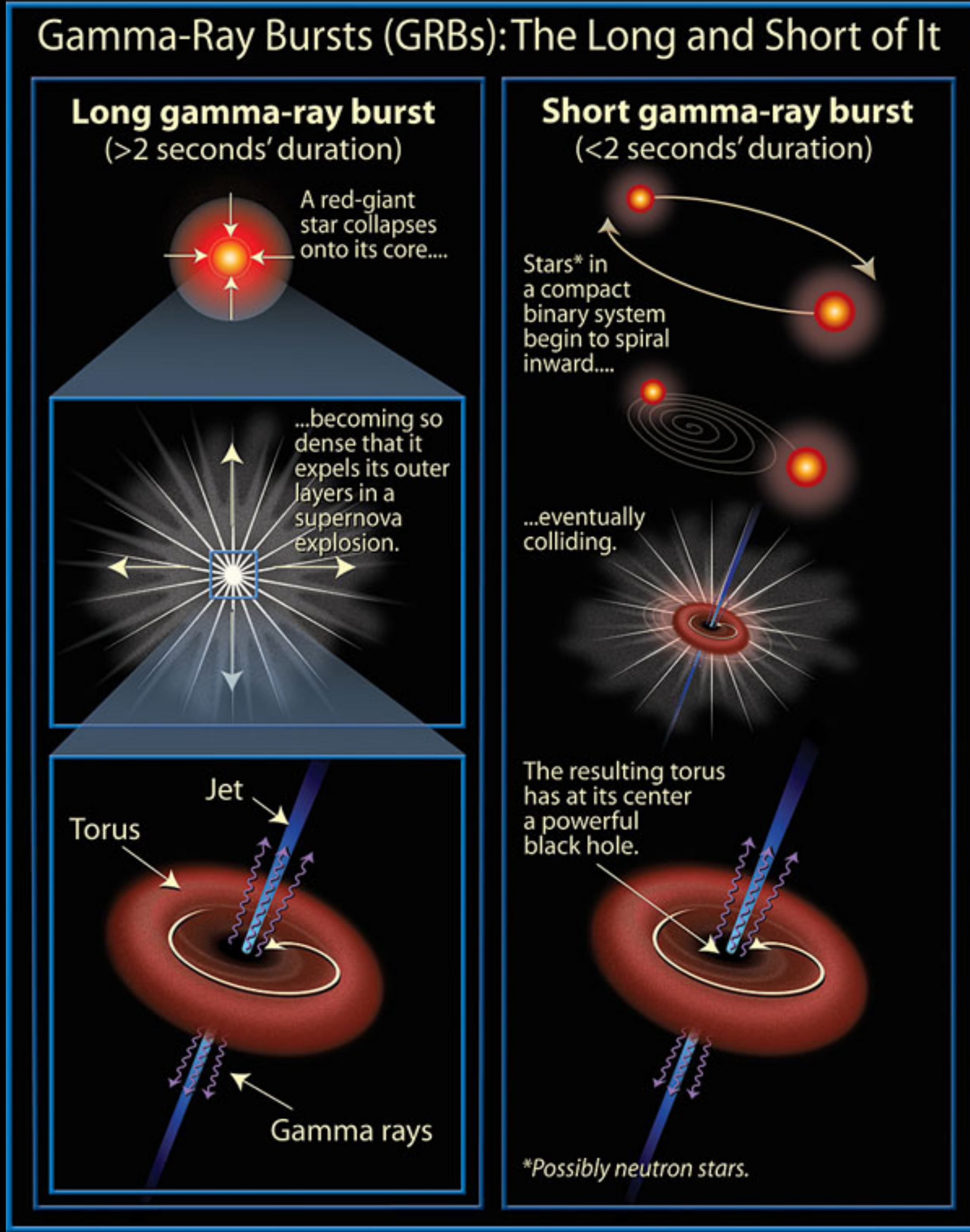

Gamma-Ray Bursts (GRBs): The Long and Short of It
Long gamma-ray burst
(>2 seconds' duration)
A red-giant star collapses onto its core....
...becoming so dense that it expels its outer layers in a supernova explosion.
Jet
Torus
Gamma rays
Short gamma-ray burst
(<2 seconds' duration)
Stars* in a compact binary system begin to spiral inward....
...eventually colliding.
The resulting torus has at its center a powerful black hole.
*Possibly neutron stars.

Neutron Star

- supported by <u>neutron degeneracy pressure</u>

$$M_{ch} = 0.21 \left(\frac{Z}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{3/2} m_p$$

$$= 5.6\, M_\odot \quad \text{for } \frac{Z}{A} \approx 1$$

Neutron Star Properties

- mass and radius

$$M \sim 1.4\, M_\odot, \quad R \sim 11\ \text{km} \left(\frac{M}{1.4\, M_\odot}\right)^{-1/3}$$

- escape speed

$$v_{esc} = \sqrt{\frac{2GM}{R}} = 1.8\times10^{5}\ \text{km s}^{-1} \left(\frac{M}{1.4\, M_\odot}\right)^{2/3}$$

