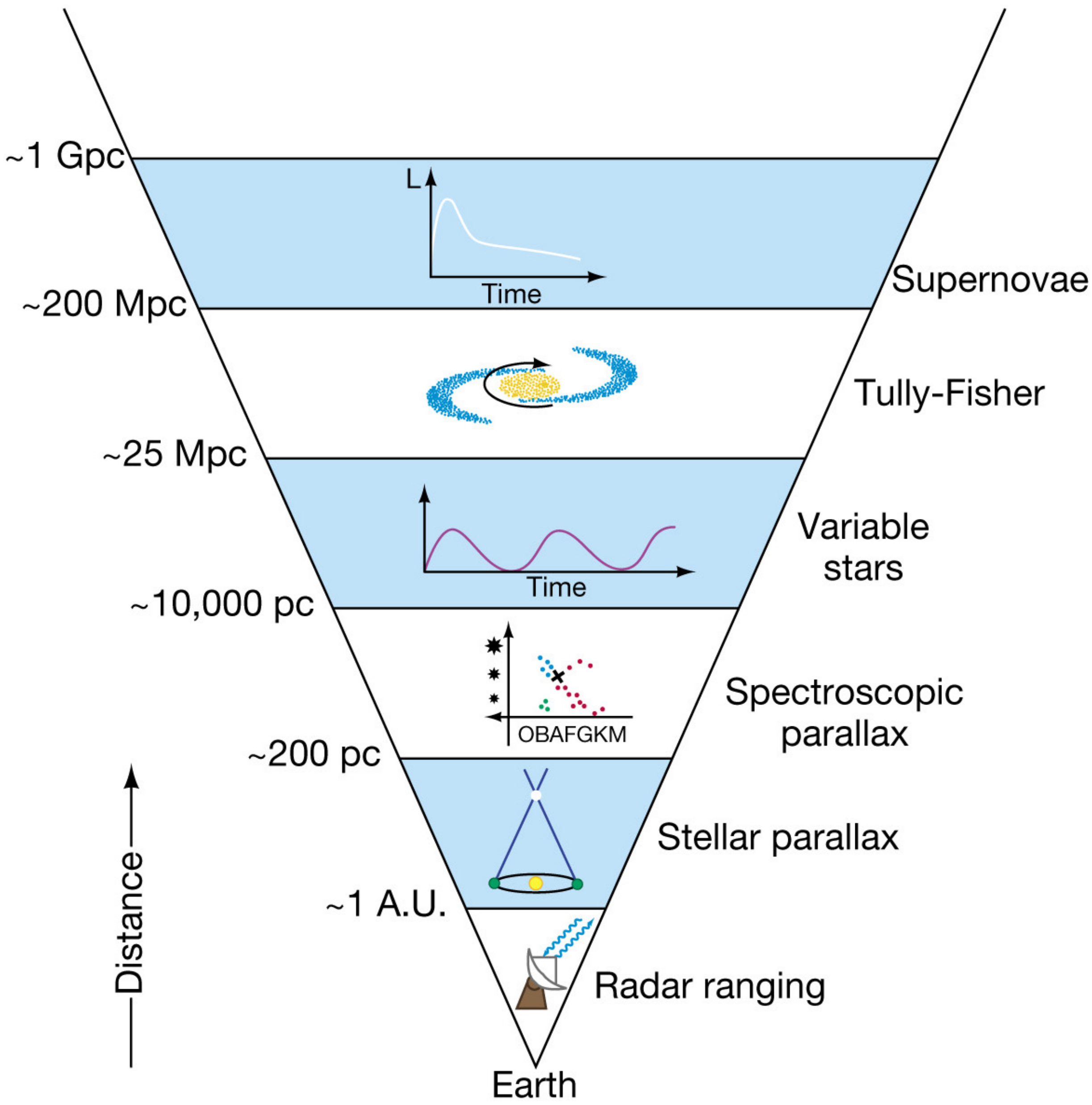
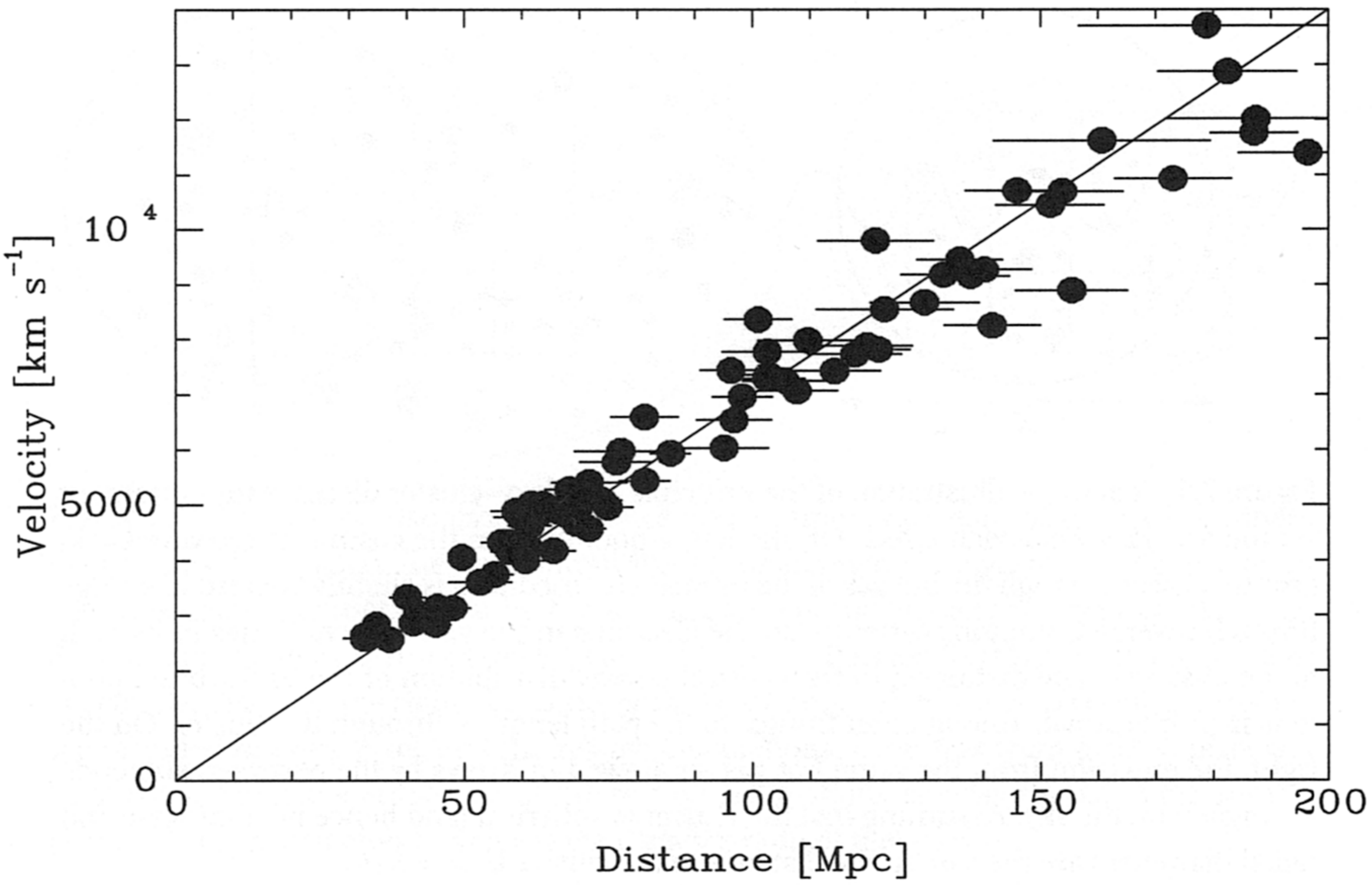
1. Hubble's law
2. Large-scale structure
3. Olbers paradox
4. Age of the universe
5. The hot Big Bang
6. Friedmann equations
7. Future of the universe
8. The early universe
   - cosmic microwave background
   - primordial nucleosynthesis
9. Growth of structure
10. Cosmic inflation

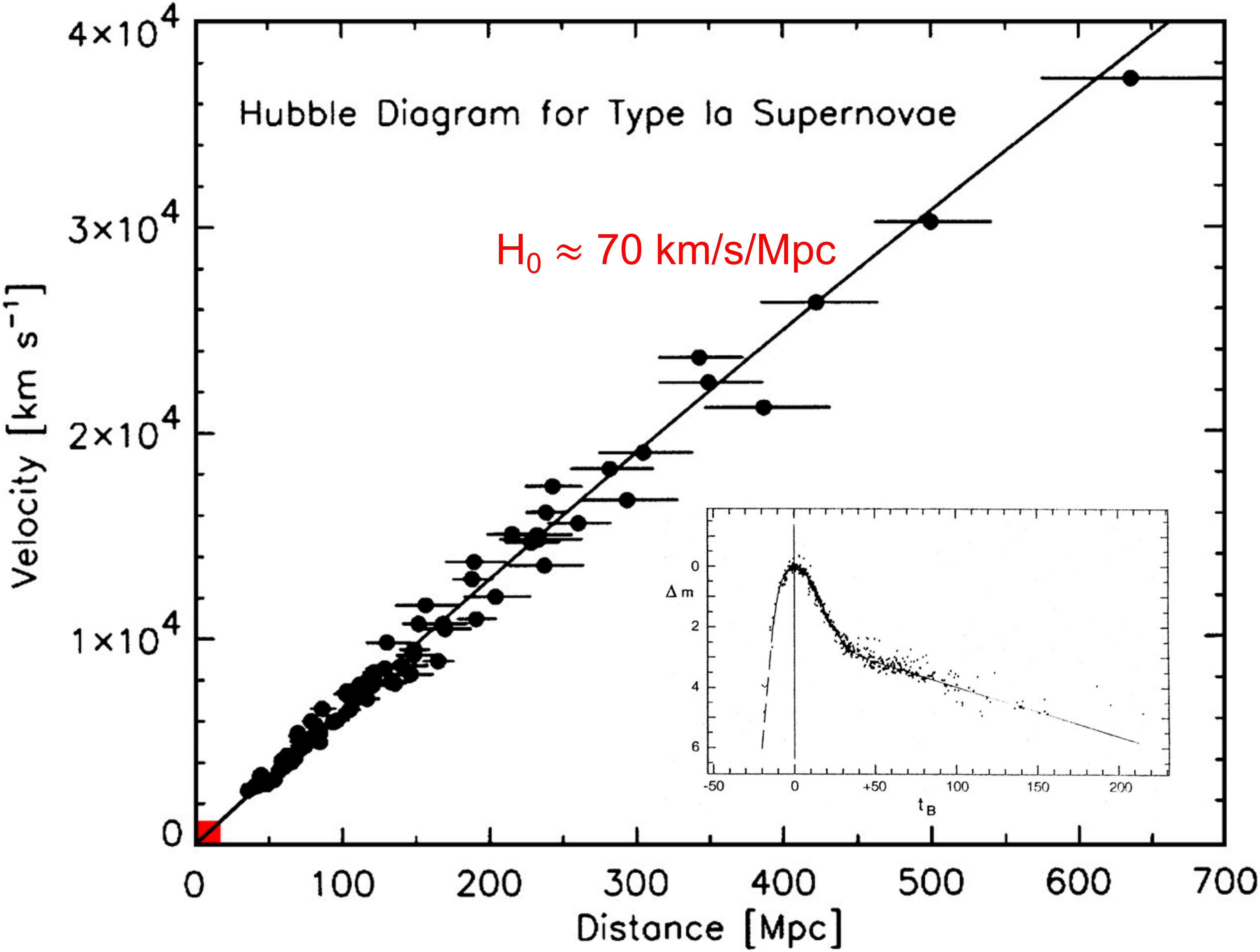
PHYS 431 Galactic Astrophysics

PHYS 432 Cosmology

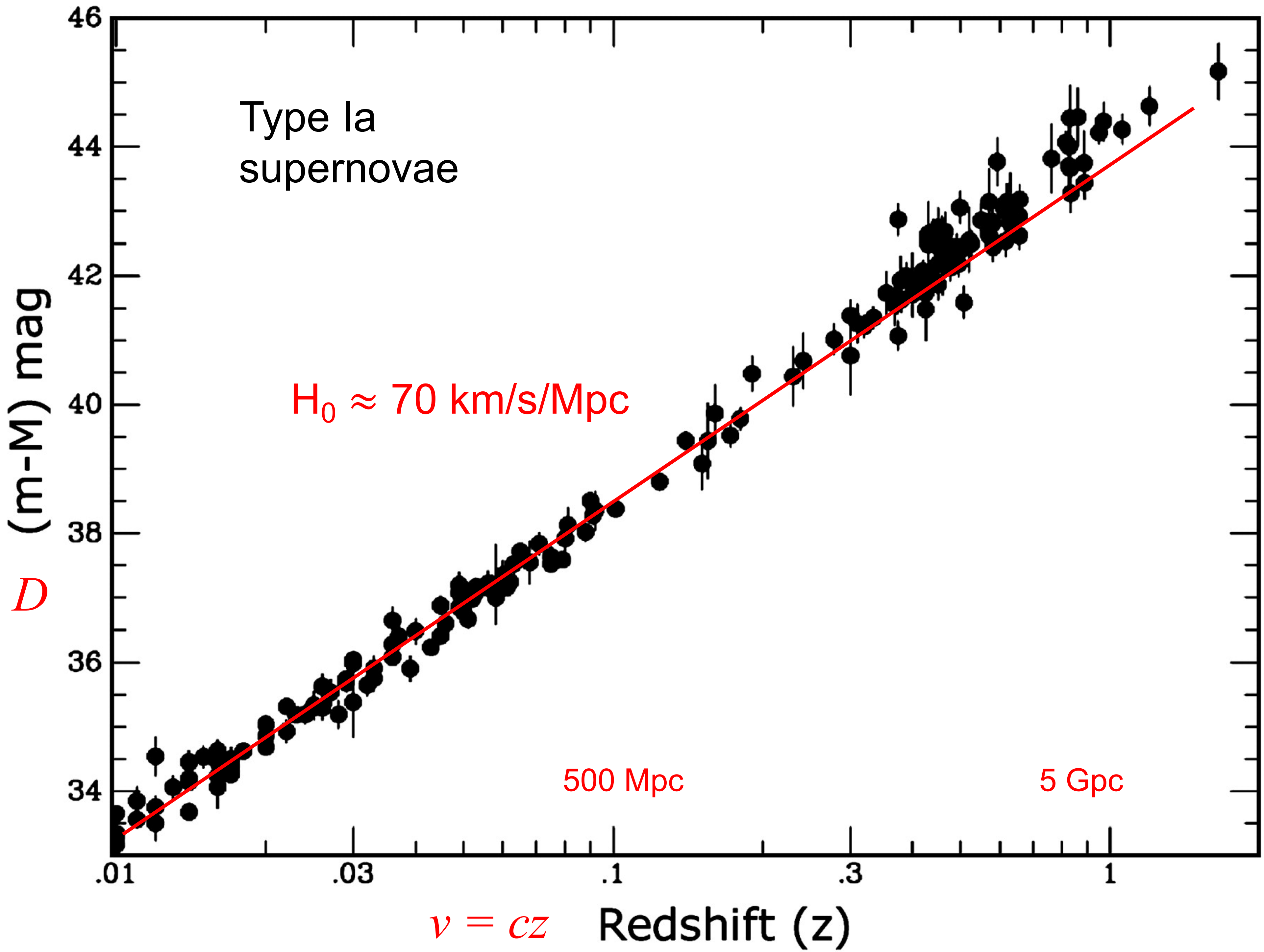

~1 Gpc
~200 Mpc
~25 Mpc
~10,000 pc
~200 pc
~1 A.U.
L
Time
Supernovae
Tully-Fisher
Time
Variable stars
OBAFGKM
Spectroscopic parallax
Stellar parallax
Radar ranging
Distance
Earth

