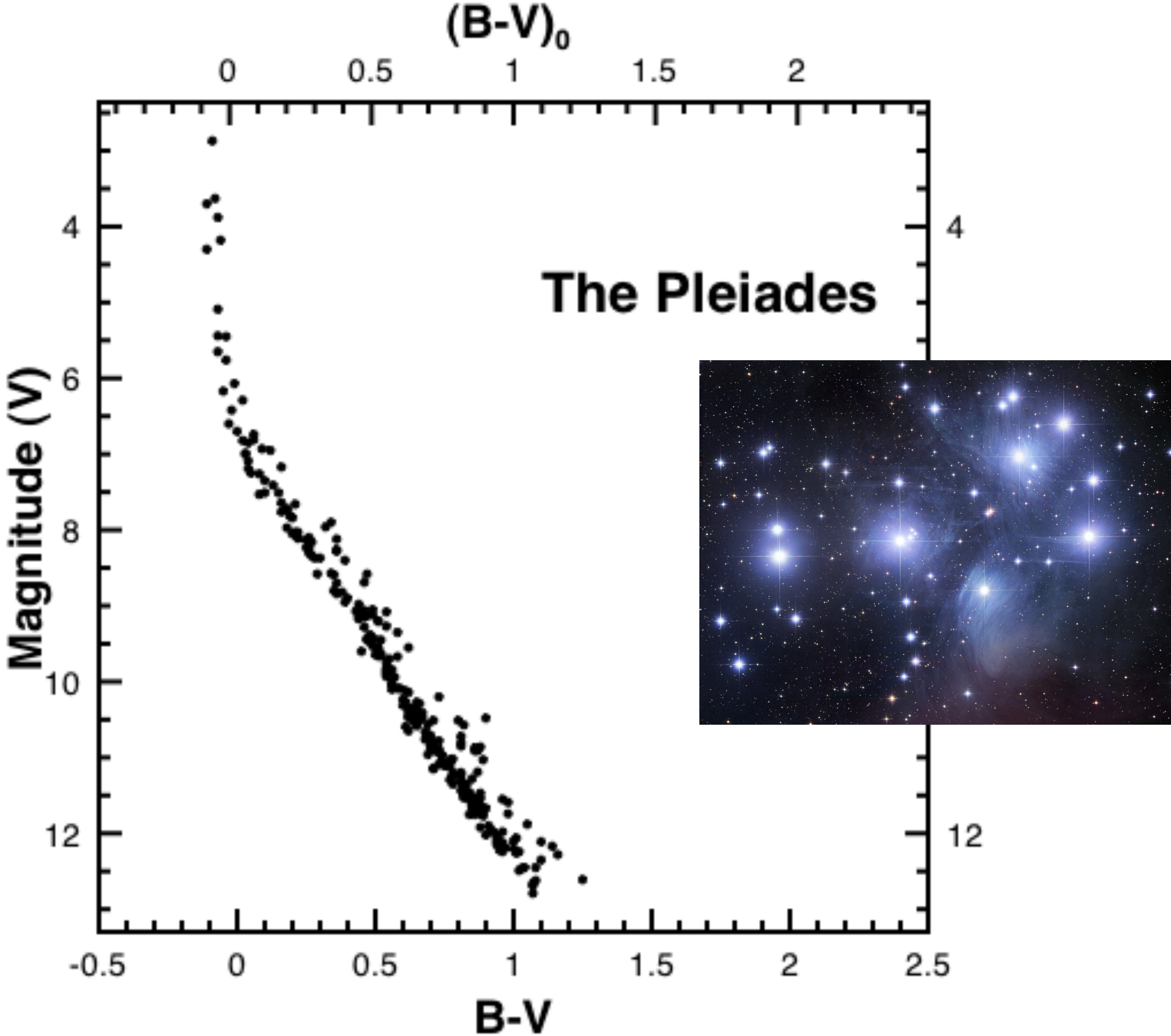


They lie along a few well-defined sequences.