$0.6\, c$

$$\rightarrow c \ \text{ when } \ M = 3.0\, M_\odot$$

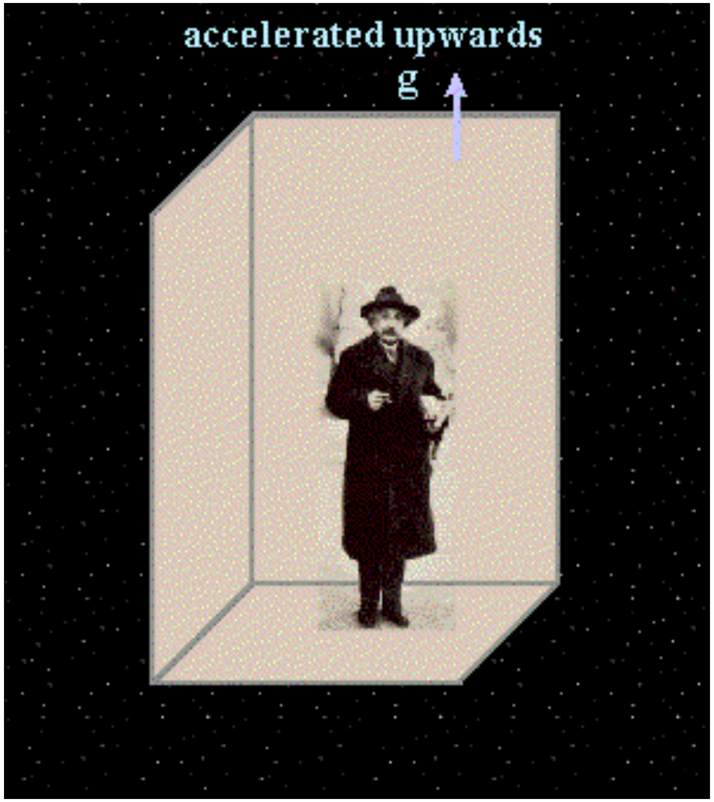
Escape Speed

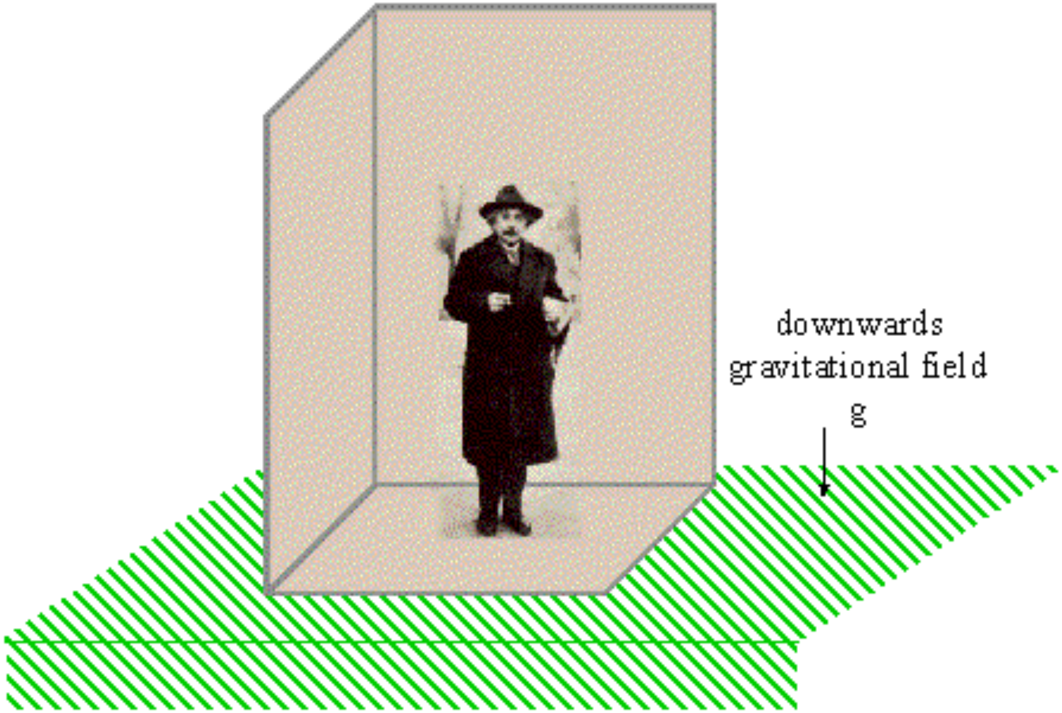
$$v_{esc} = \sqrt{\frac{2GM}{R}} = c$$

$$\Longrightarrow \quad R = \frac{2GM}{c^2} = 3\ \mathrm{km}\left(\frac{M}{M_{\odot}}\right)$$

Schwarzchild radius

Equivalence Principle





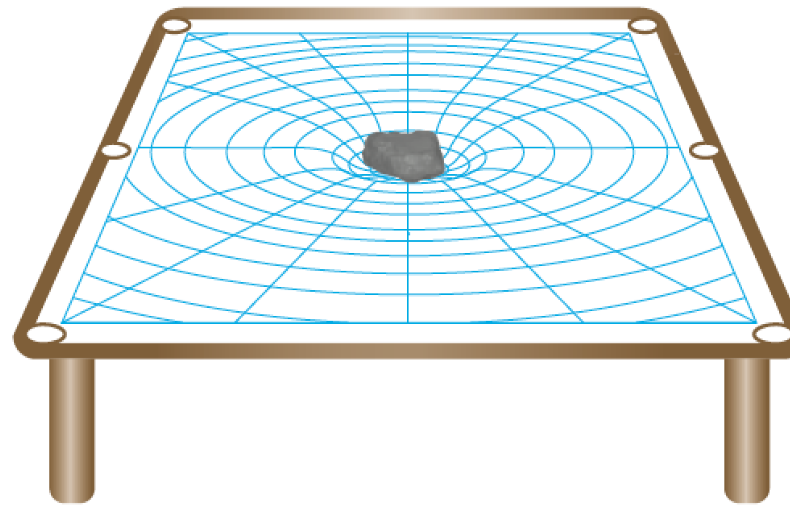
$F = ma$? $F = \frac{GmM}{r^2}$

Einstein's Gravity

$$ds^2 = dx^2 + dy^2 + dz^2$$
$$ds^2 = dr^2 + r^2 d\theta^2 + dz^2$$
$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$$