# Hubble's Law

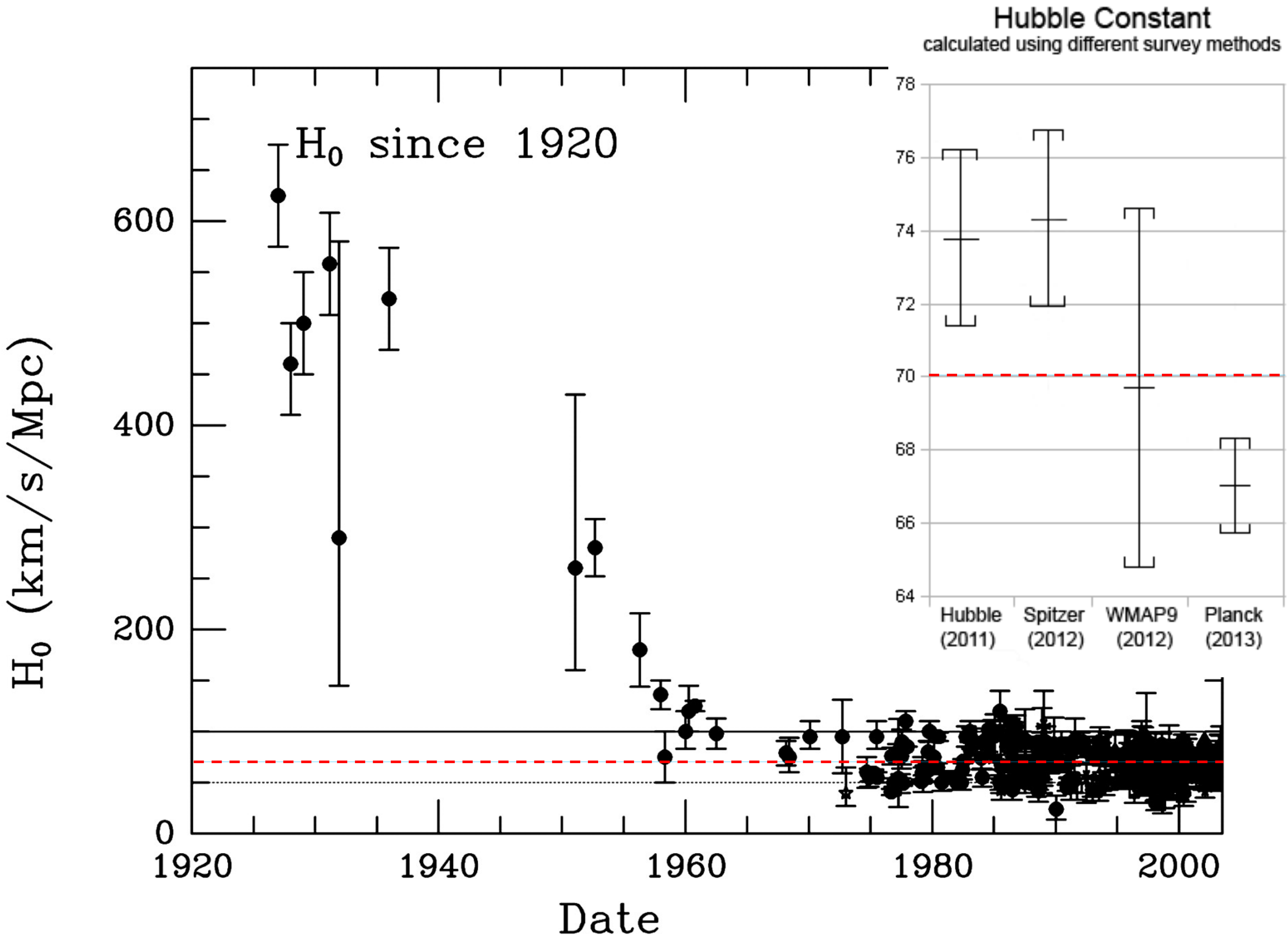




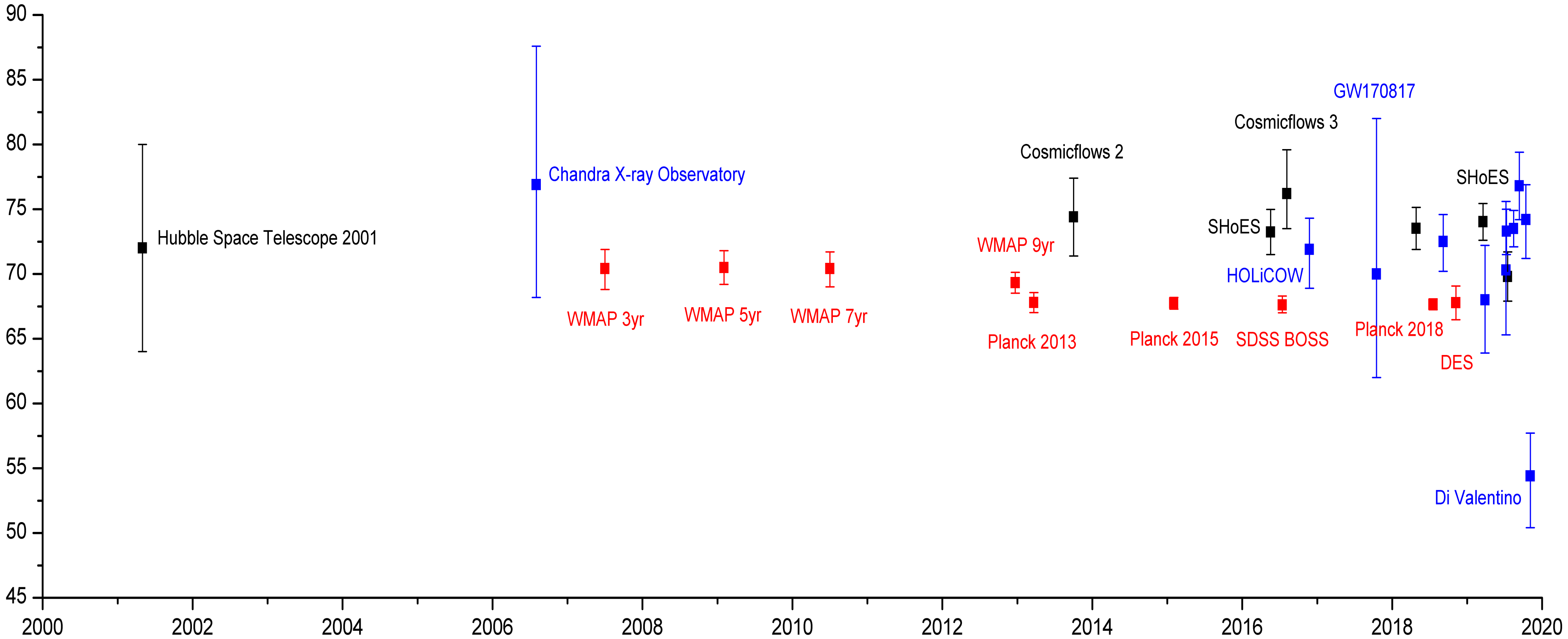
Hubble Diagram for Type Ia Supernovae
H0 ≈ 70 km/s/Mpc
Velocity [km s-1]
4×10^4
3×10^4
2×10^4
1×10^4
0
0
100
200
300
400
500
600
700
Distance [Mpc]
Δ m
0
2
4
6
-50
0
+50
100
150
200
tB



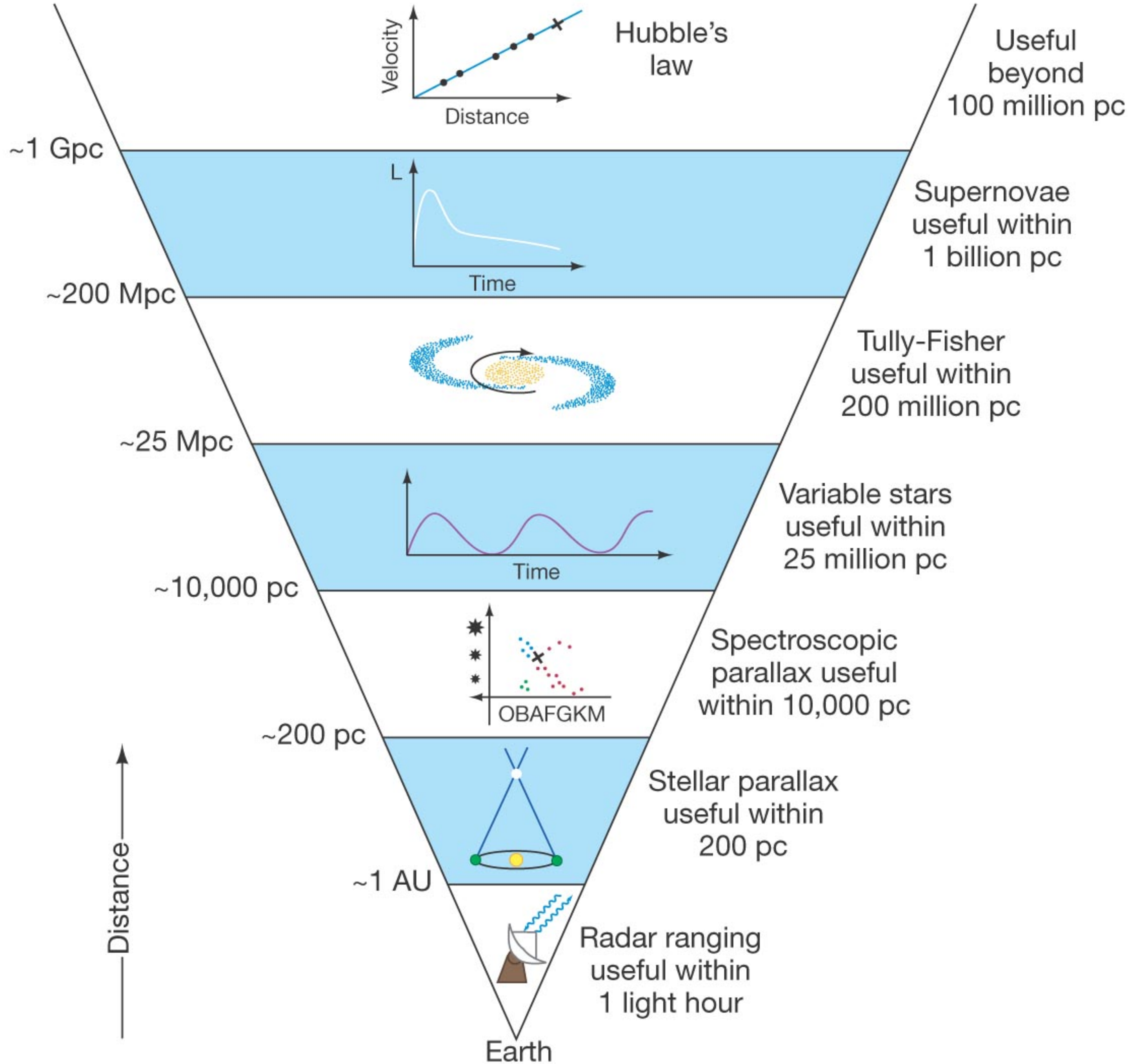
Type Ia supernovae
H0 ≈ 70 km/s/Mpc
500 Mpc
5 Gpc
(m-M) mag
D
v = cz
Redshift (z)
46
44
42
40
38
36
34
.01
.03
.1
.3
1

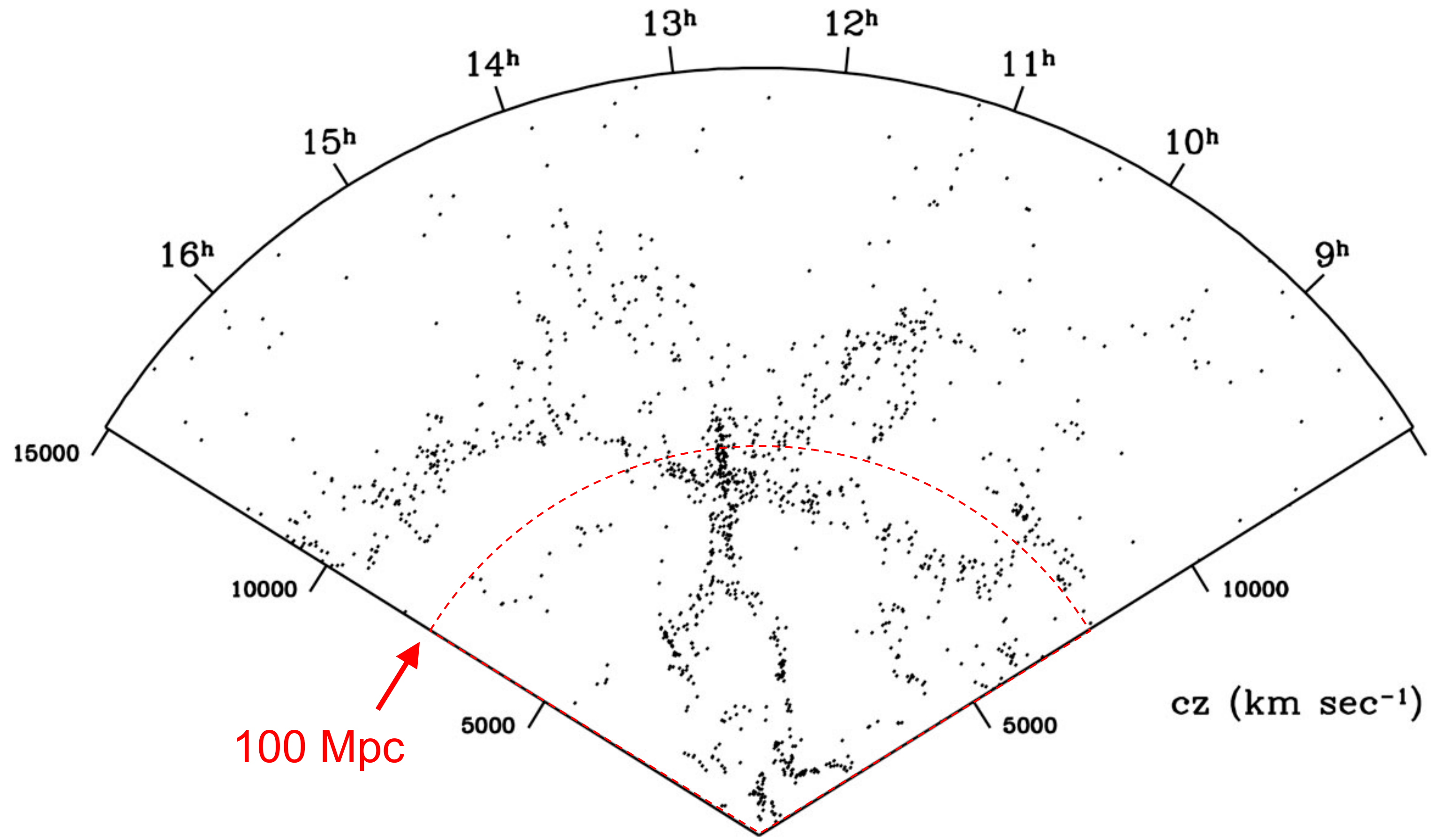


$H_0$ since 1920
$H_0$ (km/s/Mpc)
600
400
200
0
1920
1940
1960
1980
2000
Date
Hubble Constant
calculated using different survey methods
78
76
74
72
70
68
66
64
Hubble (2011)
Spitzer (2012)
WMAP9 (2012)
Planck (2013)