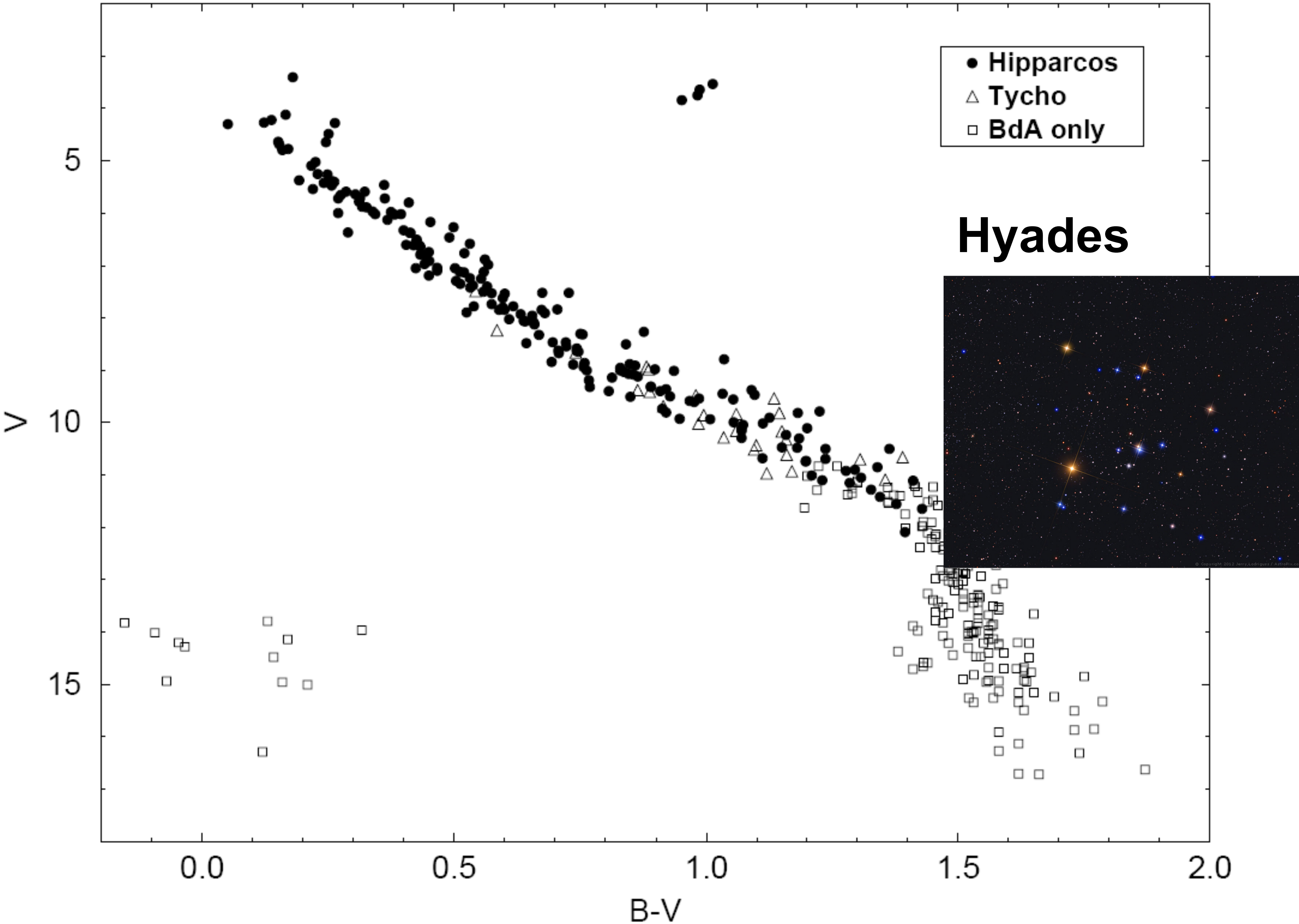


J. Anderson, STScI

The vast majority of stars lie along the
Main Sequence (MS)