gravity = spacetime curvature

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}\, dx^\mu dx^\nu$$

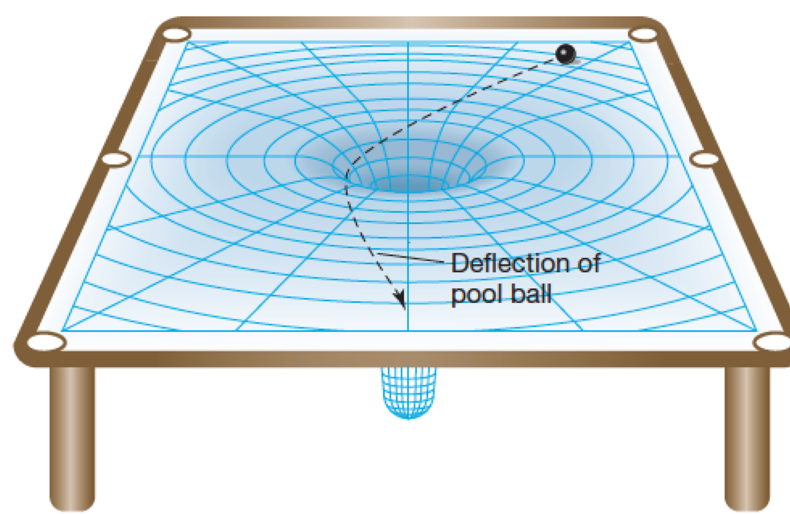


- Einstein's (1915) equations

$$G = 8\pi T$$

curvature — mass and energy

  – reduce to Newtonian gravity for small masses/low speeds



Matter tells spacetime how to curve

Spacetime tells matter how to move

J. A. Wheeler

General Relativity

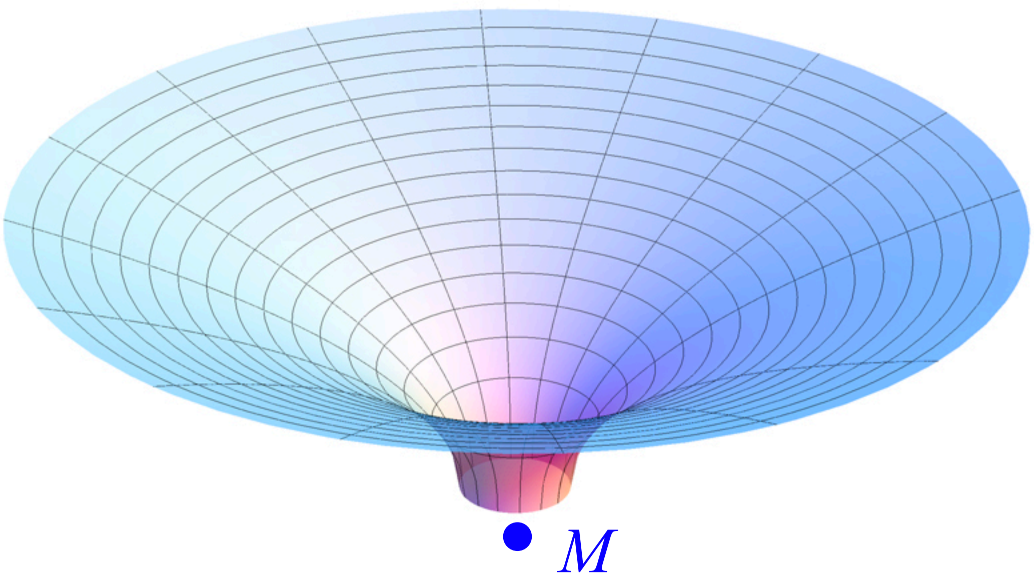
$$G \;=\; 8\,\pi\,T$$

$$R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu} + \Lambda\,g_{\mu\nu} \;=\; \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} \;=\; R^{\alpha}_{\mu\alpha\nu} \;= \frac{\partial \Gamma^{\alpha}_{\nu\mu}}{\partial x_{\alpha}} - \frac{\partial \Gamma^{\alpha}_{\alpha\mu}}{\partial x_{\nu}} + \Gamma^{\beta}_{\beta\alpha}\,\Gamma^{\alpha}_{\nu\mu} - \Gamma^{\alpha}_{\nu\beta}\,\Gamma^{\beta}_{\alpha\mu}$$

$$\Gamma^{\alpha}_{\beta\gamma} \;=\; \tfrac{1}{2}\left(\frac{\partial g_{\alpha\beta}}{\partial x_{\gamma}} + \frac{\partial g_{\alpha\gamma}}{\partial x_{\beta}} + \frac{\partial g_{\beta\gamma}}{\partial x_{\alpha}}\right)$$

Schwarzschild Geometry



$$c^2 d\tau^2 \;=\; \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$r_s \;=\; \frac{2GM}{c^2}$$

$$v_{esc}^2 = \frac{2GM}{r_S} = c^2$$

Schwarzschild radius

Schwarzschild Geometry

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

clock at rest: $dr = d\theta = d\phi = 0$

$$\Longrightarrow \; d\tau = \left(1 - \frac{r_s}{r}\right)^{1/2} dt$$

$$= \left(1 - \frac{2GM}{rc^2}\right)^{1/2} dt$$

$$\approx \left(1 - \frac{GM}{rc^2}\right) dt$$

$$\Longrightarrow \; \frac{d\tau}{dt} = 1 - \frac{GM}{rc^2} = 1 - \frac{\Delta\Phi}{c^2}$$

time dilation

$d\tau \to 0$

as $r \to r_s$

Schwarzschild Geometry

$$c^2 d\tau^2 \;=\; \left(1-\frac{r_s}{r}\right) c^2 dt^2 - \left(1-\frac{r_s}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

frequency:

$$\frac{\nu(r)}{\nu(\infty)} = \frac{dt}{d\tau} = \left(1-\frac{r_s}{r}\right)^{-1/2}$$

$$\Longrightarrow \frac{\nu(\infty)}{\nu(r)} = \left(1-\frac{r_s}{r}\right)^{1/2}$$

$$\approx 1 - \frac{GM}{rc^2}$$

gravitational redshift

$\nu(\infty) \to 0$ as $r \to r_s$

$r = r_s$ is the <u>event horizon</u>

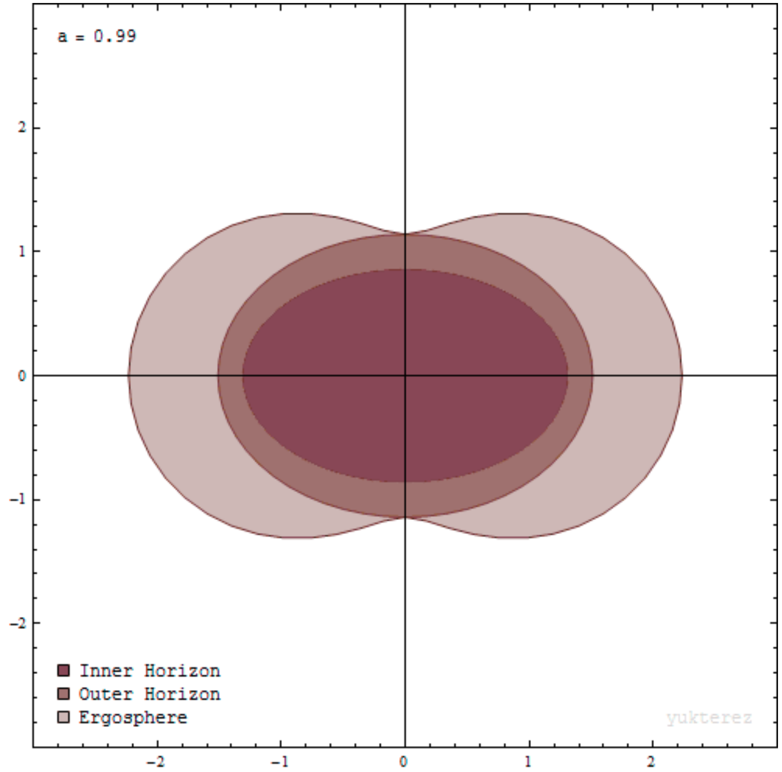
Kerr Geometry

$$-c^2 d\tau^2 = -\left(1 - \frac{r_s r}{\Sigma}\right) c^2 dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{r_s r a^2}{\Sigma} \sin^2\theta\right) \sin^2\theta \, d\phi^2 - \frac{2 r_s r a \sin^2\theta}{\Sigma} c \, dt \, d\phi$$