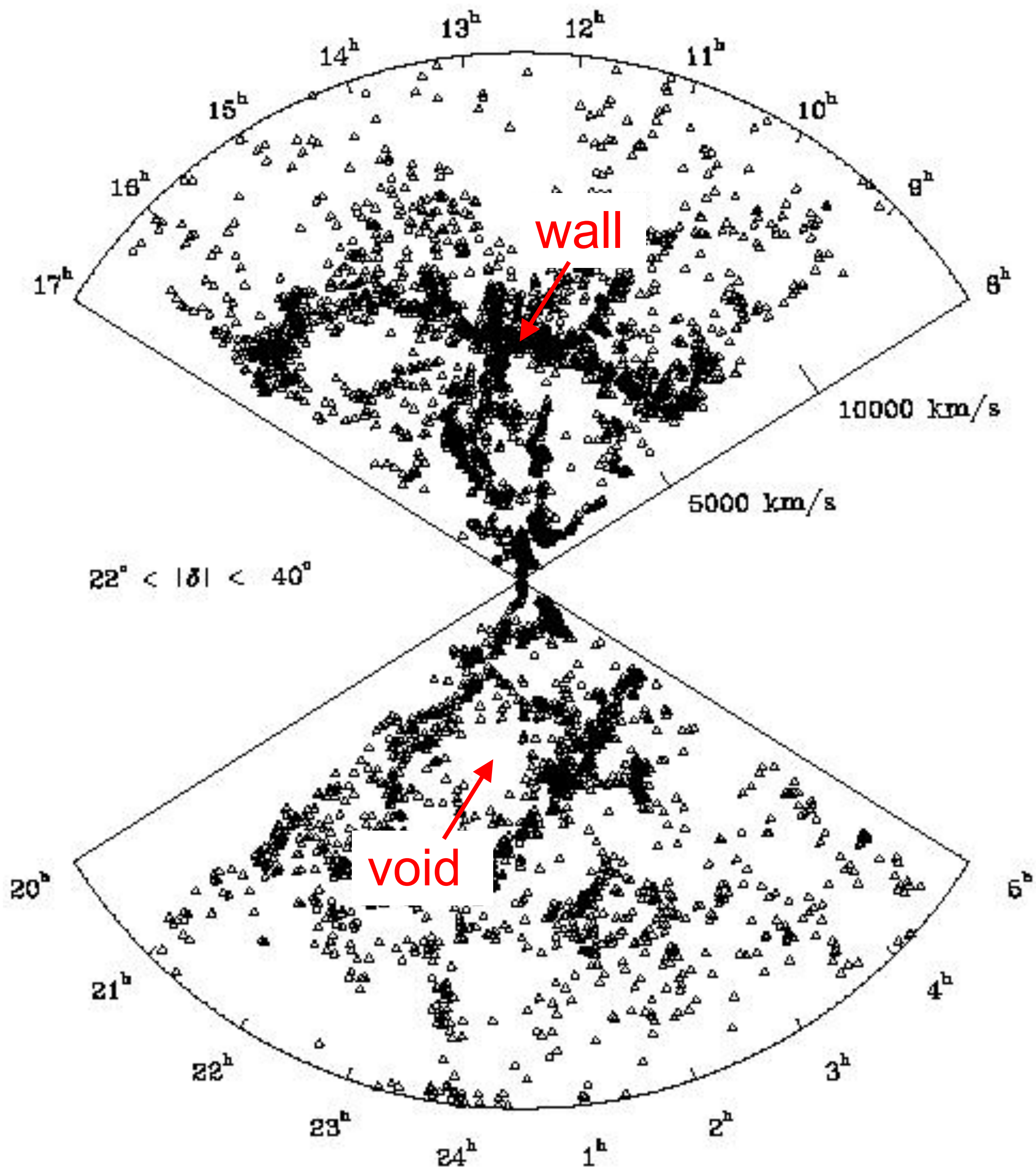


Hubble Space Telescope 2001
Chandra X-ray Observatory
WMAP 3yr
WMAP 5yr
WMAP 7yr
WMAP 9yr
Planck 2013
Cosmicflows 2
Planck 2015
SHoES
HOLiCOW
Cosmicflows 3
SDSS BOSS
GW170817
Planck 2018
DES
SHoES
Di Valentino
90
85
80
75
70
65
60
55
50
45
2000
2002
2004
2006
2008
2010
2012
2014
2016
2018
2020

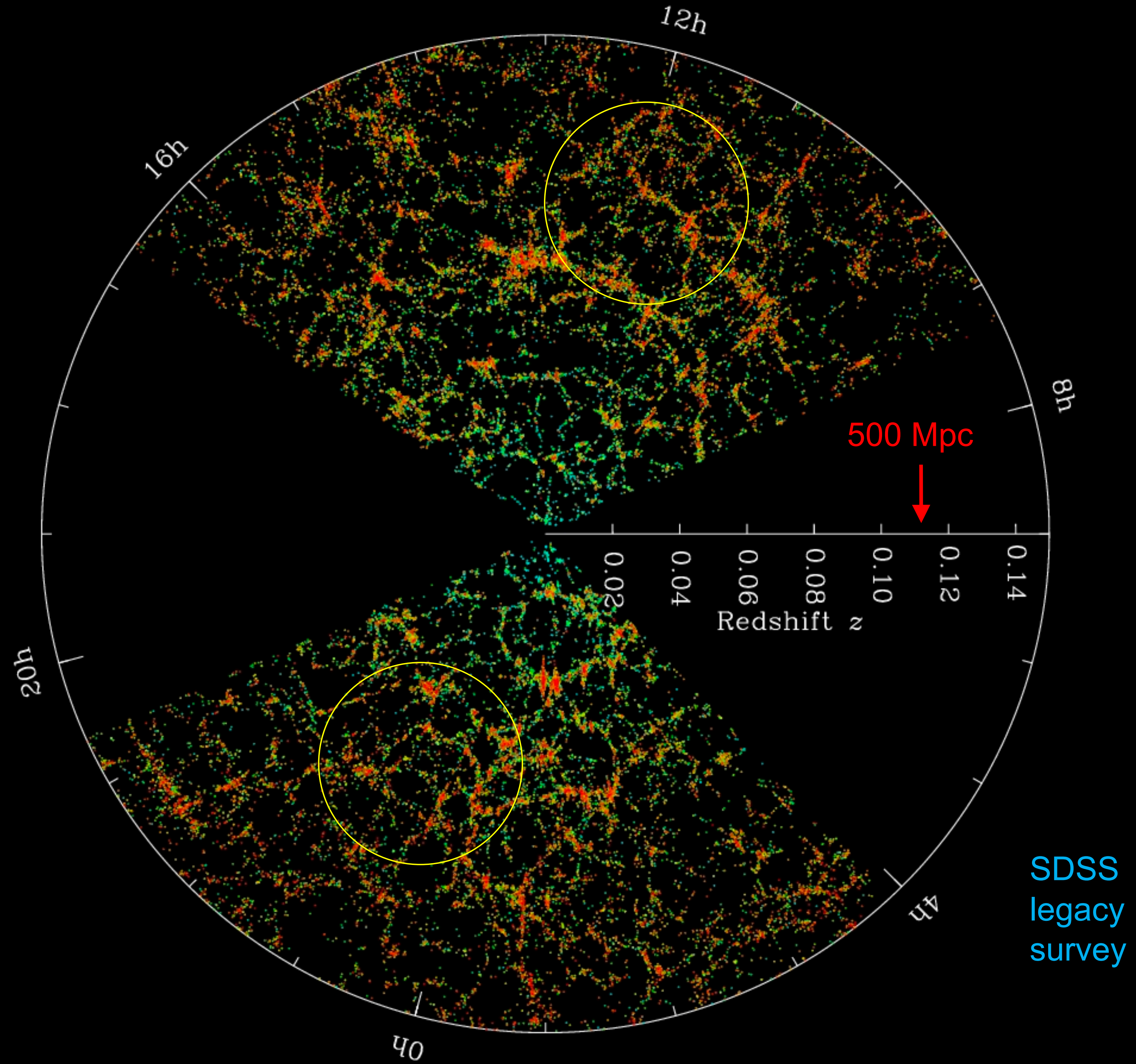

Velocity
Distance
Hubble's law
Useful beyond 100 million pc
~1 Gpc
L
Time
Supernovae useful within 1 billion pc
~200 Mpc
Tully-Fisher useful within 200 million pc
~25 Mpc
Time
Variable stars useful within 25 million pc
~10,000 pc
OBAFGKM
Spectroscopic parallax useful within 10,000 pc
~200 pc
Stellar parallax useful within 200 pc
~1 AU
Radar ranging useful within 1 light hour
Earth
Distance