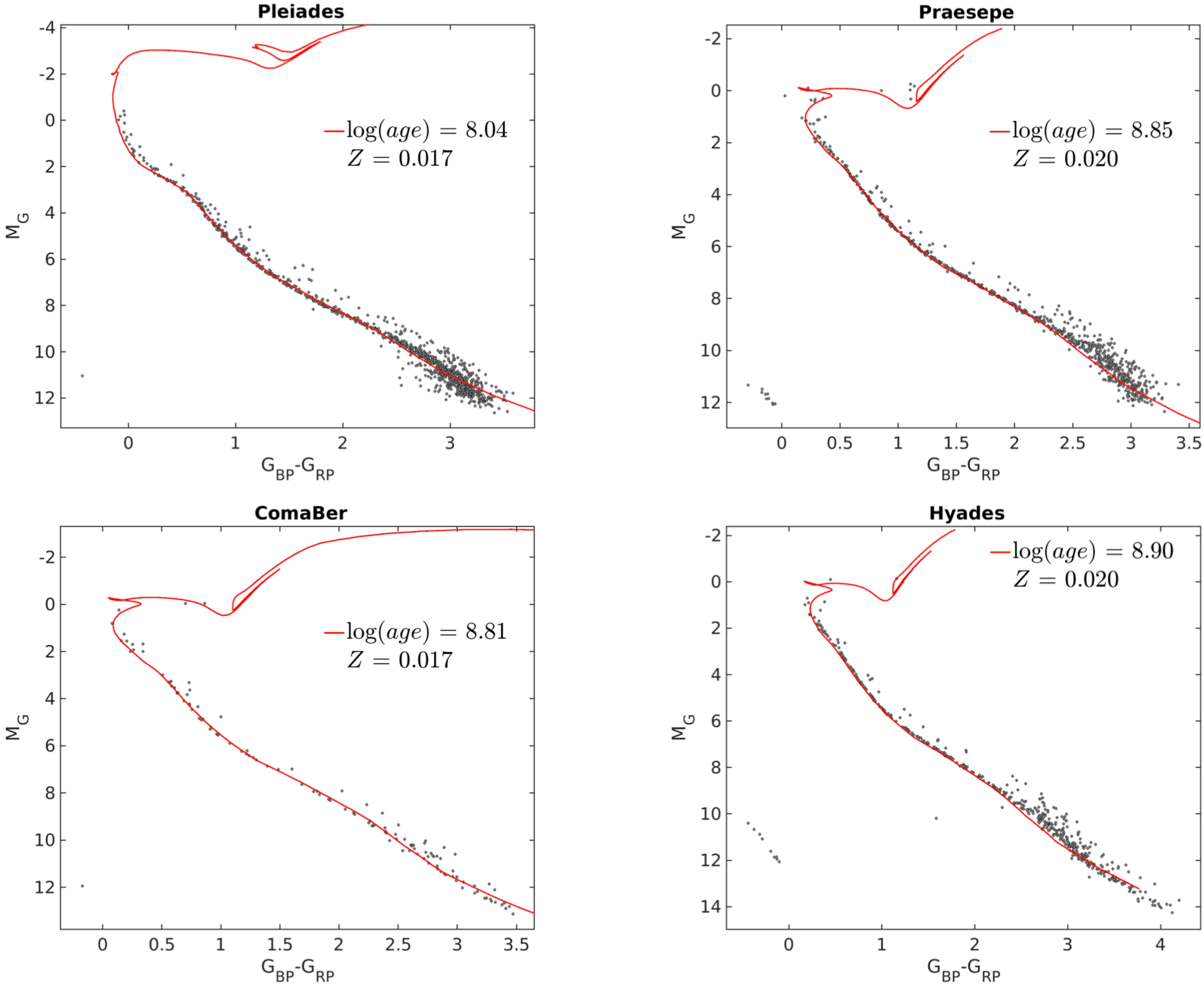

(B-V)$_0$
0
0.5
1
1.5
2
4
6
8
10
12
Magnitude (V)
4
12
The Pleiades
-0.5
0
0.5
1
1.5
2
2.5
B-V