$$a = \frac{J}{Mc},$$

$$\Sigma = r^2 + a^2 \cos^2\theta$$

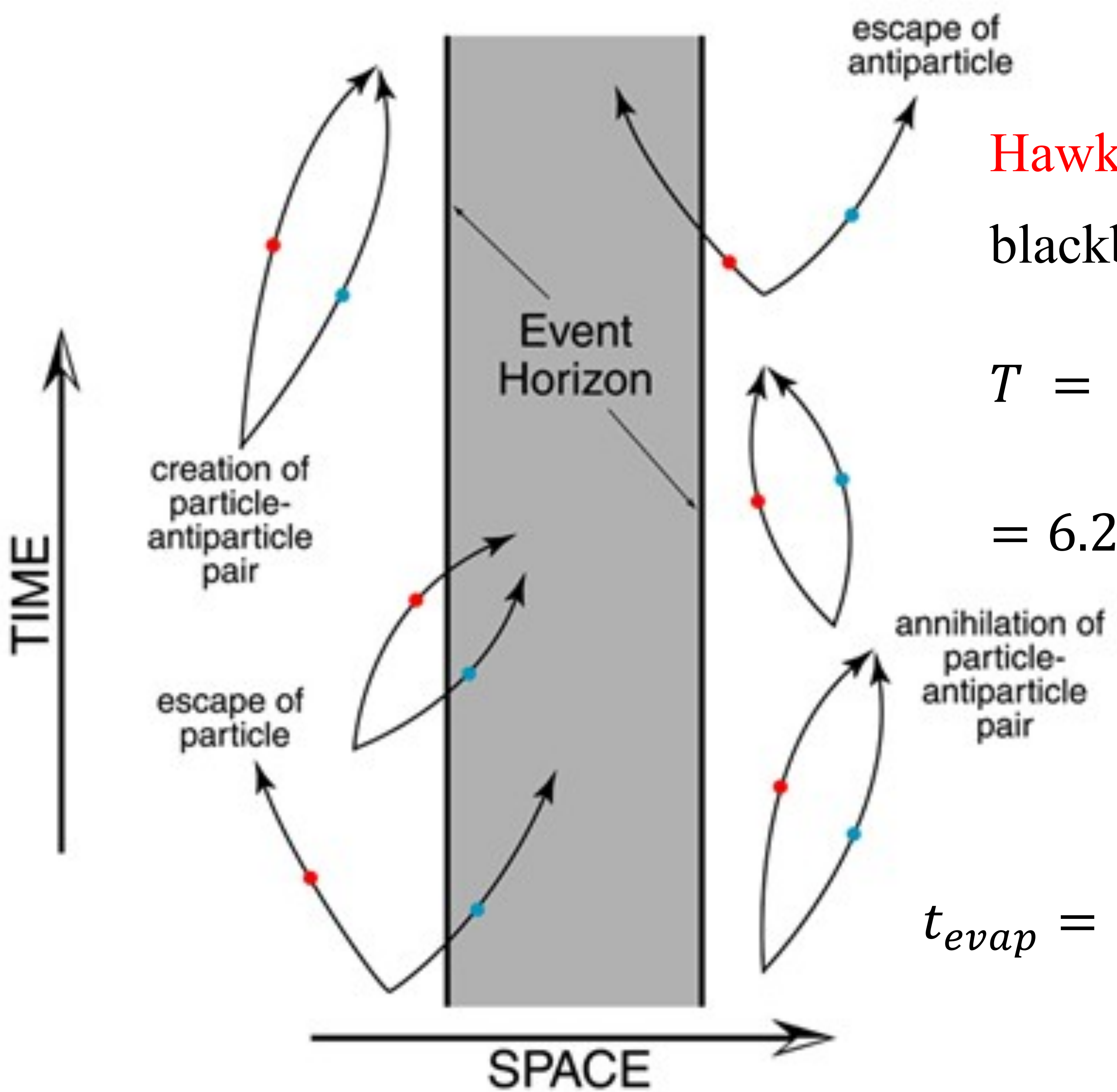
$$\Delta = r^2 - r_s r + a^2$$



frame dragging

Planck Scales

$$\begin{aligned} \ell_p &= \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35}\ \mathrm{m} \\ \tau_p &= \frac{\ell_p}{c} = 5.3 \times 10^{-44}\ \mathrm{s} \\ m_p &= \sqrt{\frac{\hbar c}{G}} = 1.2 \times 10^{-8}\ \mathrm{kg} \end{aligned}$$