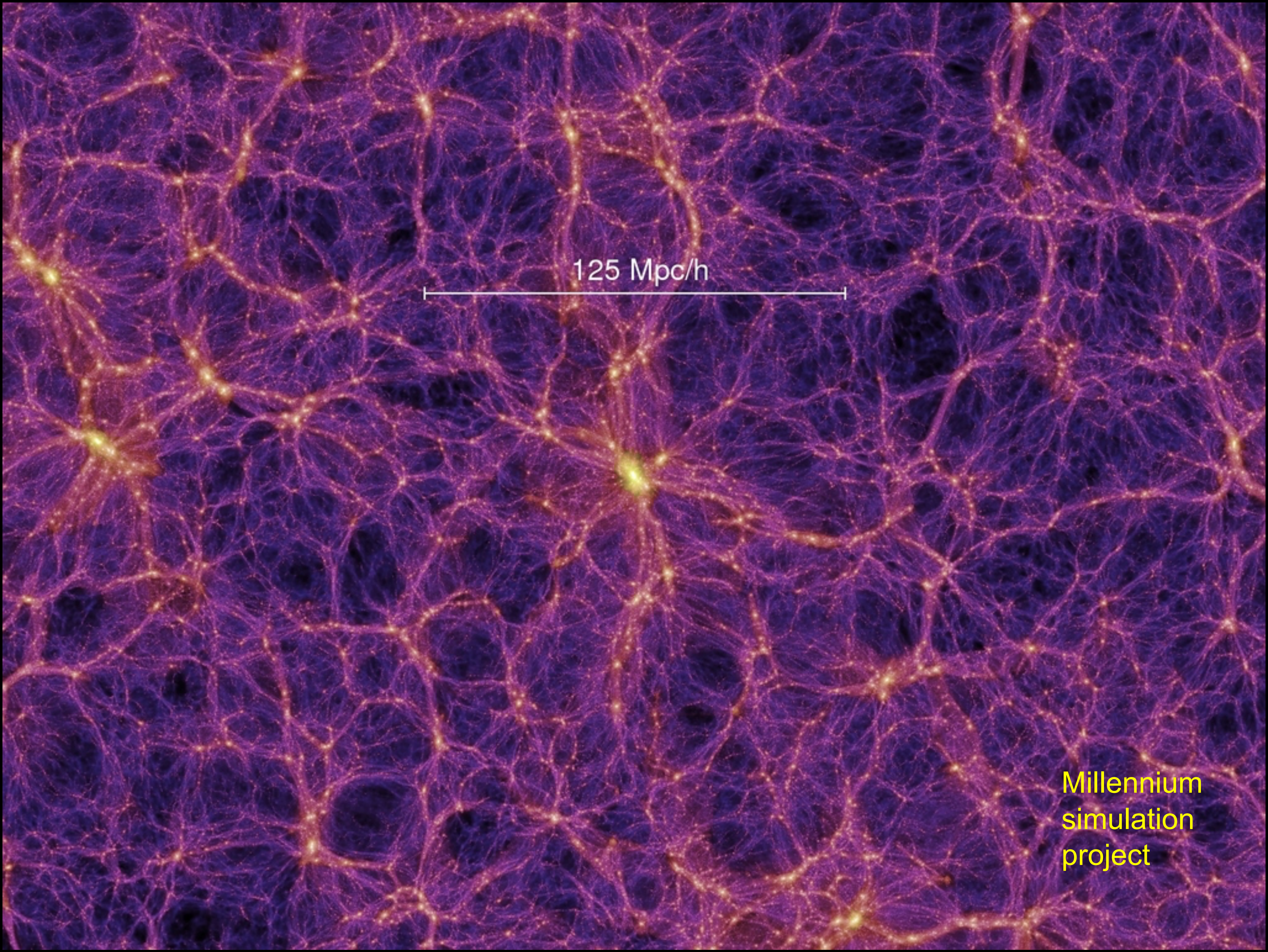

CfA
redshift
survey

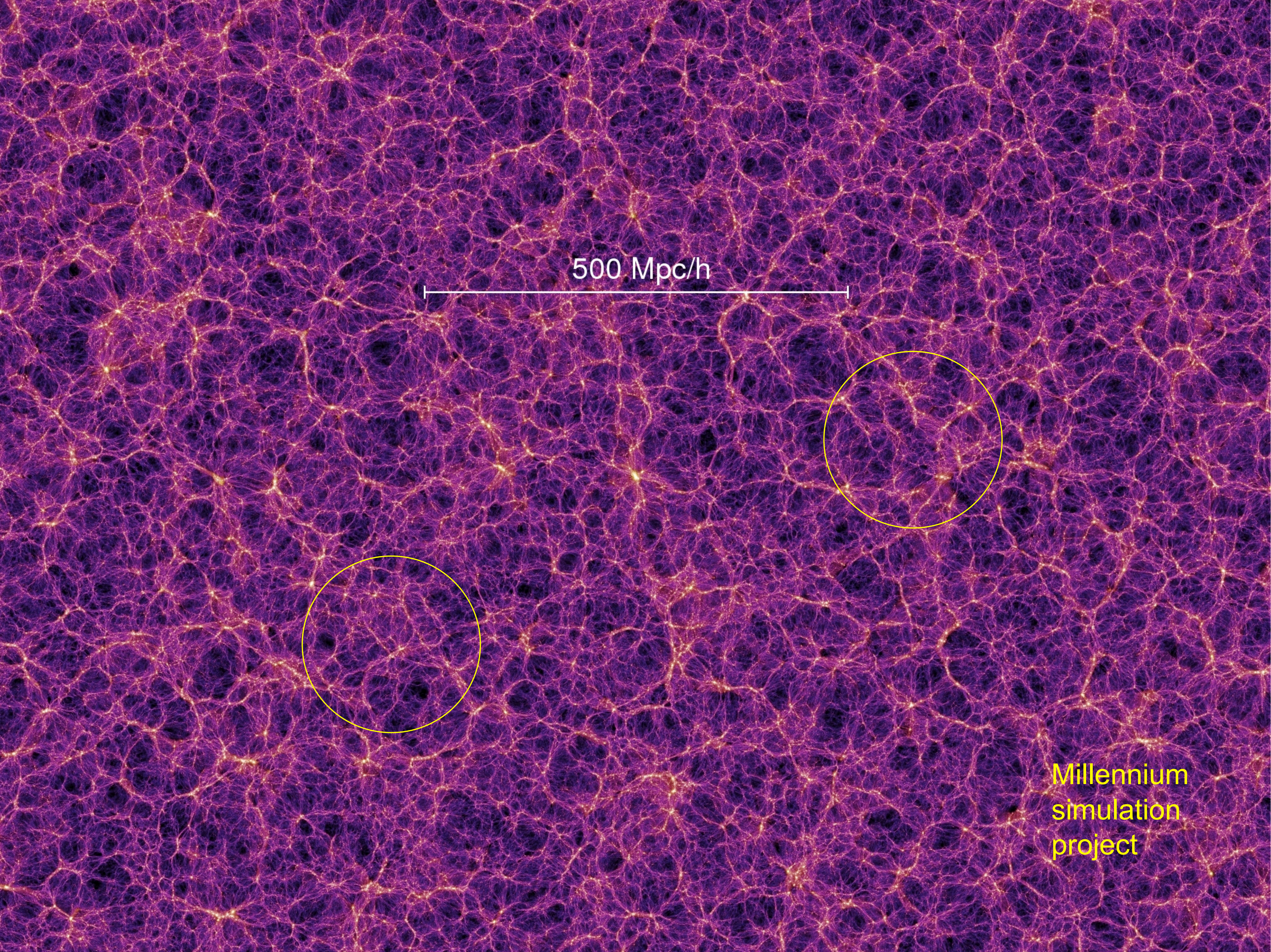



CfA redshift survey



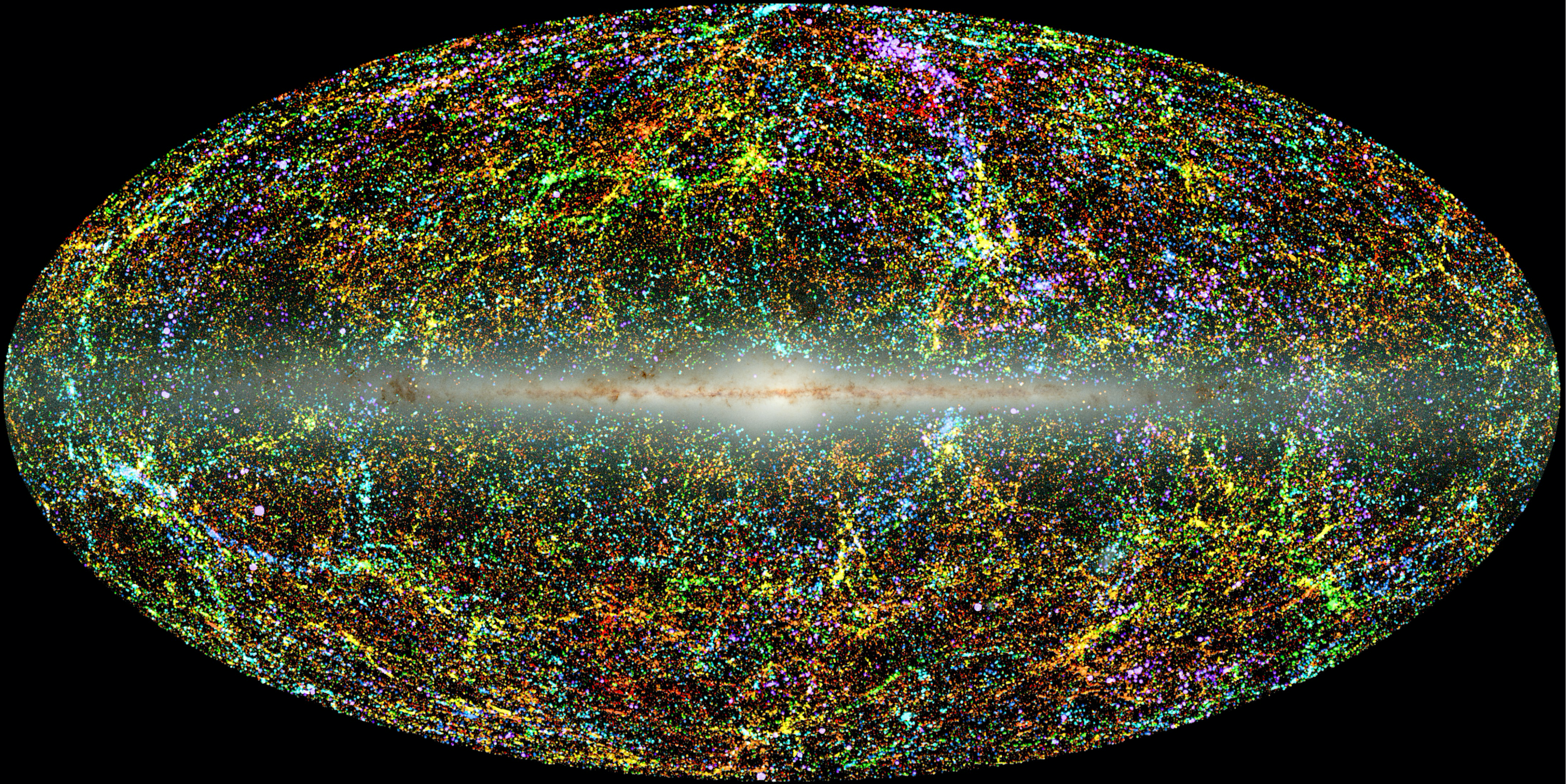
12h
16h
8h
20h
4h
0h
500 Mpc
0.02
0.04
0.06
0.08
0.10
0.12
0.14
Redshift z
SDSS
legacy
survey

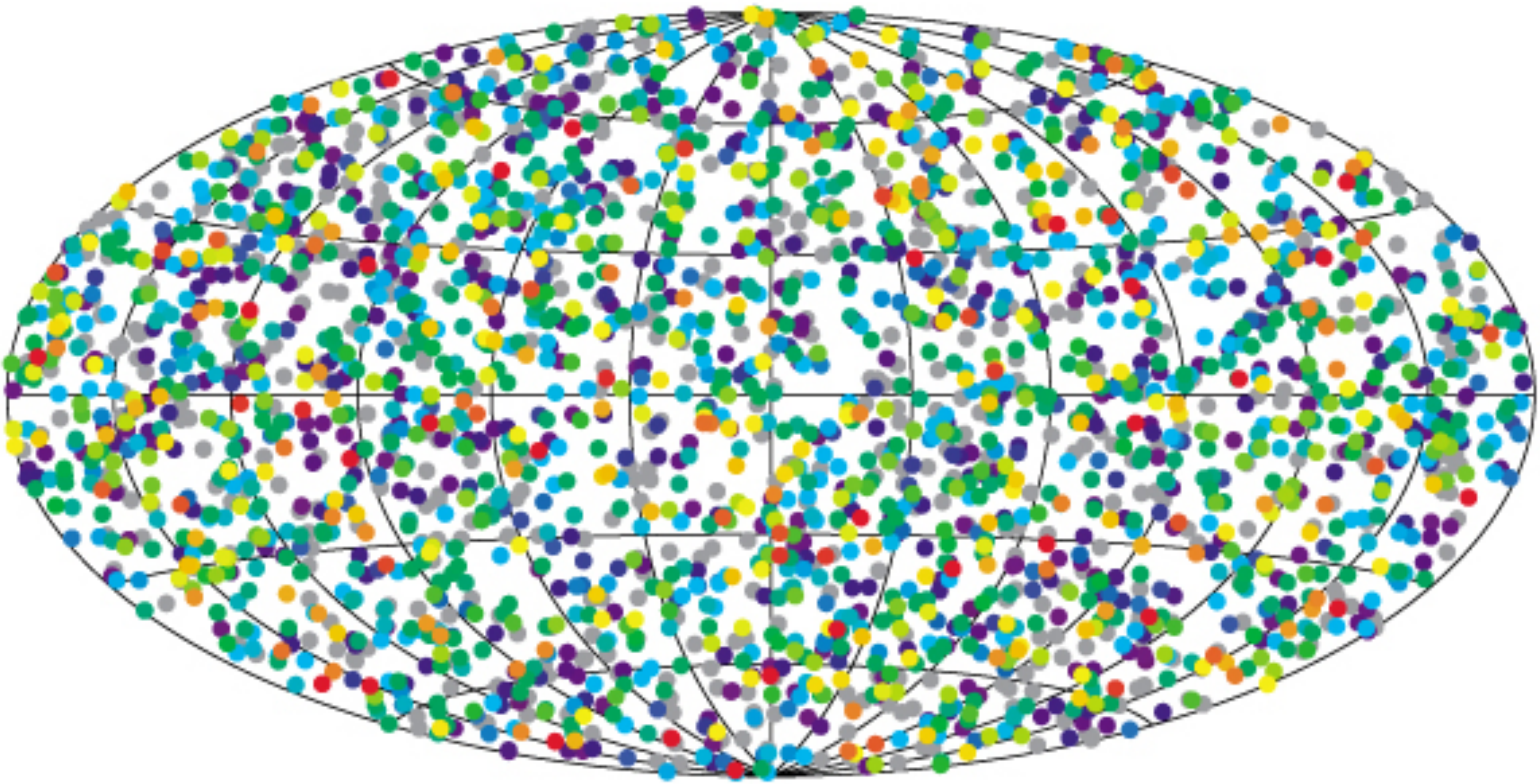

125 Mpc/h
Millennium
simulation
project


500 Mpc/h
Millennium
simulation
project

# Cosmological Principle

- on very large ("cosmological") scales, the universe is
  - homogeneous
  - isotropic

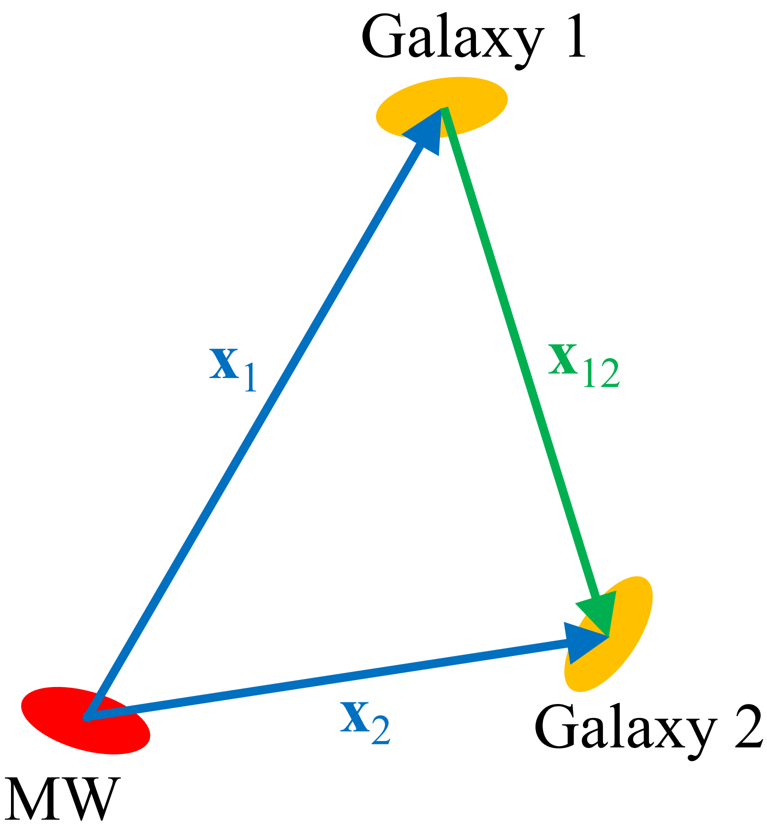




# Cosmological Principle

- on very large (“cosmological”) scales, the universe is
  - homogeneous
  - isotropic

- “very large” means $\gg$ 100 Mpc




Hubble's Law:

$$\mathbf{v}_1 = H_0\,\mathbf{x}_1$$

$$\mathbf{v}_2 = H_0\,\mathbf{x}_2$$

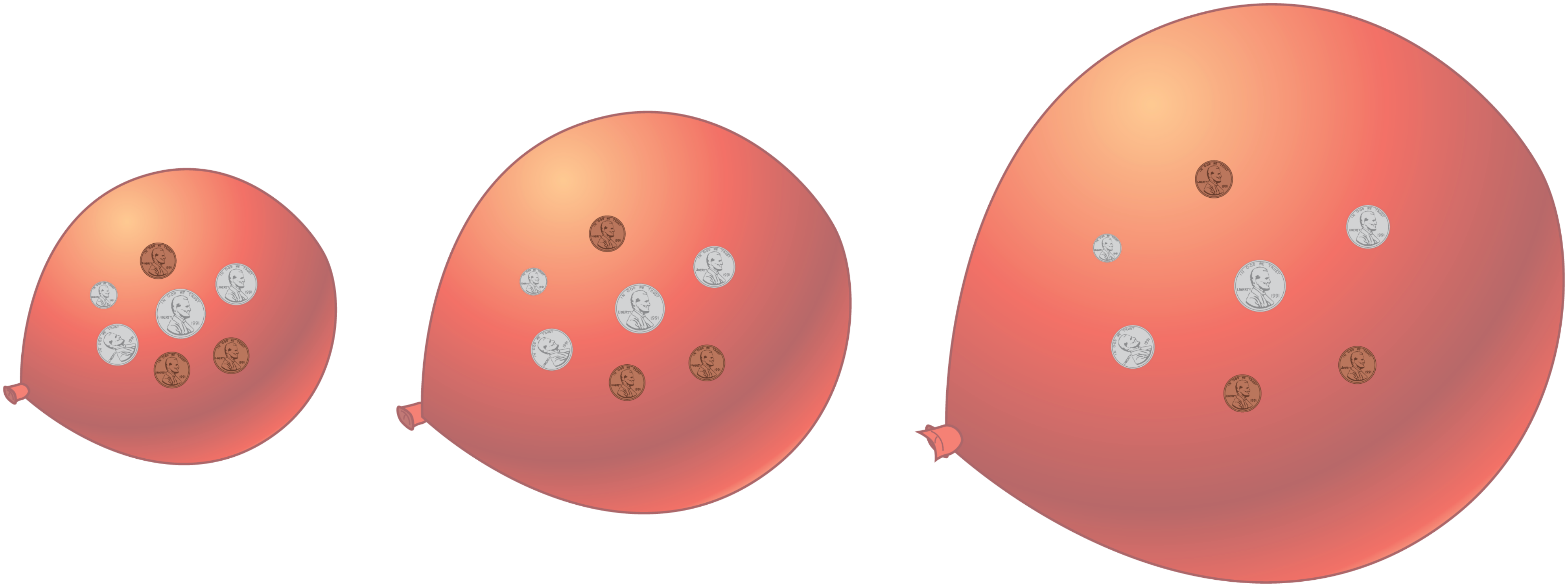
From Galaxy 1:

$$\mathbf{x}_{12} = \mathbf{x}_2 - \mathbf{x}_1$$

$$\begin{aligned}\mathbf{v}_{12} &= \mathbf{v}_2 - \mathbf{v}_1 \\ &= H_0\,\mathbf{x}_2 - H_0\,\mathbf{x}_1 \\ &= H_0\,(\mathbf{x}_2 - \mathbf{x}_1) \\ &= H_0\,\mathbf{x}_{12}\end{aligned}$$