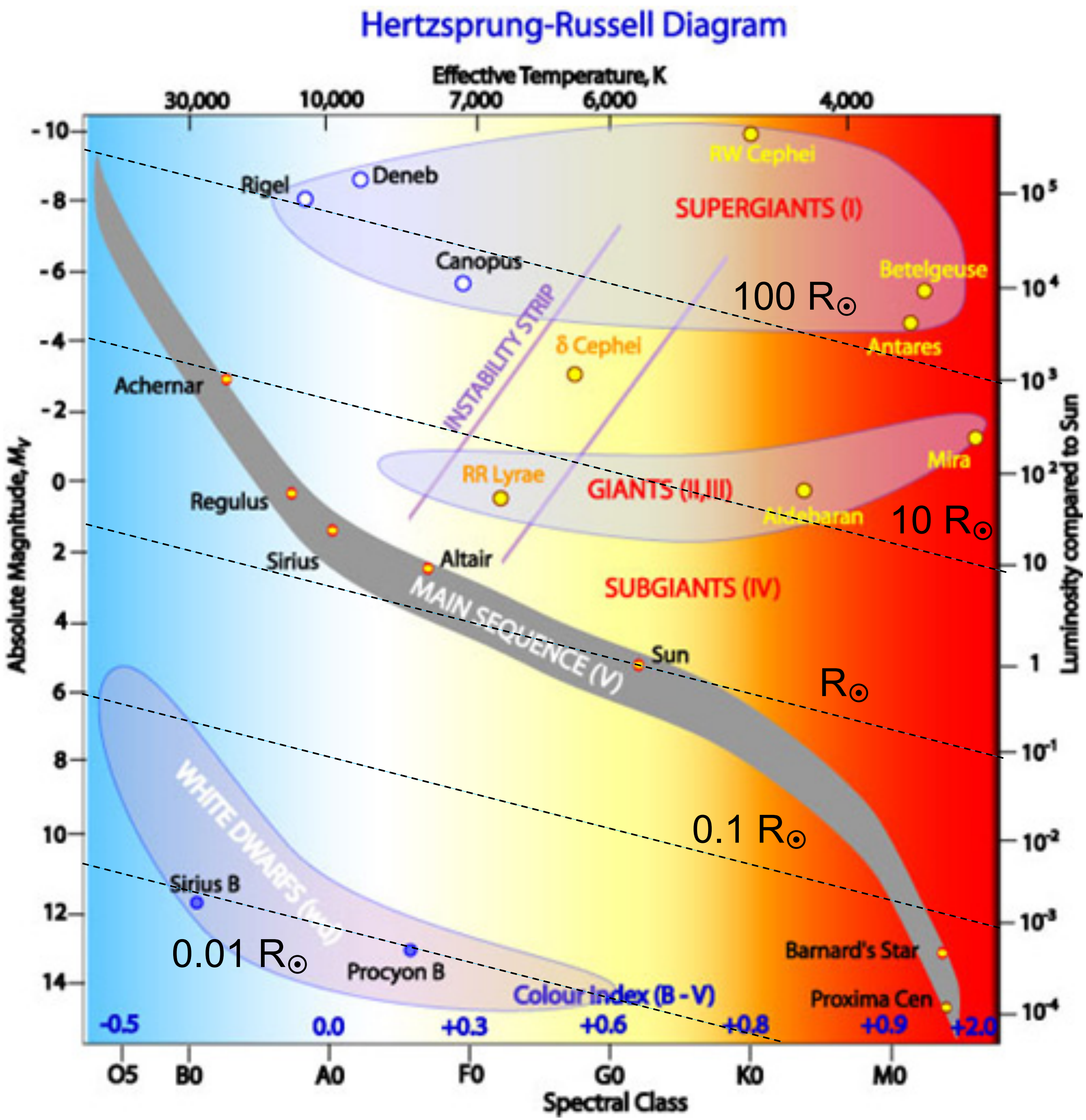
Hyades