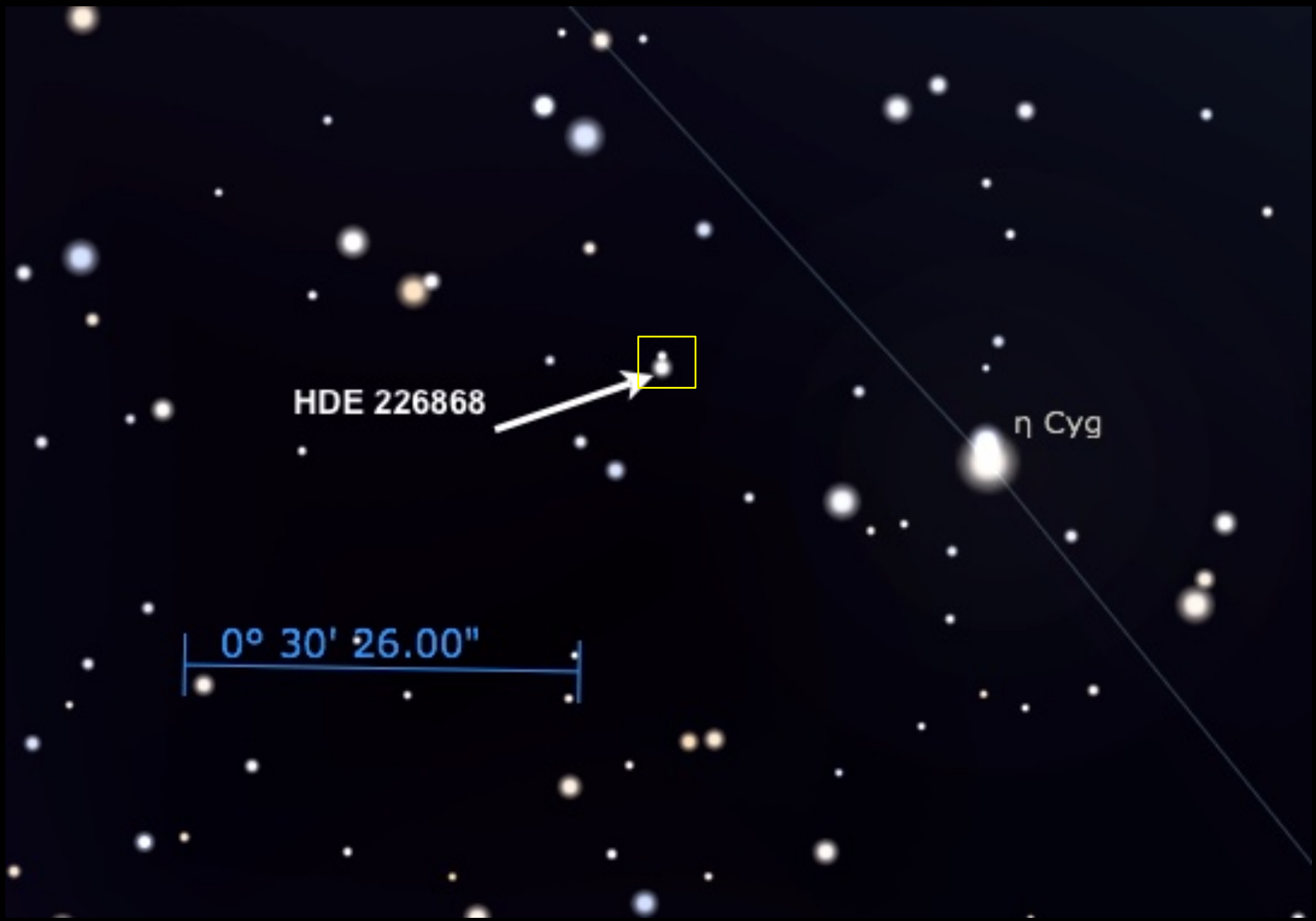
Hawking radiation

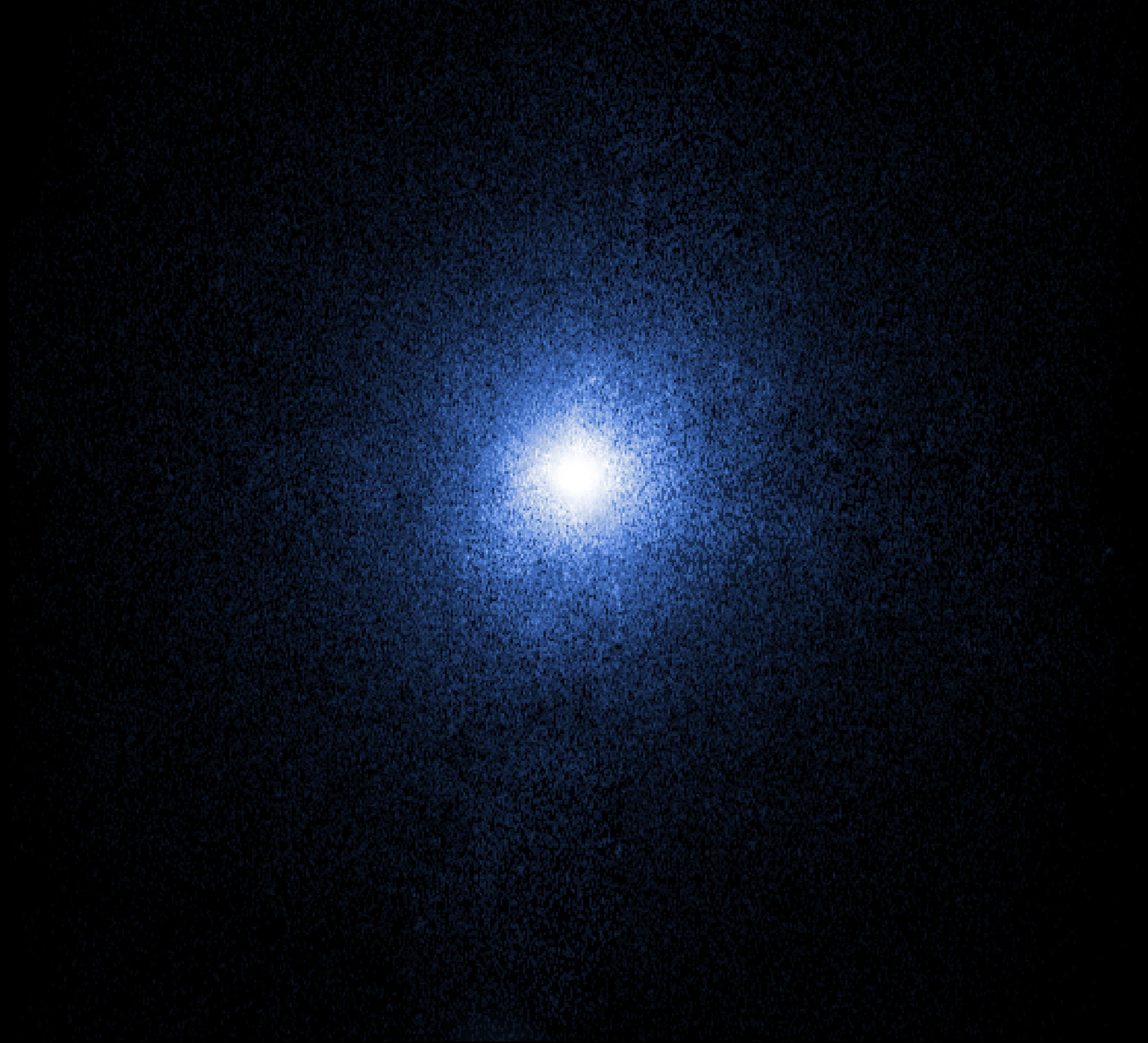
blackbody

$$T = \frac{\hbar c^2}{8\pi G M k}$$

$$= 6.2\times10^{-8} K \left(\frac{M}{M_\odot}\right)^{-1}$$

$$t_{evap} = 2\times10^{67}\mathrm{yr}\left(\frac{M}{M_\odot}\right)^{3}$$


HDE 226868
η Cyg
0° 30' 26.00"