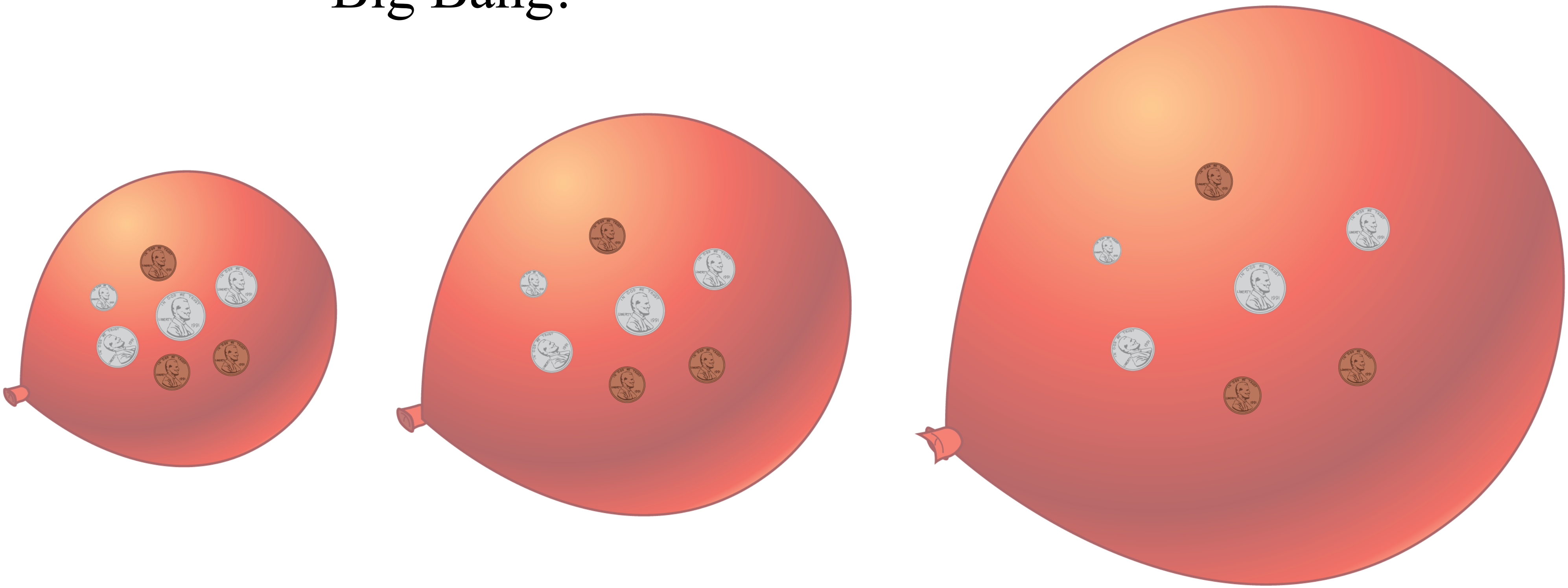
# Hubble Expansion

- is there an “edge” to the expansion?
- NO – expansion is a stretching of space, not the motion of galaxies through space

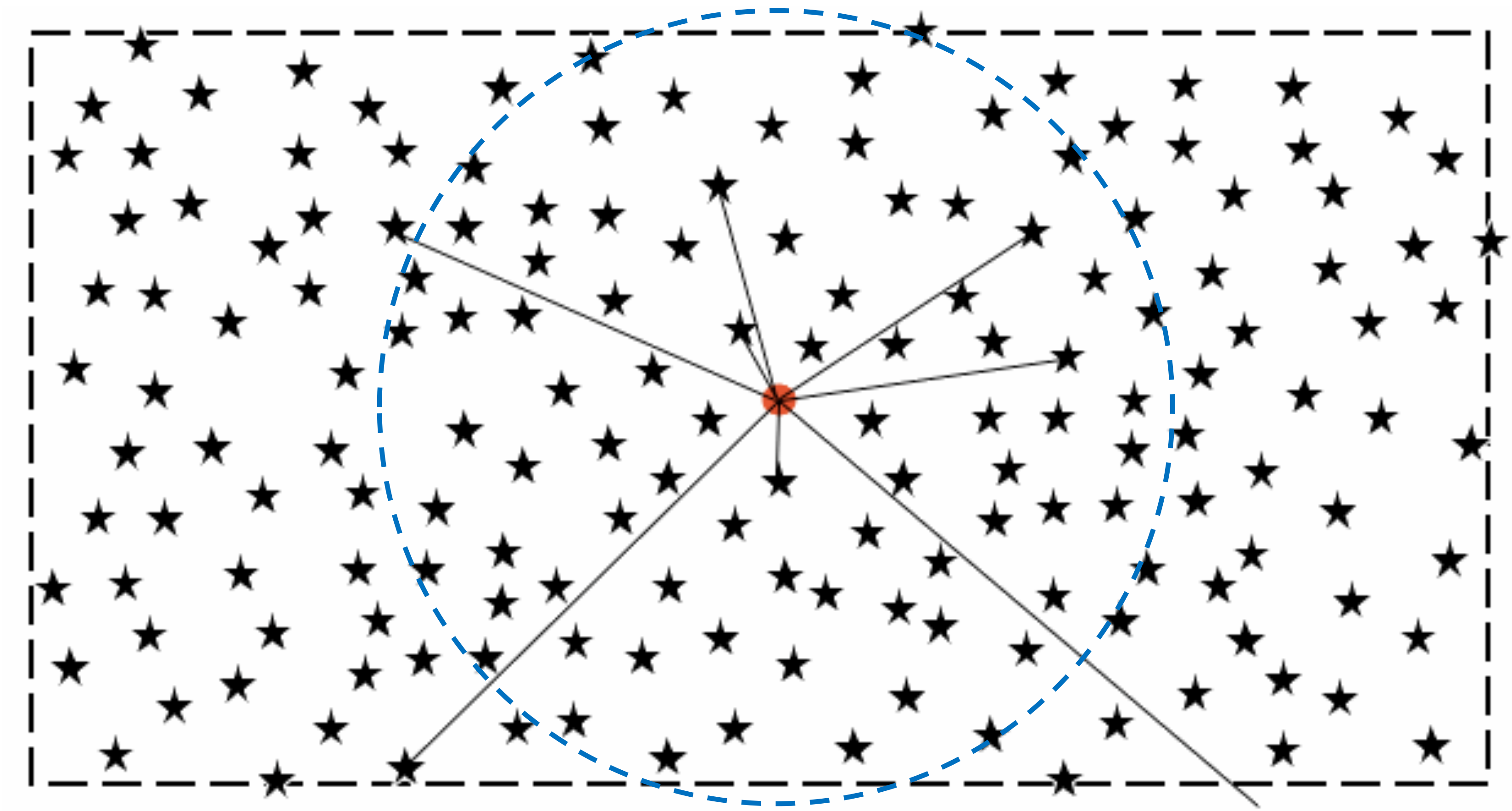


# The Big Bang

- $V = H_0 D \implies t = D/V = H_0^{-1}$
  - <u>same</u> for all galaxies
  - Big Bang!



# Olbers Paradox



Resolution: the visible universe is finite

the universe is not infinitely old

# Age of the Universe

- oldest globular clusters: 10–12 Gyr
- oldest white dwarfs: 10 Gyr
- radioactive decay of $^{235}$U and $^{238}$U: 10 Gyr

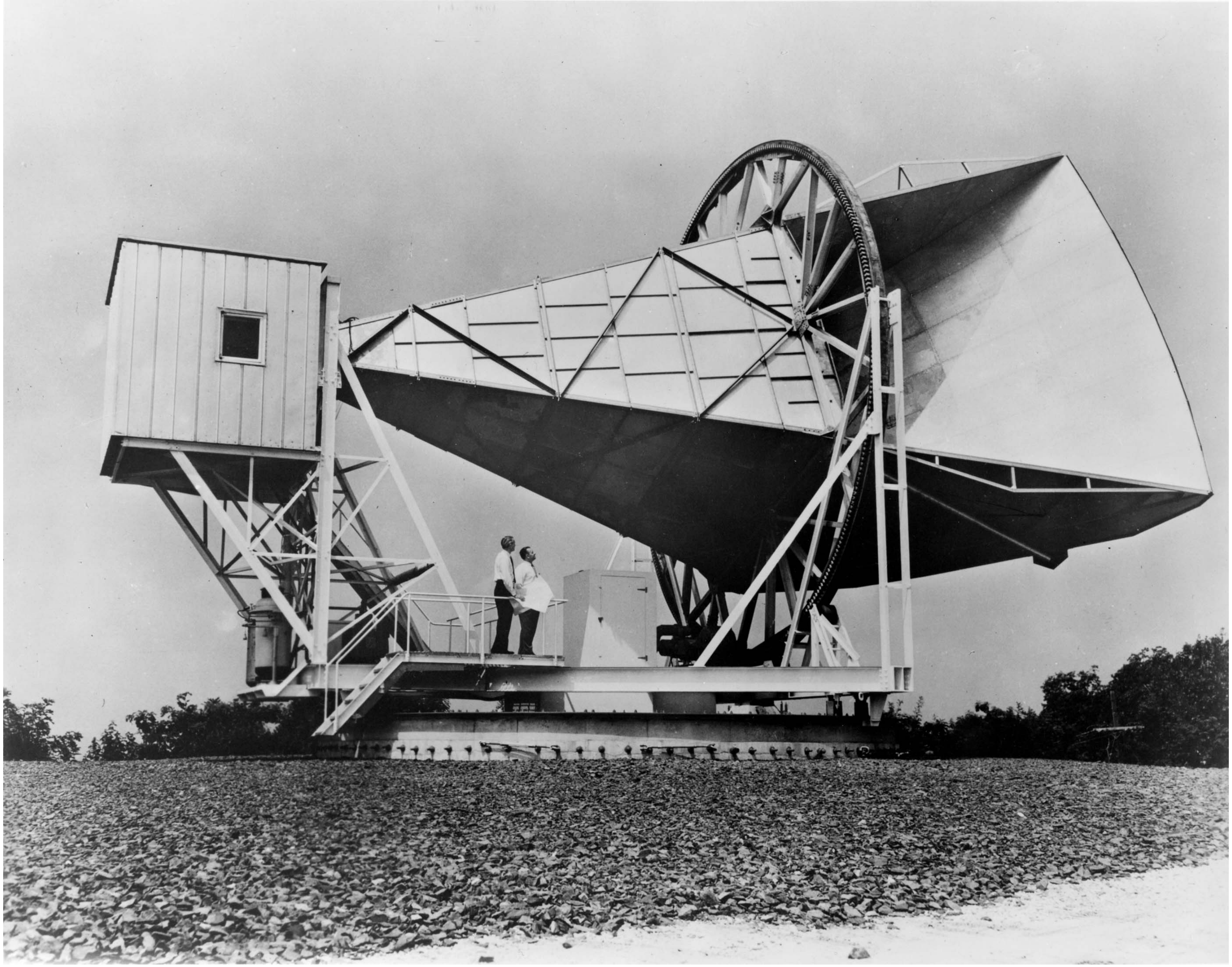
$$\left.\frac{N\left({}^{235}U\right)}{N\left({}^{238}U\right)}\right|_{obs} = \left.\frac{N\left({}^{235}U\right)}{N\left({}^{238}U\right)}\right|_{SN} e^{\left(\frac{1}{\tau_{238}}-\frac{1}{\tau_{235}}\right)t}$$