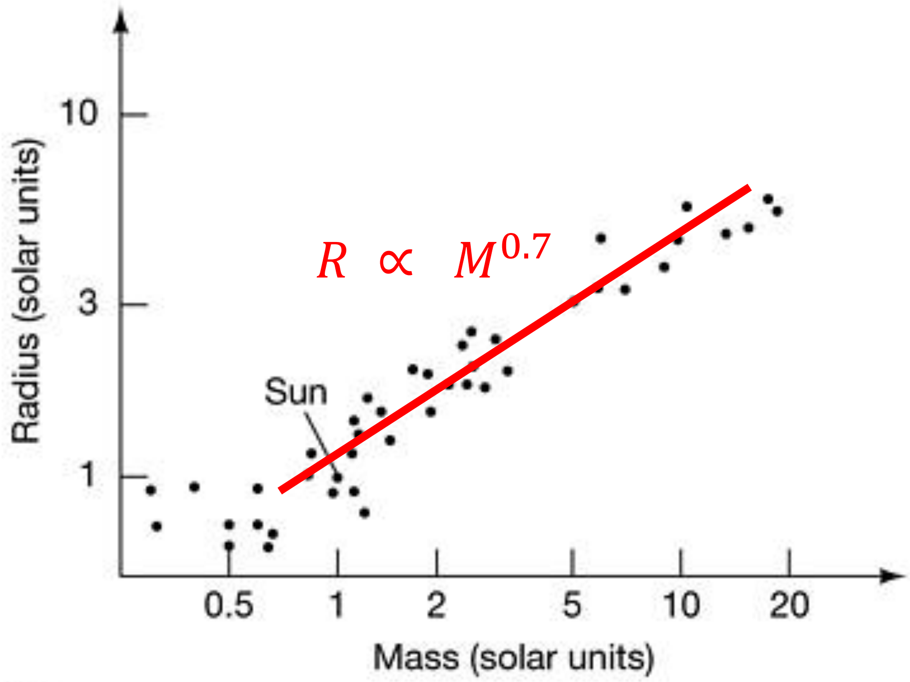


A&A 616, A10 (2018)