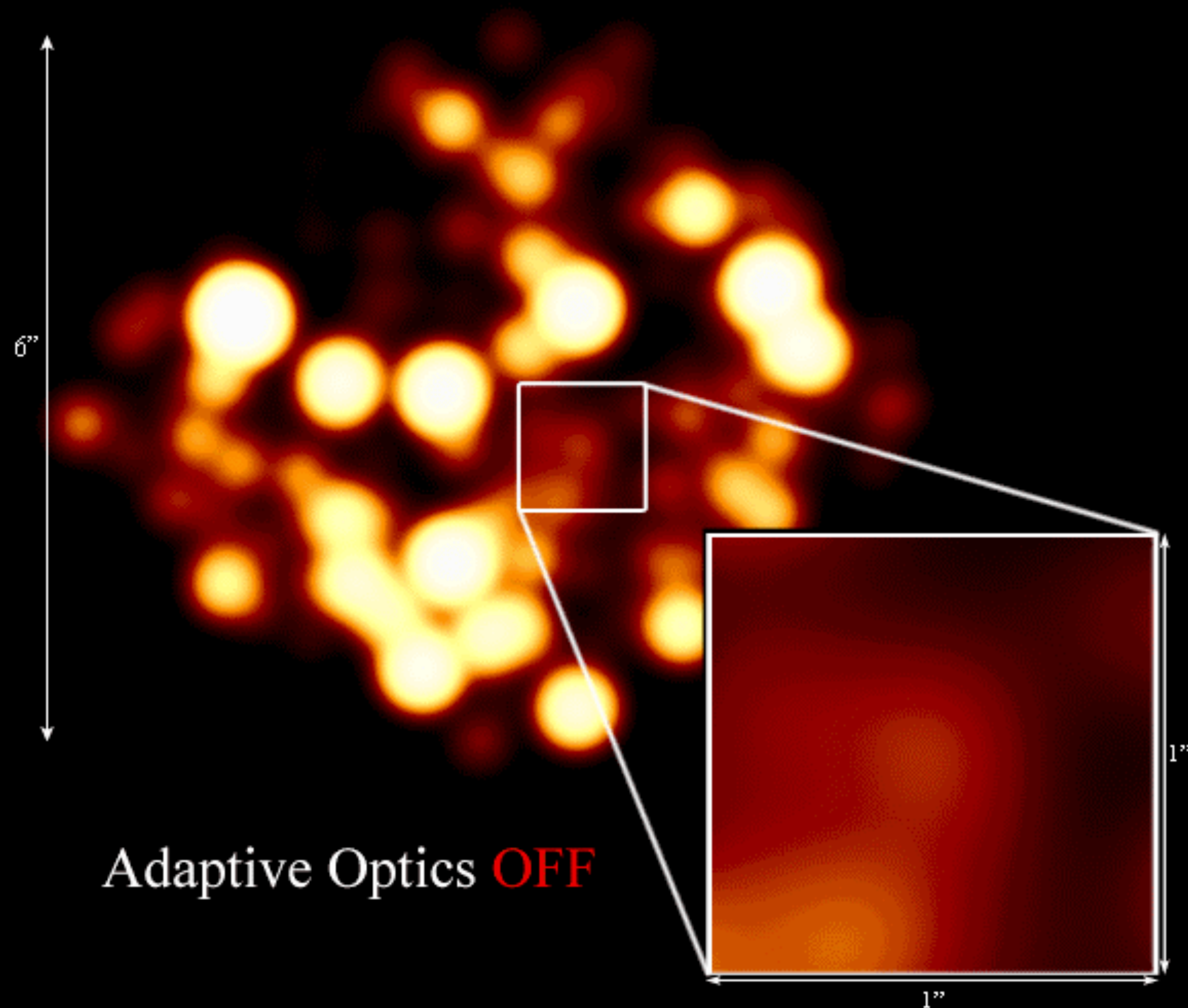



HDE 226868
X-ray source

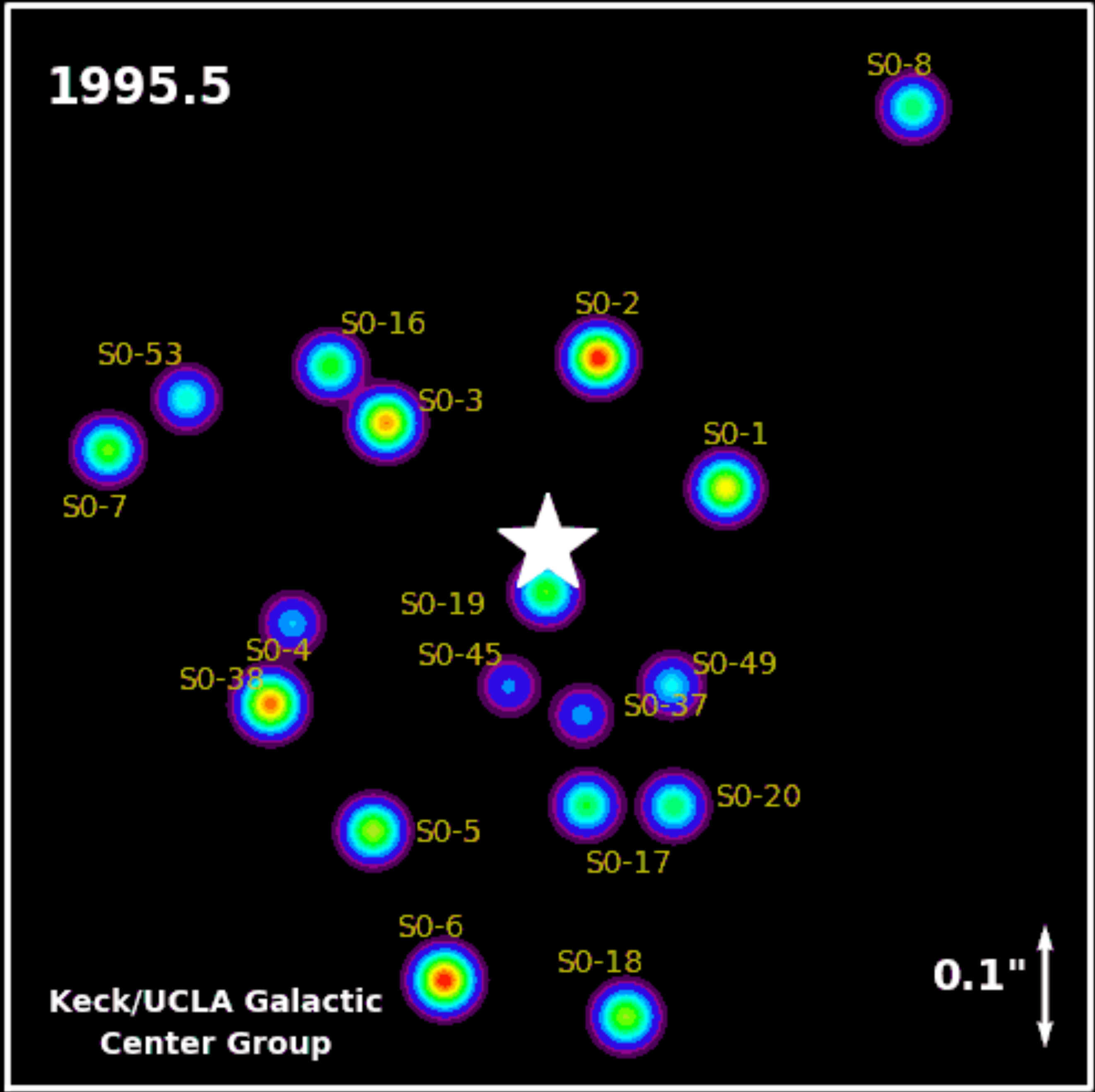

globular cluster
G1 (in M31)


irregular
galaxy M82

The Galactic Center at 2.2 microns



Adaptive Optics OFF


1995.5
S0-8
S0-2
S0-16
S0-53
S0-3
S0-1
S0-7
S0-19
S0-4
S0-45
S0-49
S0-38
S0-37
S0-20
S0-5
S0-17
S0-6
S0-18
0.1"
Keck/UCLA Galactic Center Group