- inverse Hubble constant: $H_0^{-1} = 13.9$ Gyr

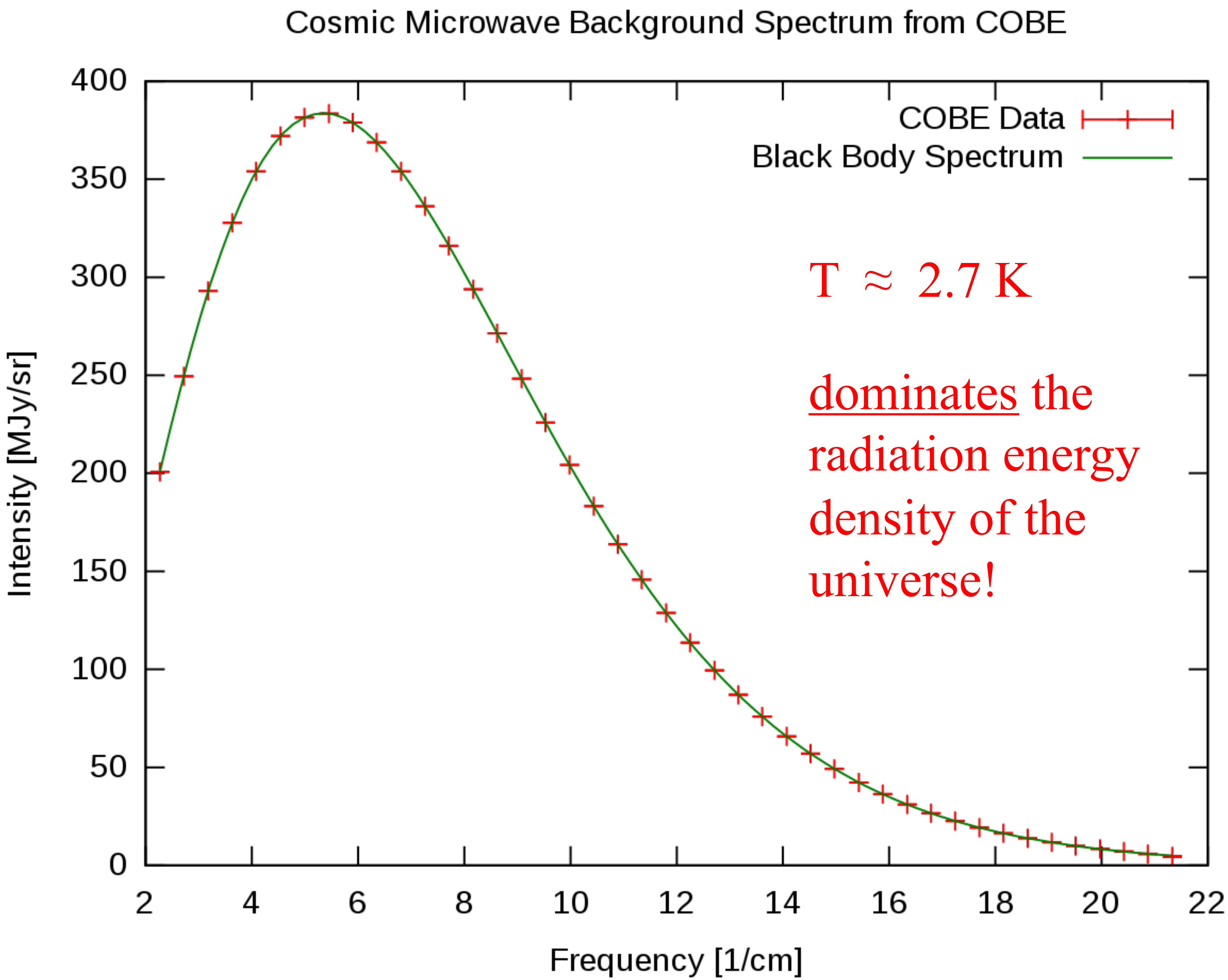
# Cosmic Scale Factor

- cosmological principle means that the relative expansion of the distance between any two galaxies must be the same
- everyone sees the same Hubble constant
- expansion is described by a single quantity $R(t)$
  - scale factor of the universe
  - defines past and future cosmic expansion
  - $H = \dot{R}/R$

# The Hot Big Bang



# The Hot Big Bang

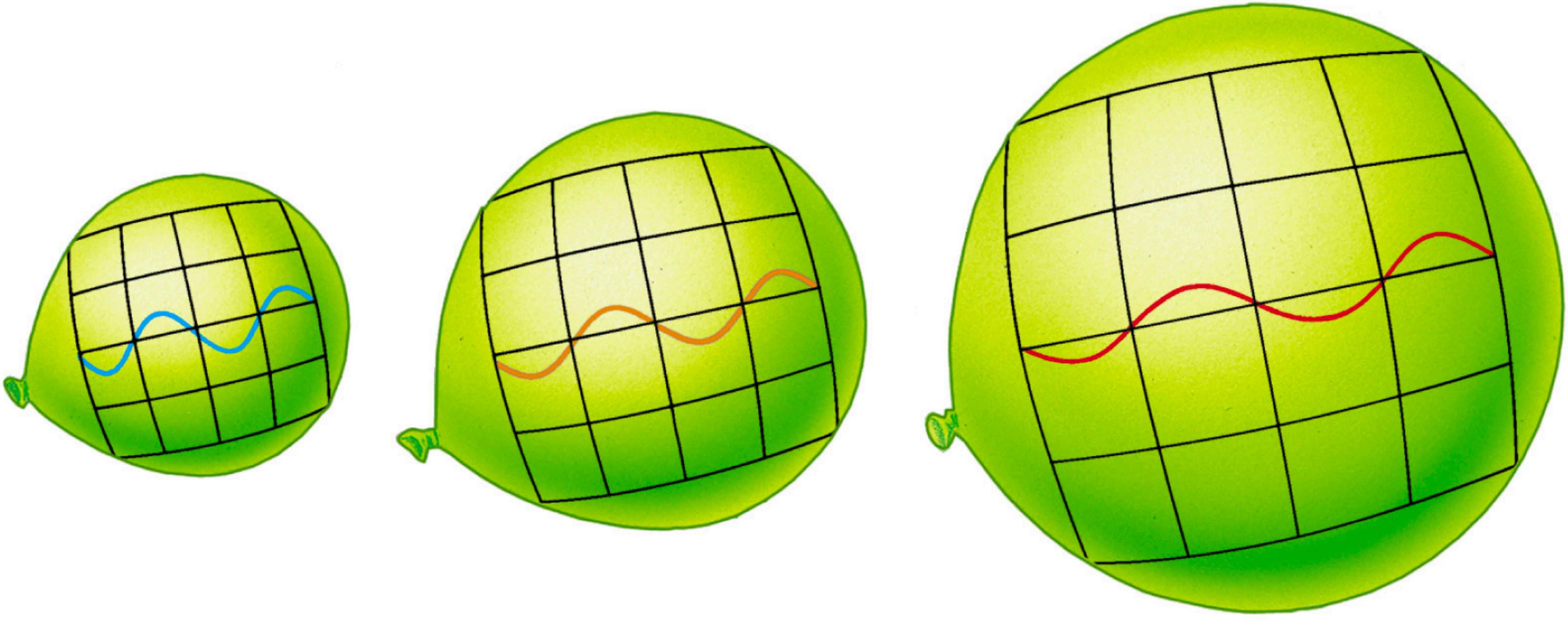



$T \approx 2.7$ K

<u>dominates</u> the radiation energy density of the universe!

# Cosmological Redshift

$$\frac{\lambda_0}{\lambda_e} = \frac{R(t_0)}{R(t_e)} = 1 + z$$



# The Hot Big Bang

- matter density: $\rho_{matter} \propto R^{-3}$
- radiation density: $\rho_{radiation} \propto R^{-4}$
- radiation temperature: $T_{radiation} \propto R^{-1}$

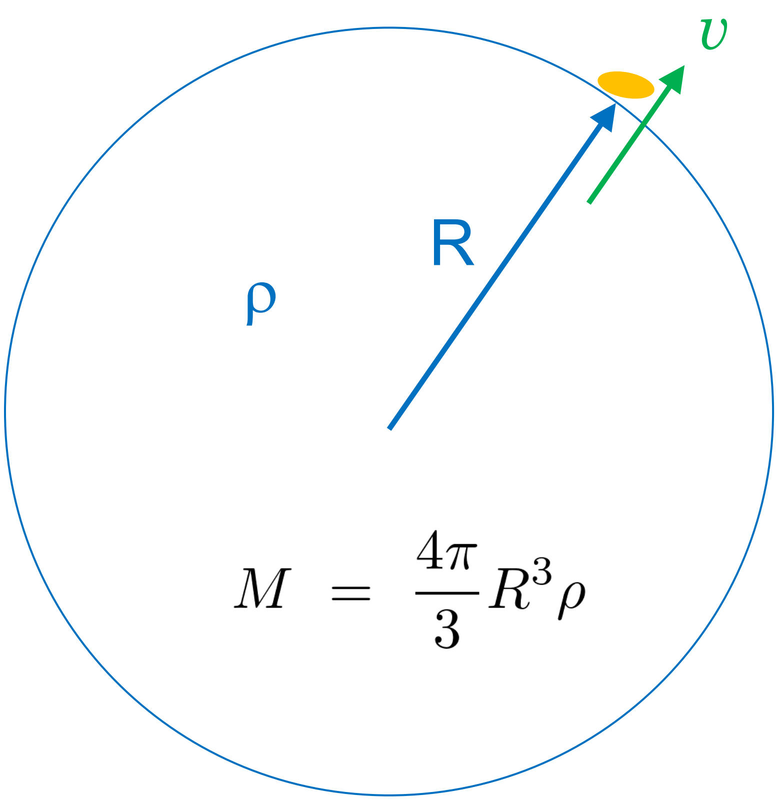
- early universe was hot and dense
- earliest universe was radiation dominated

# Friedmann Equations (Newton)

$$v = \dot{R} = \frac{dR}{dt}$$

$$\tfrac{1}{2}\dot{R}^2 - \frac{GM}{R} = E$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho + \frac{2E}{R^2}$$




$$M = \frac{4\pi}{3}R^3\rho$$

$$\ddot{R} = -\frac{GM}{R^2} = -\frac{4\pi}{3}GR\rho$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3}G\rho$$

$$\dot{\rho}c^2V + \rho c^2\dot{V} = P\dot{V}$$

$$\dot{\rho}c^2 = -\frac{\dot{V}}{V}\left(\rho c^2 + P\right)$$

$$dU = P\,dV$$

$$U = \rho c^2 V$$

$$\dot{\rho}c^2 = -3\frac{\dot{R}}{R}\left(\rho c^2 + P\right)$$

# Friedmann Equations (GR)

$$ds^2 = c^2 dt^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

$R$ = scale factor of the universe

$k \sim$ curvature of the universe

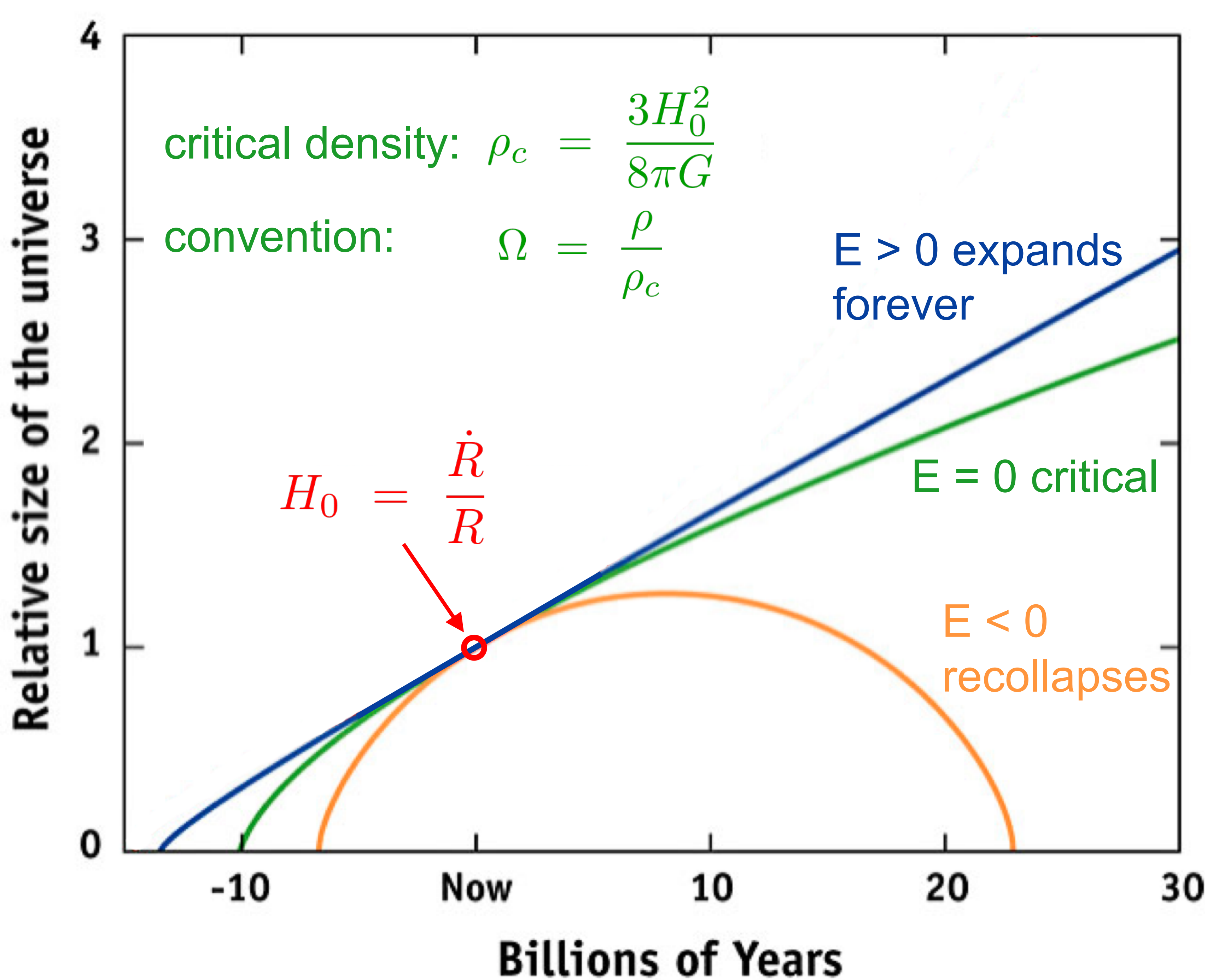
$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}$$