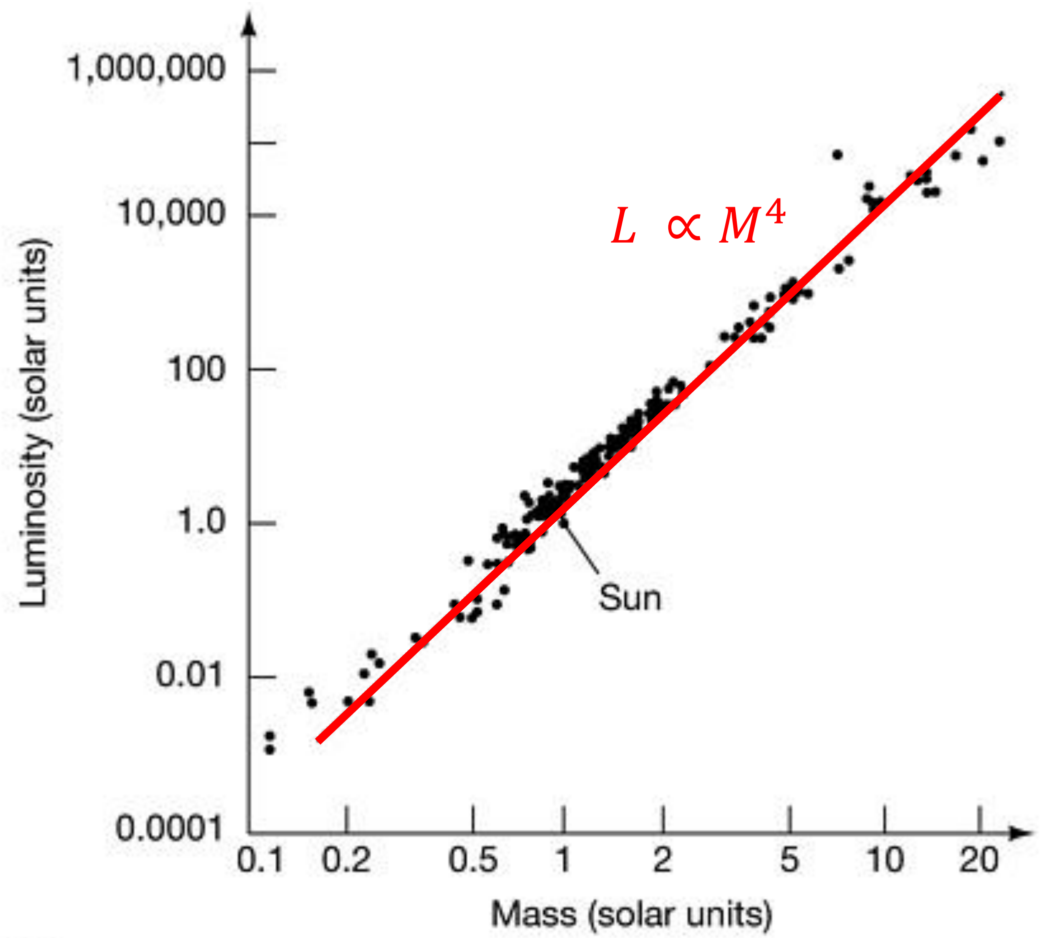
Gaia DR2



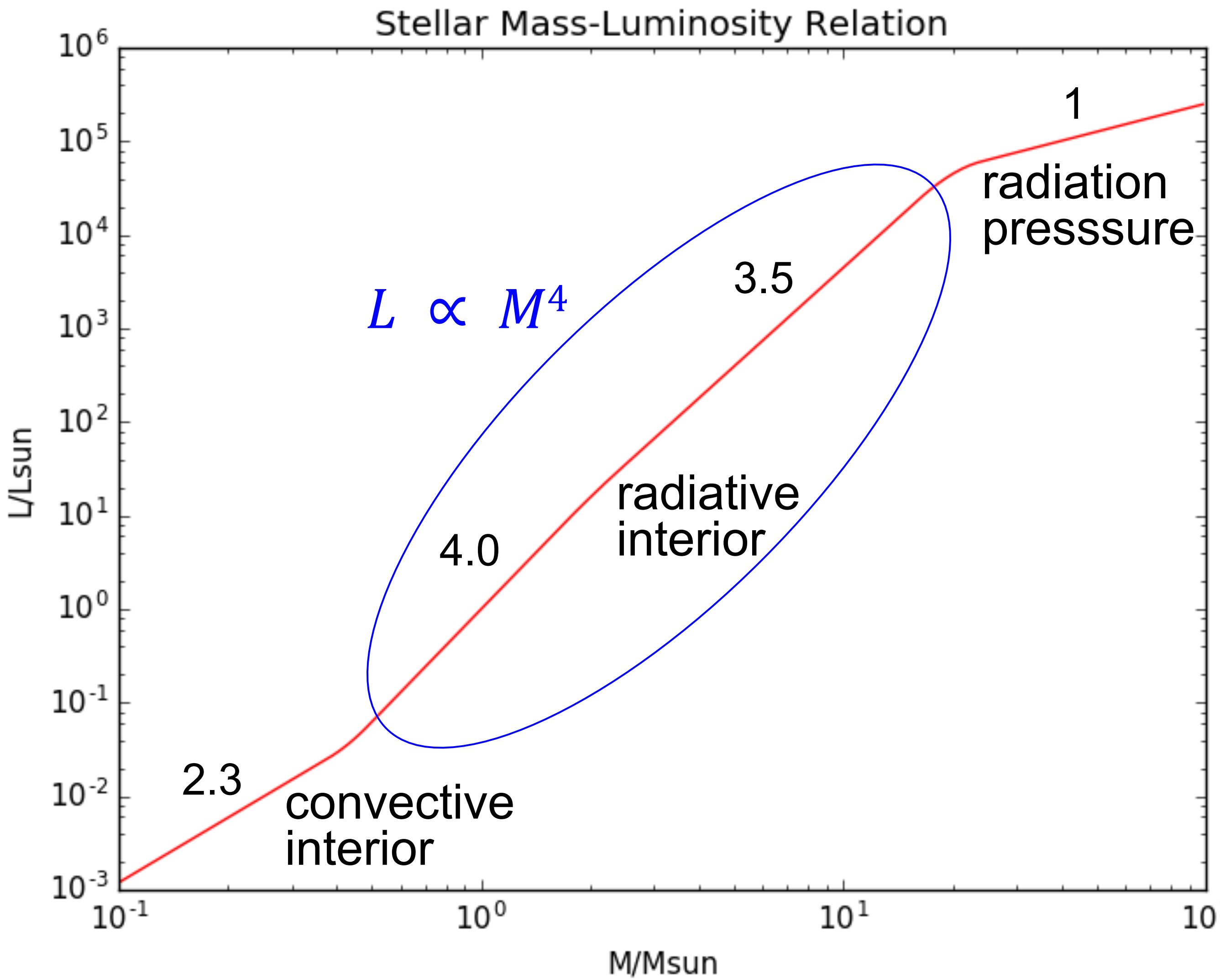

Hertzsprung-Russell Diagram
Effective Temperature, K
30,000
10,000
7,000
6,000
4,000
Absolute Magnitude, M_V
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
Luminosity compared to Sun
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
Rigel
Deneb
RW Cephei
SUPERGIANTS (I)
Canopus
Betelgeuse
$100\ R_\odot$
Antares
INSTABILITY STRIP
δ Cephei
Achernar
Mira
RR Lyrae
GIANTS (II,III)
Regulus
Aldebaran
$10\ R_\odot$
Sirius
Altair
SUBGIANTS (IV)
MAIN SEQUENCE (V)
Sun
$R_\odot$
WHITE DWARFS (wd)
$0.1\ R_\odot$
Sirius B
$0.01\ R_\odot$
Procyon B
Barnard's Star
Colour Index (B - V)
Proxima Cen
-0.5
0.0
+0.3
+0.6
+0.8
+0.9
+2.0
O5
B0
A0
F0
G0
K0
M0
Spectral Class



10
3
1
Radius (solar units)
Sun
$R \propto M^{0.7}$
0.5
1
2
5
10
20
Mass (solar units)



1,000,000
10,000
100
1.0
0.01
0.0001
Luminosity (solar units)
$L \propto M^4$
Sun
0.1
0.2
0.5
1
2
5
10
20
Mass (solar units)


Stellar Mass-Luminosity Relation
L/Lsun
M/Msun
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-1}
10^{0}
10^{1}
10^{2}
$L \propto M^4$
1
radiation
presssure
3.5
radiative
interior
4.0
2.3
convective
interior

$$t_{MS} \sim \frac{M}{dM/dt} \propto \frac{M}{L} \propto M^{-3}$$

➔ stars must evolve

➔ the most massive stars evolve <u>fastest</u>!

$$t_{MS} \approx 10^{10} \left(\frac{M}{M_\odot}\right)^{-3} \text{ years}$$

➔ stars must evolve

➔ the most massive stars evolve fastest!