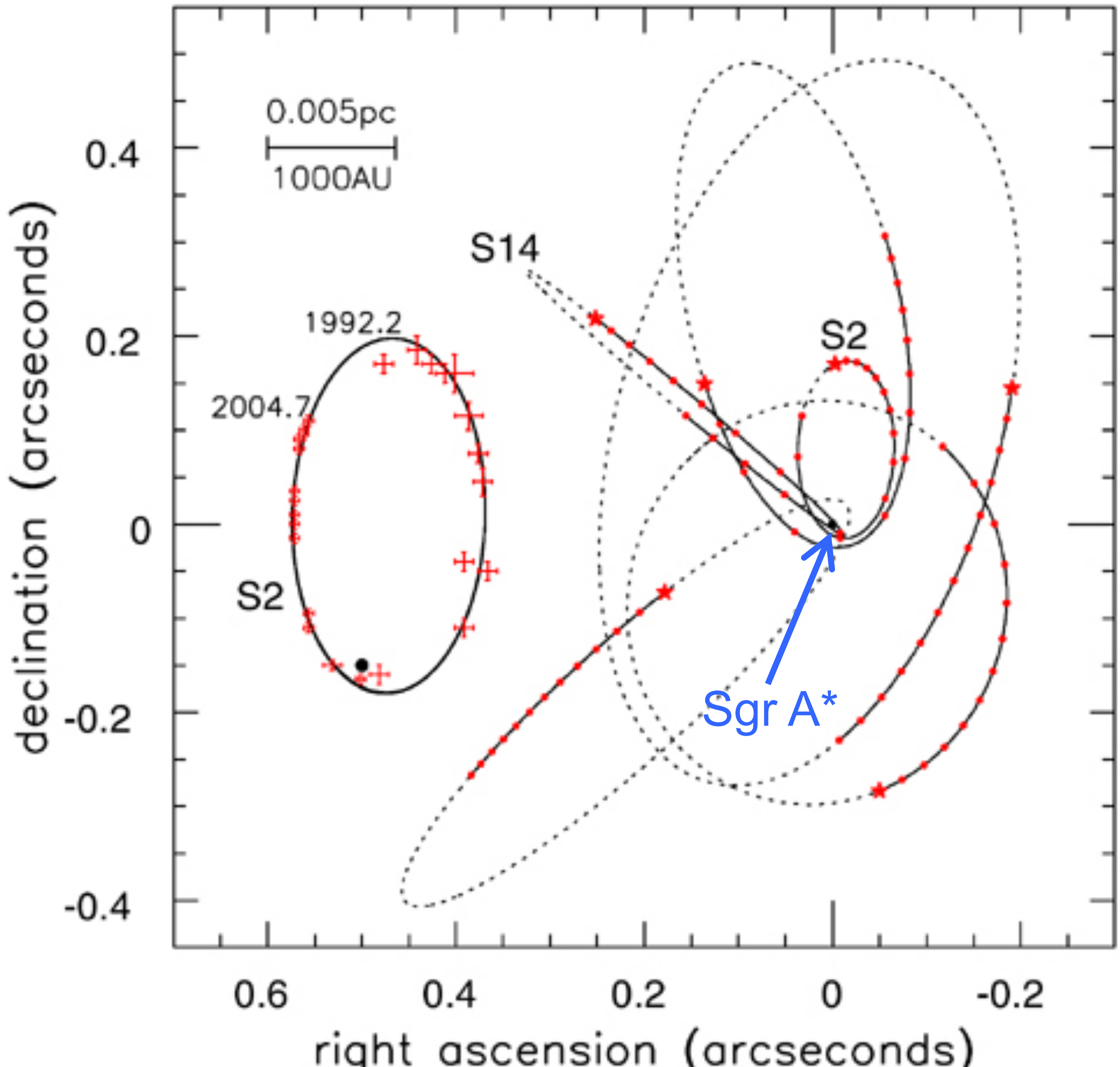
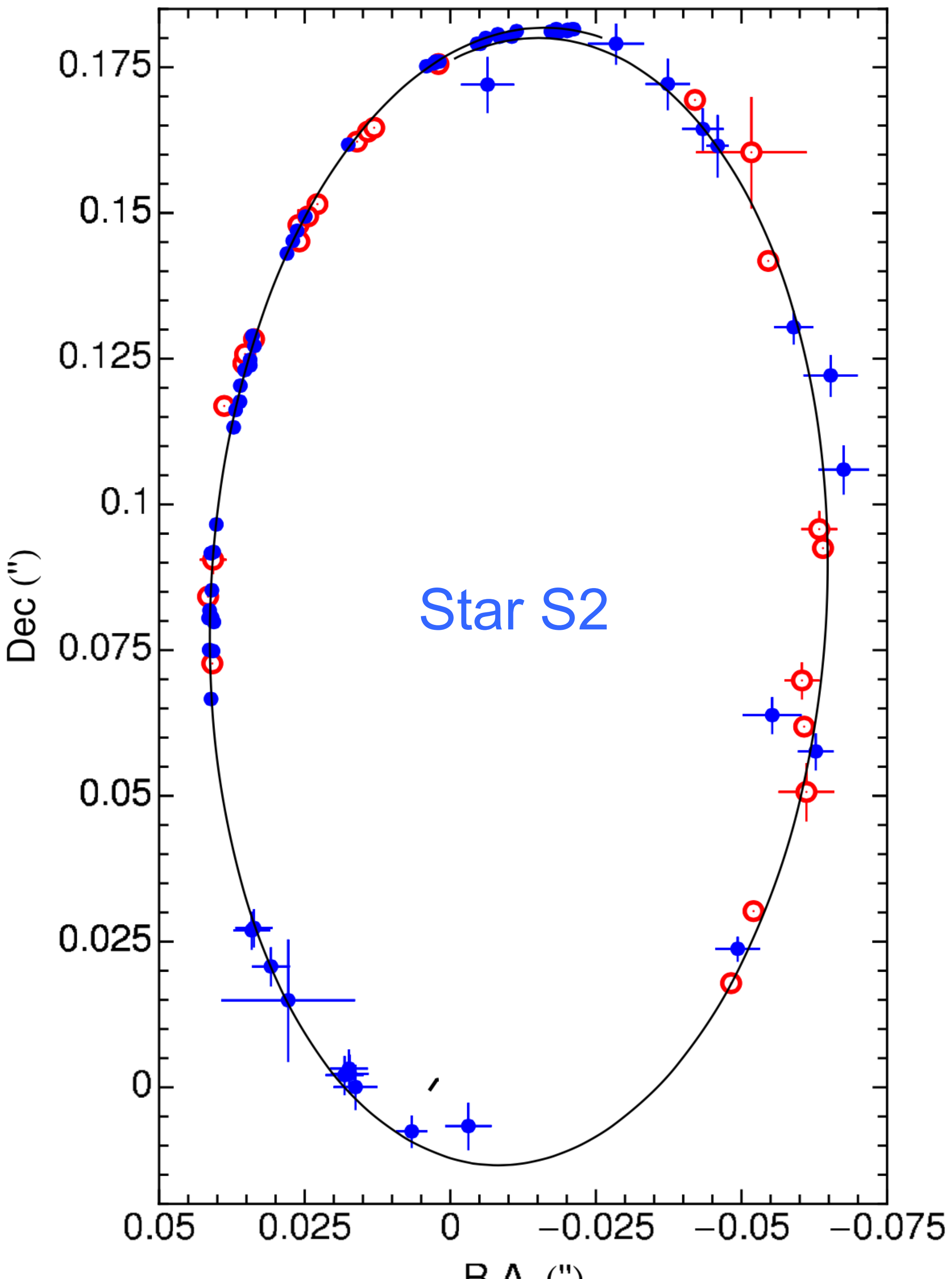


Fig 2.17 (Eisenhauer/MPE) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007


Star S2
Dec (")
R.A. (")
0.175
0.15
0.125
0.1
0.075
0.05
0.025
0
0.05
0.025
0
−0.025
−0.05
−0.075