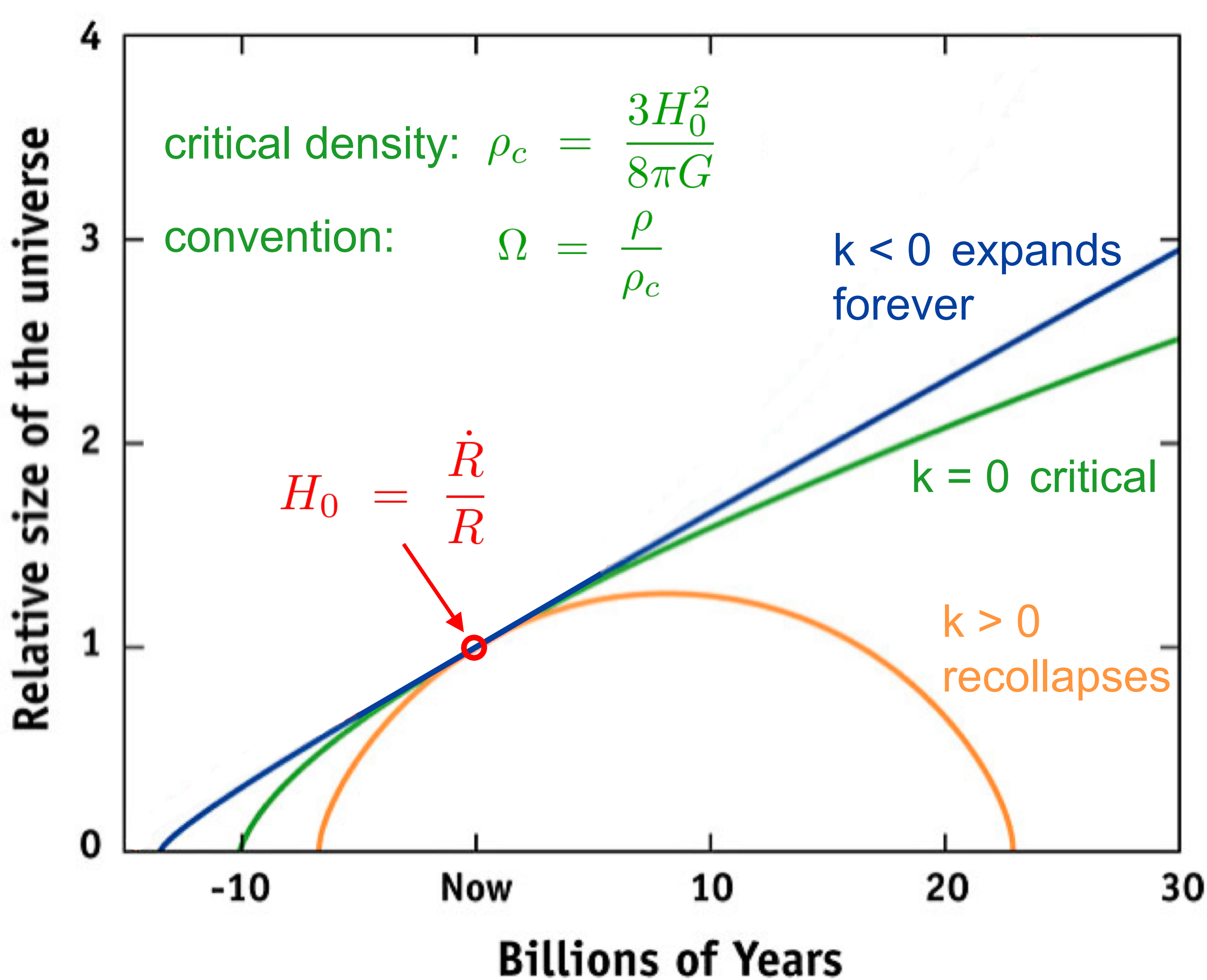
geometry affects dynamics

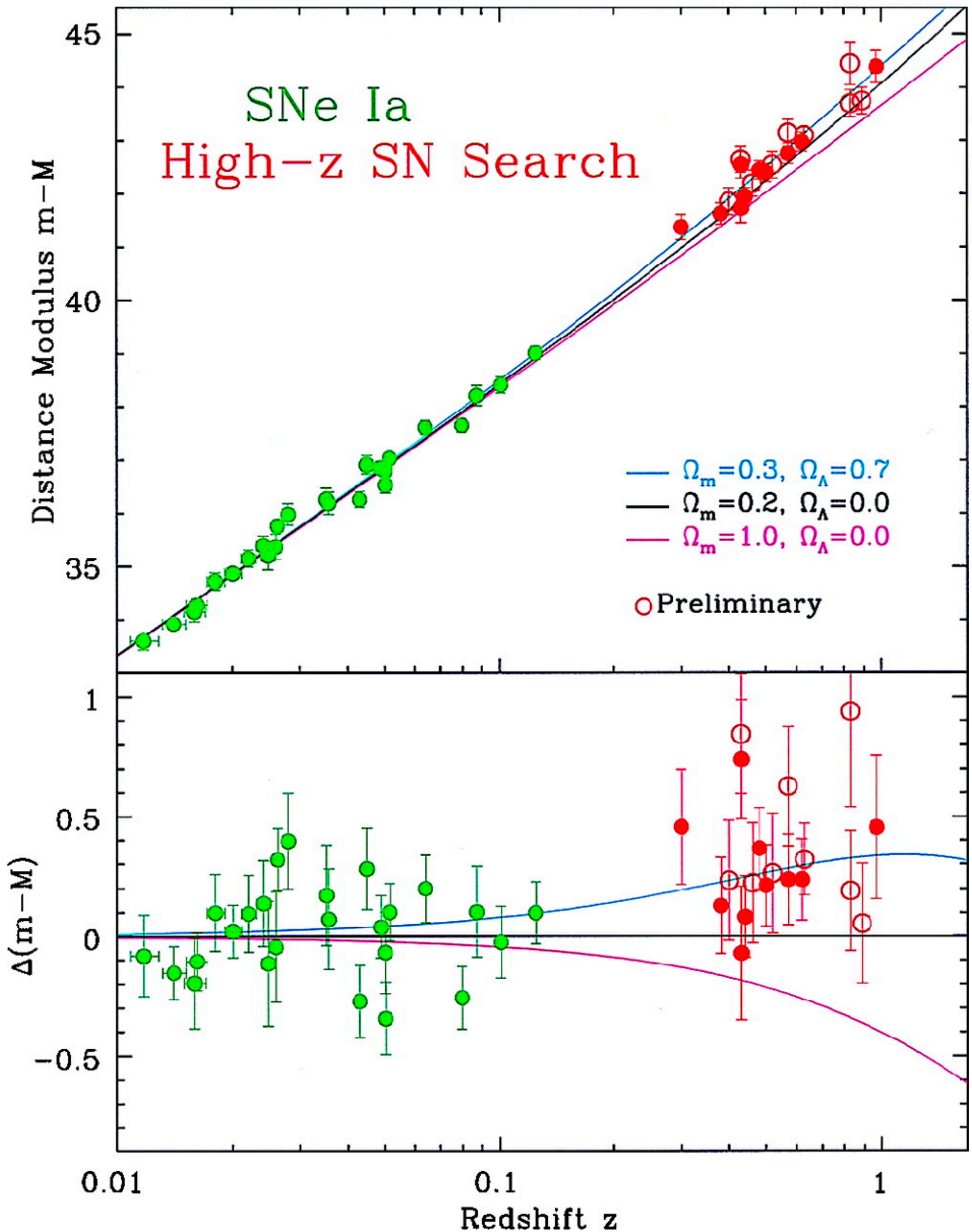
$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3}G\left(\rho + \frac{3P}{c^2}\right)$$

pressure gravitates!

$$\dot{\rho}c^2 = -3\frac{\dot{R}}{R}\left(\rho c^2 + P\right)$$


critical density: $\rho_c = \frac{3H_0^2}{8\pi G}$
convention: $\Omega = \frac{\rho}{\rho_c}$
$H_0 = \frac{\dot{R}}{R}$
E > 0 expands forever
E = 0 critical
E < 0 recollapses
Relative size of the universe
Billions of Years
0
1
2
3
4
-10
Now
10
20
30


critical density: $\rho_c = \frac{3H_0^2}{8\pi G}$
convention: $\Omega = \frac{\rho}{\rho_c}$
$H_0 = \frac{\dot{R}}{R}$
k < 0 expands forever
k = 0 critical
k > 0 recollapses
Relative size of the universe
0
1
2
3
4
-10
Now
10
20
30
Billions of Years


SNe Ia
High−z SN Search
Ωm=0.3, ΩΛ=0.7
Ωm=0.2, ΩΛ=0.0
Ωm=1.0, ΩΛ=0.0
Preliminary
Distance Modulus m−M
45
40
35
Δ(m−M)
1
0.5
0
−0.5
0.01
0.1
1
Redshift z

Dark energy!

# Friedmann Equations (GR)

$$ds^2 = c^2 dt^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

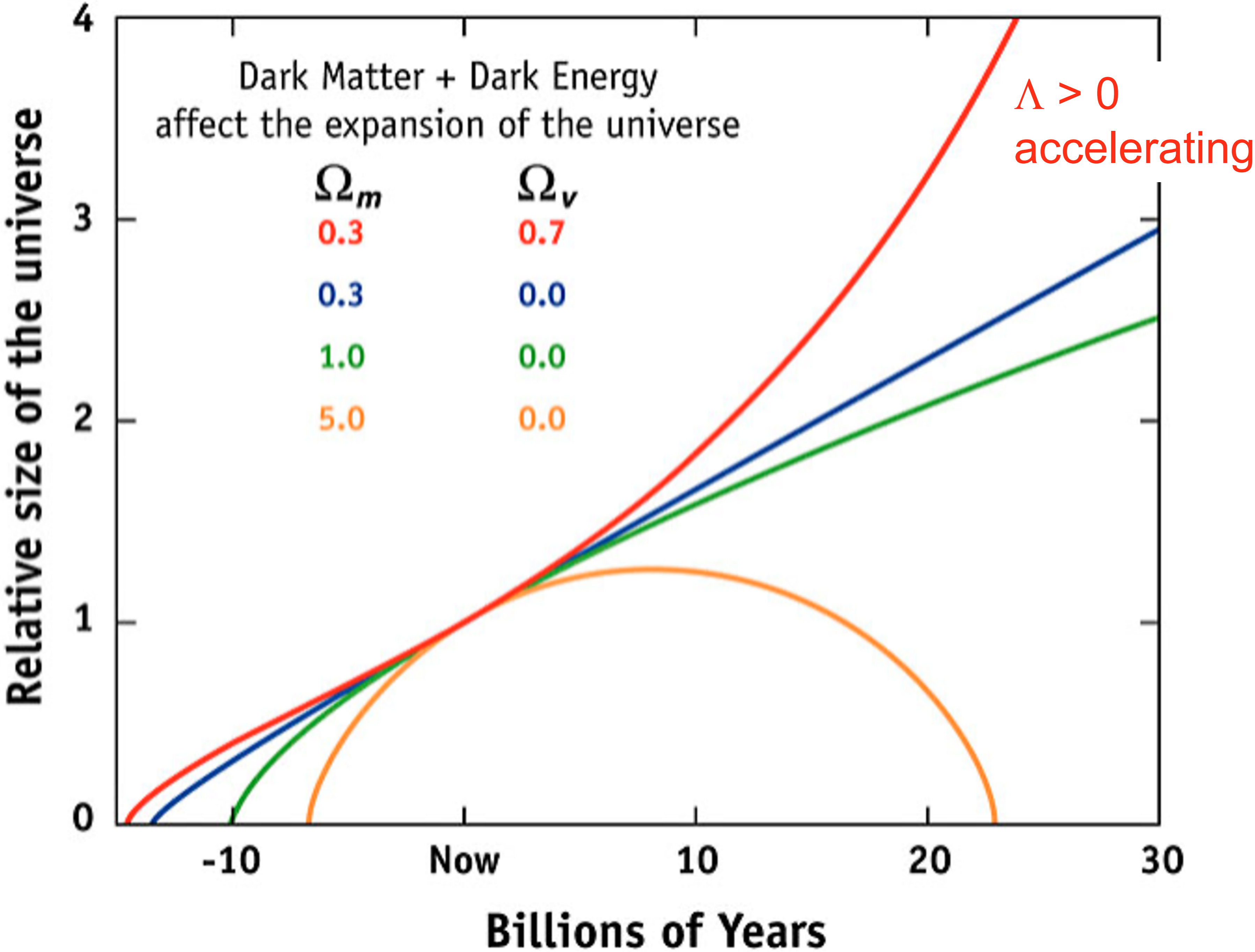
$R$ = scale factor of the universe

$k \sim$ curvature of the universe

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3}G\left(\rho + \frac{3P}{c^2}\right) + \frac{\Lambda}{3}$$

$$\dot{\rho}c^2 = -3\frac{\dot{R}}{R}\left(\rho c^2 + P\right)$$



Dark Matter + Dark Energy
affect the expansion of the universe
Ωm Ωv
0.3 0.7
0.3 0.0
1.0 0.0
5.0 0.0
Λ > 0
accelerating
Relative size of the universe
0
1
2
3
4
-10
Now
10
20
30
Billions of Years

# Three Universes

- dark energy dominated

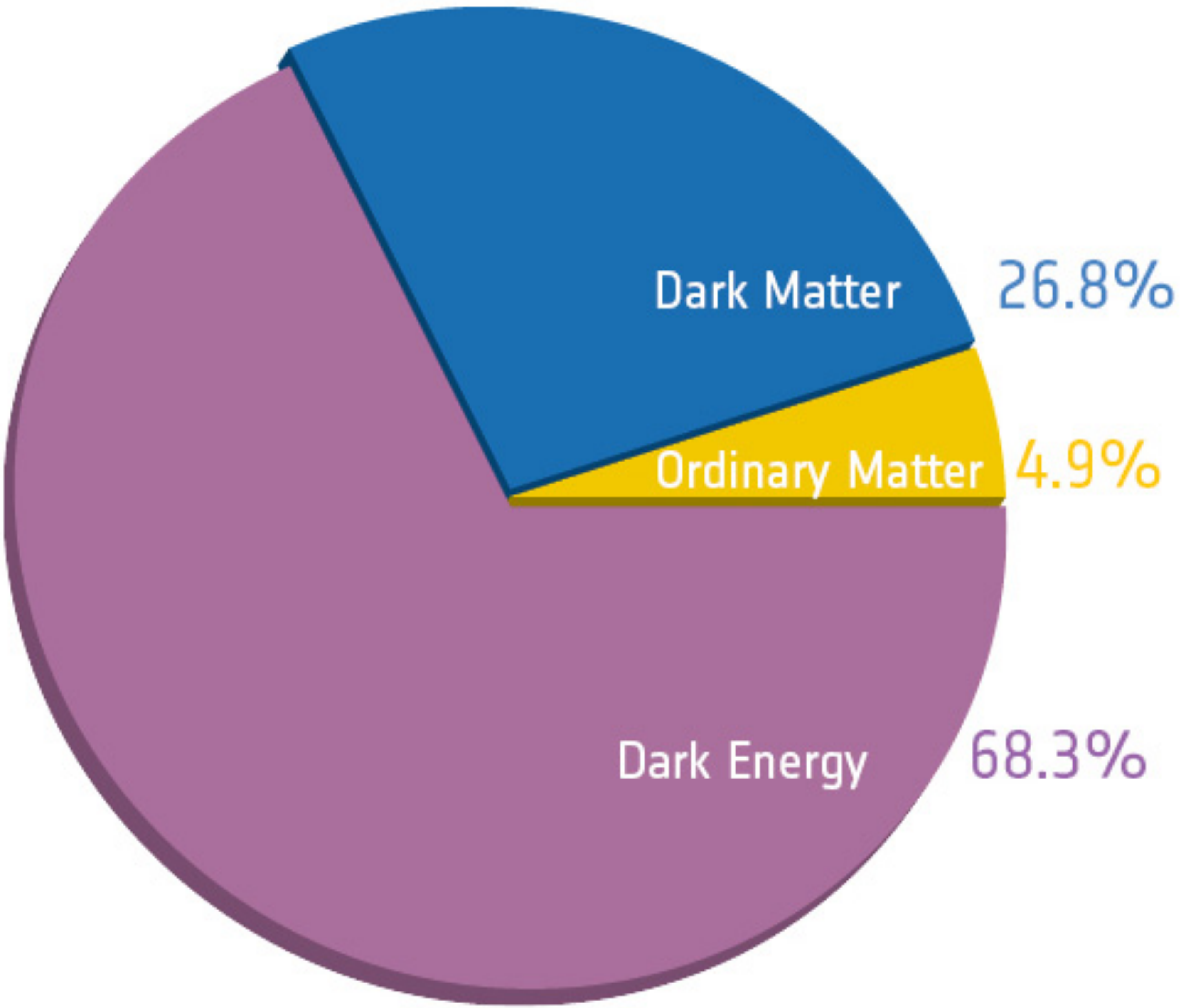
$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{1}{3}\Lambda \implies R = R_0 e^{\sqrt{\Lambda/3}\,t}$$

- matter dominated

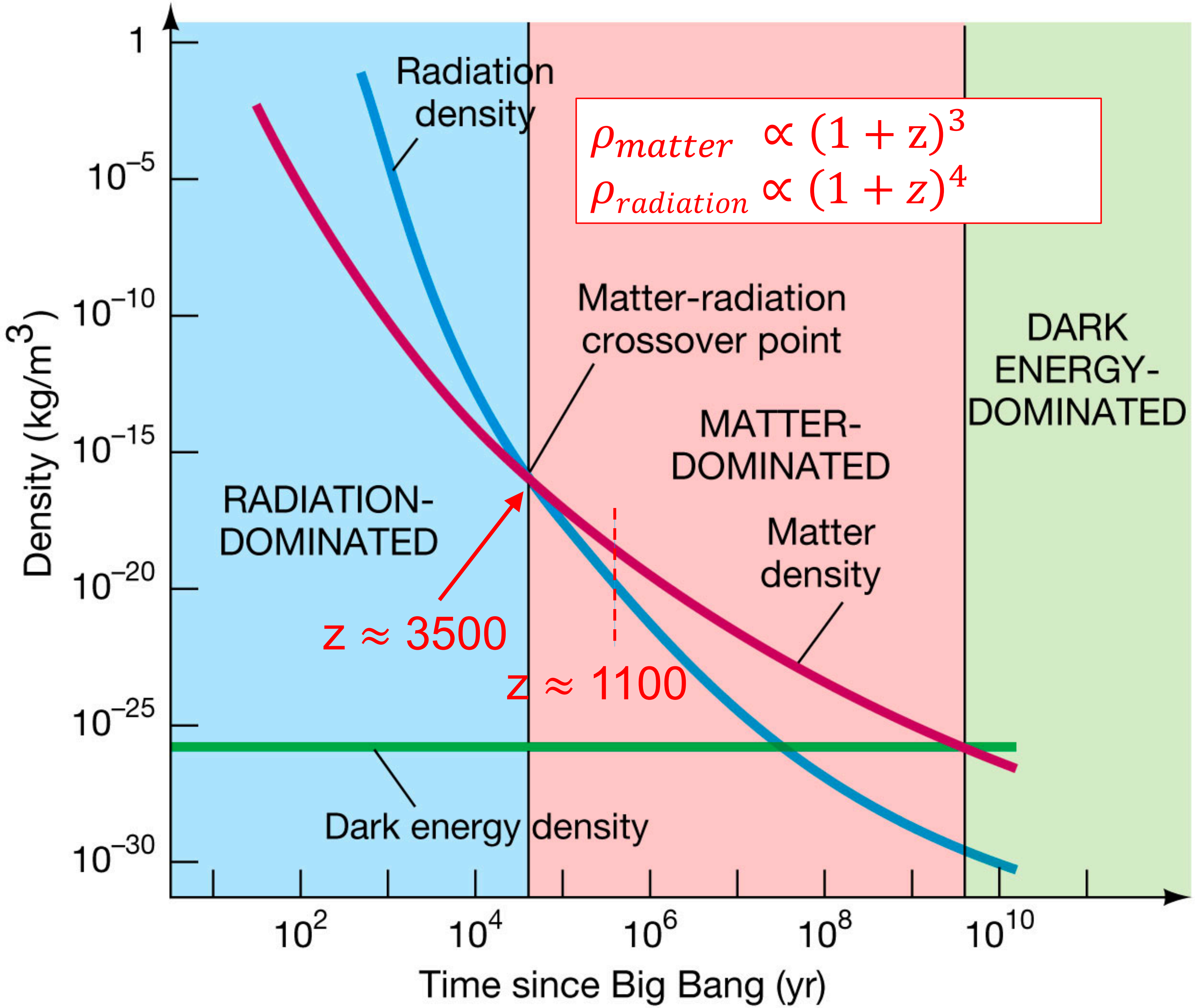
$$\left(\frac{\dot{R}}{R}\right)^2 = A\left(\frac{R_0}{R}\right)^3 \implies R = R_0 (At)^{2/3}$$

- radiation dominated

$$\left(\frac{\dot{R}}{R}\right)^2 = B\left(\frac{R_0}{R}\right)^4 \implies R = R_0 \left(2\sqrt{B}t\right)^{1/2}$$




Planck
project


Radiation density
$\rho_{matter} \propto (1+z)^3$
$\rho_{radiation} \propto (1+z)^4$
Matter-radiation crossover point
DARK ENERGY-DOMINATED
MATTER-DOMINATED
RADIATION-DOMINATED
Matter density
z ≈ 3500
z ≈ 1100
Dark energy density
Density (kg/m$^3$)
1
$10^{-5}$
$10^{-10}$
$10^{-15}$
$10^{-20}$
$10^{-25}$
$10^{-30}$
$10^2$
$10^4$
$10^6$
$10^8$
$10^{10}$
Time since Big Bang (yr)

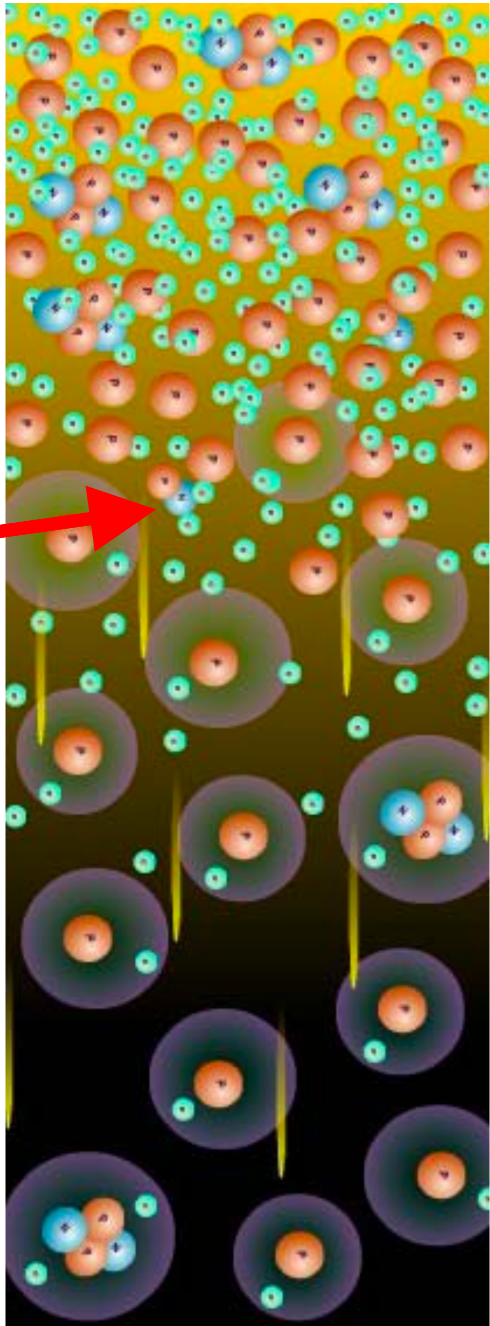
# Decoupling

- universe ionized for $T > 3000$ K
- matter and radiation strongly coupled, in thermal equilibrium

  $p + e^{-} \rightarrow H$ at T ~ 3000 K

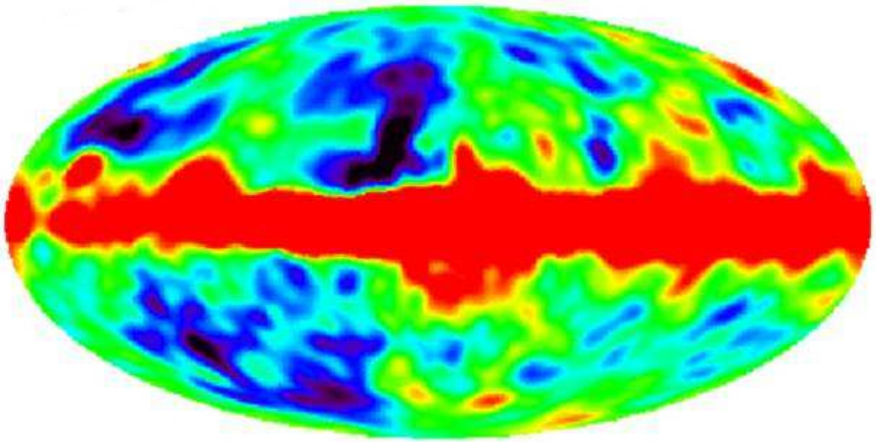
  $z = 1100$

- universe became transparent; radiation has propagated freely through space ever since
- now redshifted to 2.7 K — microwave background

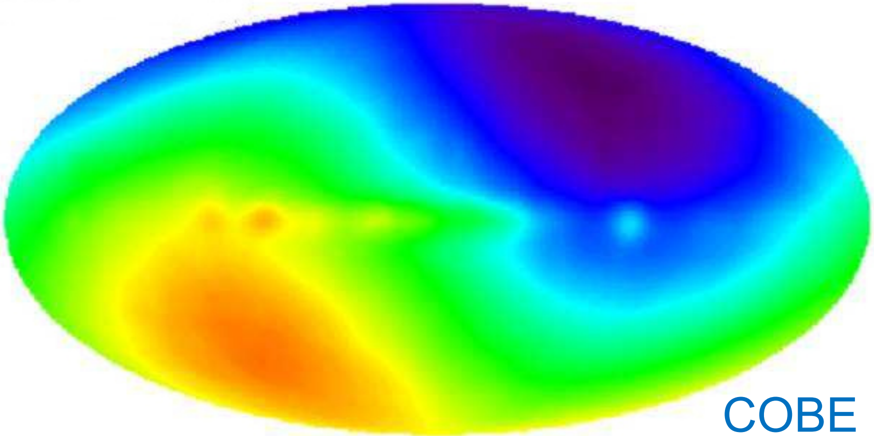


# Cosmic Microwave Background

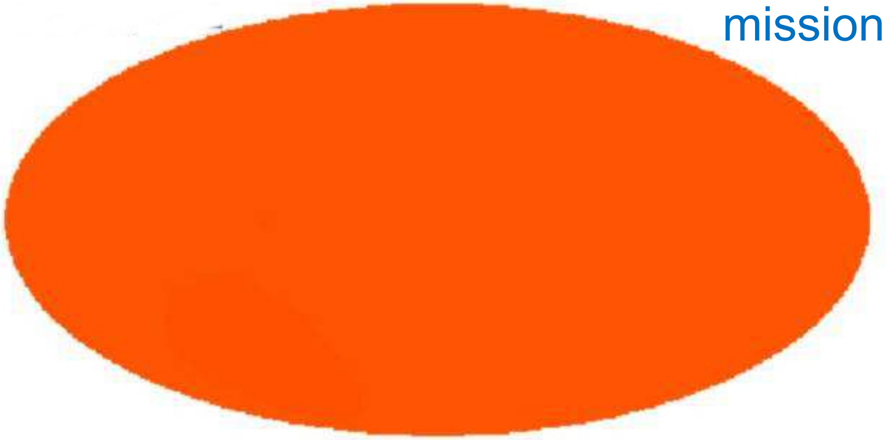
- remove the Galaxy



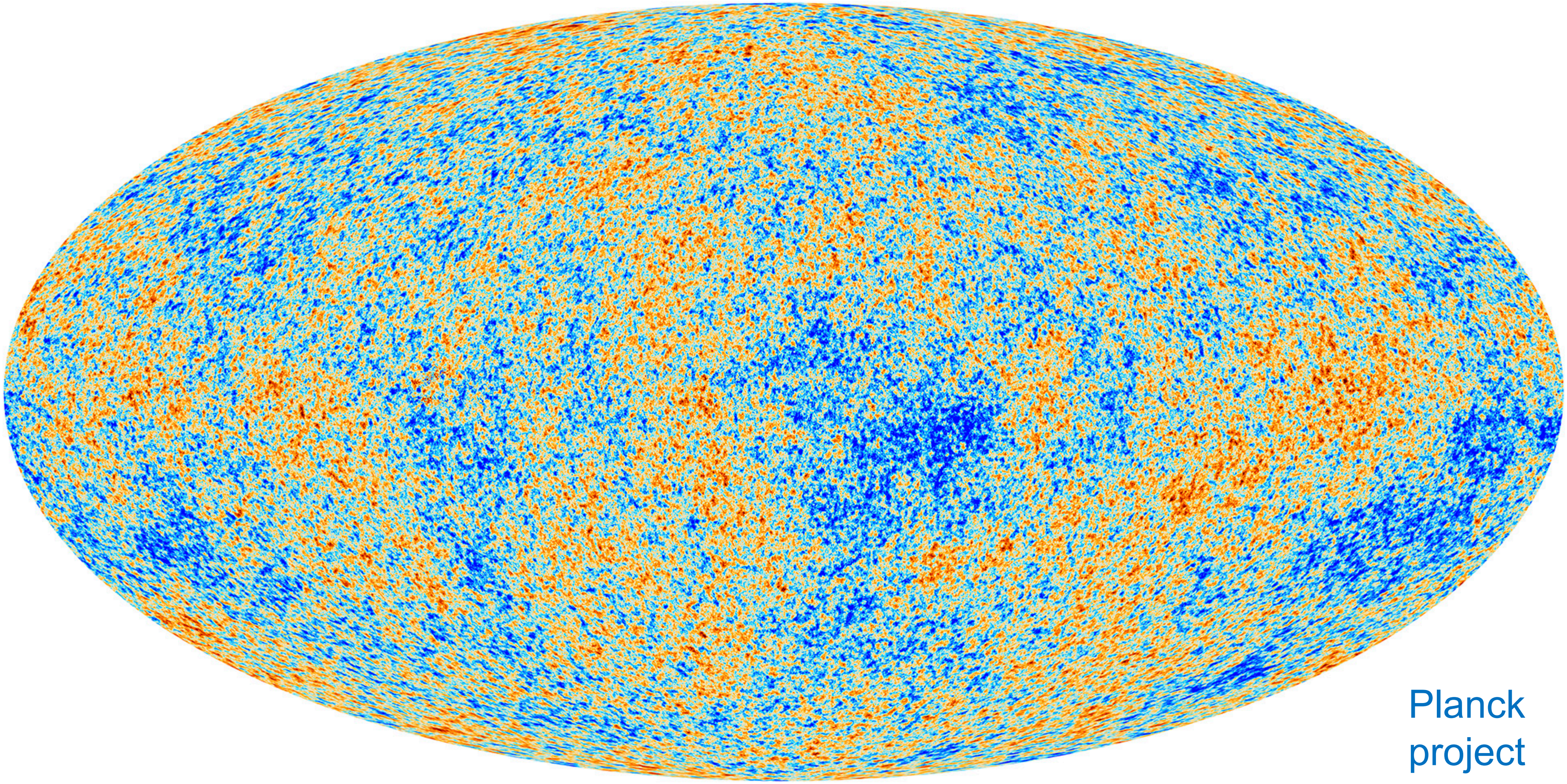
- …and the dipole signal due to our 371 km/s motion through space



- …the remaining signal is isotropic to 1 part in $10^5$



COBE mission



Planck project

- $\mu$K temperature variations map density fluctuations at the epoch of decoupling
- fluctuations grew to become the large-scale structure seen today