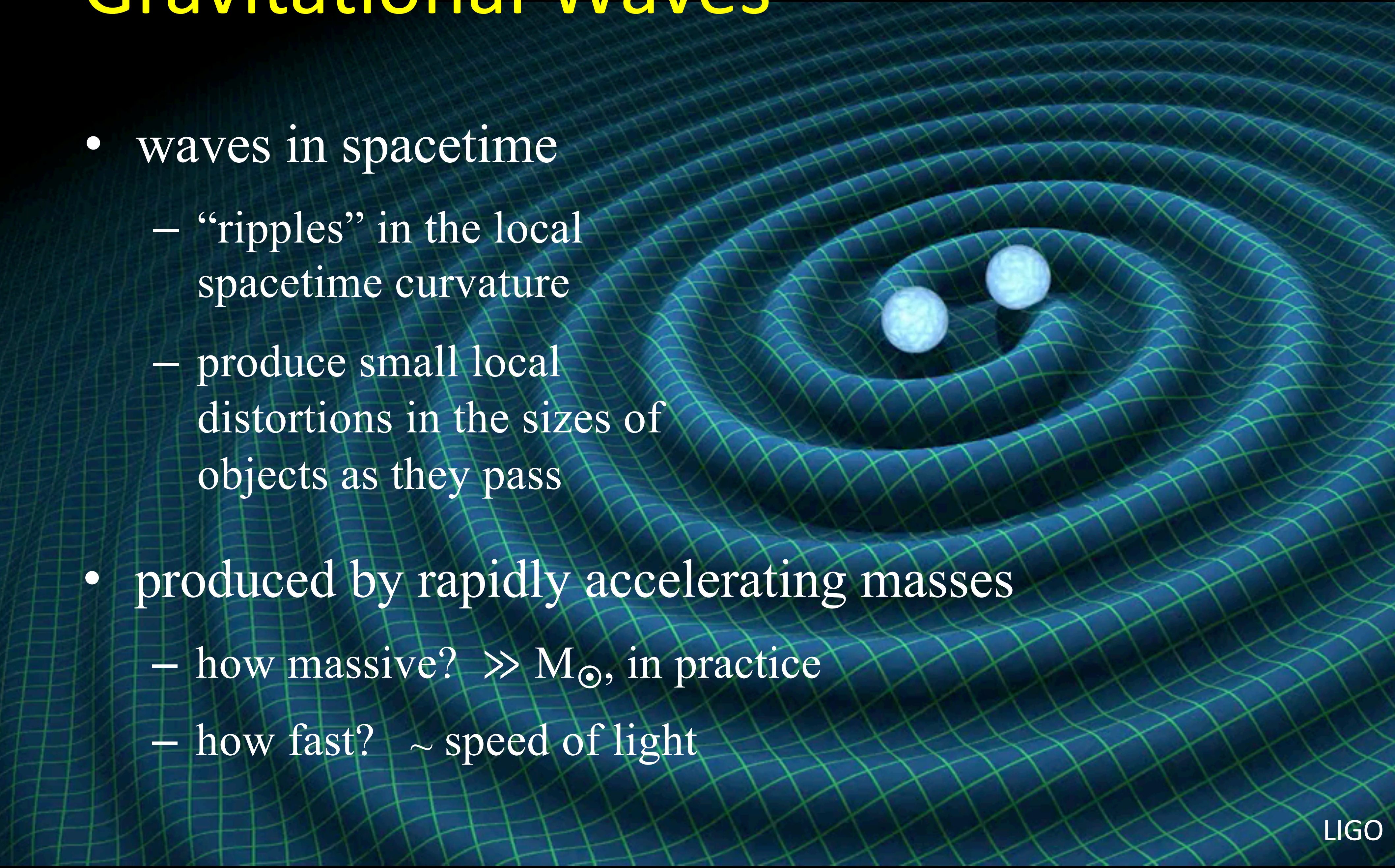
$$g_{\mu\nu} \; = \; \eta_{\mu\nu} \; + \; h_{\mu\nu}$$

$$\nabla^2 h_{\mu\nu} \; = \; - \, \frac{16\pi G}{c^4} \left(T_{\mu\nu} - \tfrac{1}{2} T_{\alpha\beta} \eta^{\alpha\beta} \eta_{\mu\nu} \right) + \frac{1}{c^2} \frac{\partial^2 h_{\mu\nu}}{\partial t^2}$$

$$\nabla^2 h \; = \; \frac{1}{c^2} \frac{\partial^2 h}{\partial t^2}$$

Gravitational Waves

- waves in spacetime
  - "ripples" in the local spacetime curvature
  - produce small local distortions in the sizes of objects as they pass
- produced by rapidly accelerating masses
  - how massive? $\gg M_{\odot}$, in practice
  - how fast? ~ speed of light



LIGO

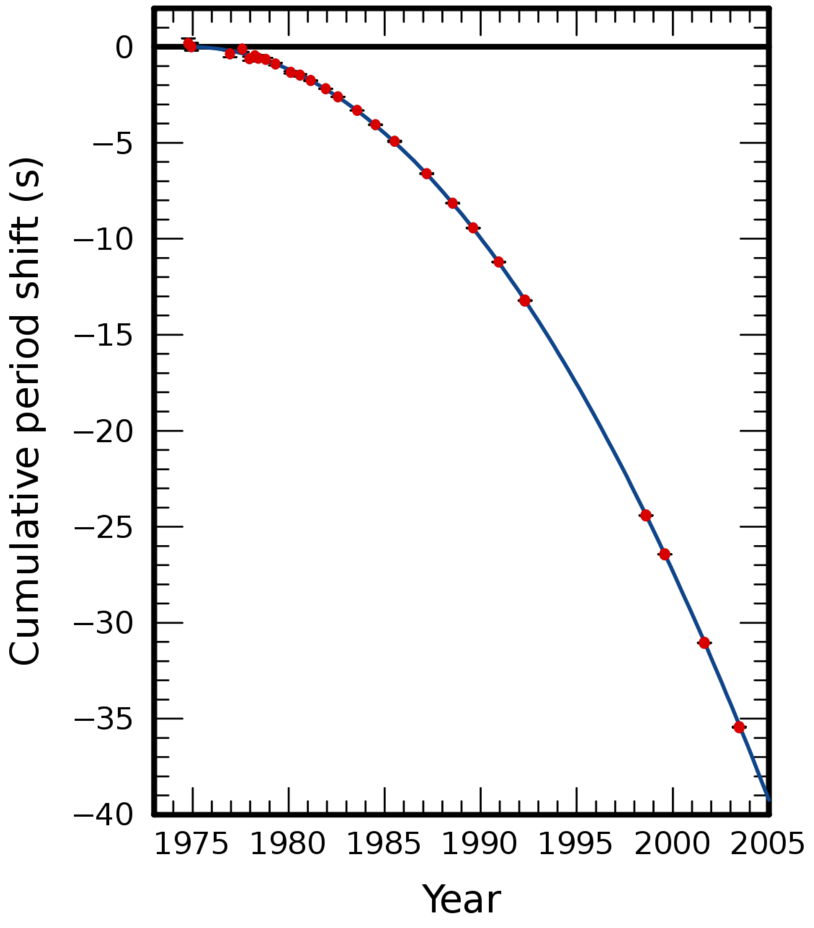
Neutron Star Inspiral

$$t_{\rm gwr} \simeq 0.02 \, \frac{c^5 a^4 (1-e^2)^{7/2}}{G^3 M m (M+m)}$$

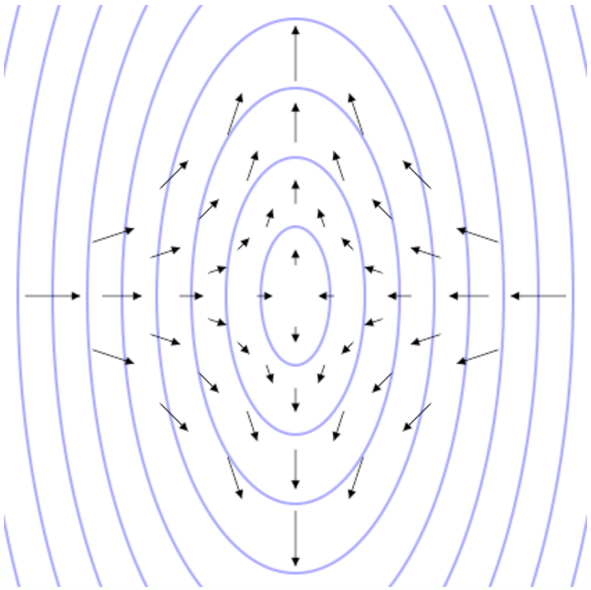
(Peters 1964)

neutron stars, $e = 0.62$,
$M, m = 1.44, 1.39 \, M_\odot$,
$a = 2.8 \, R_\odot$
(Hulse and Taylor 1975)

→ $t_{\rm gwr} = 300$ Myr



Wave Properties



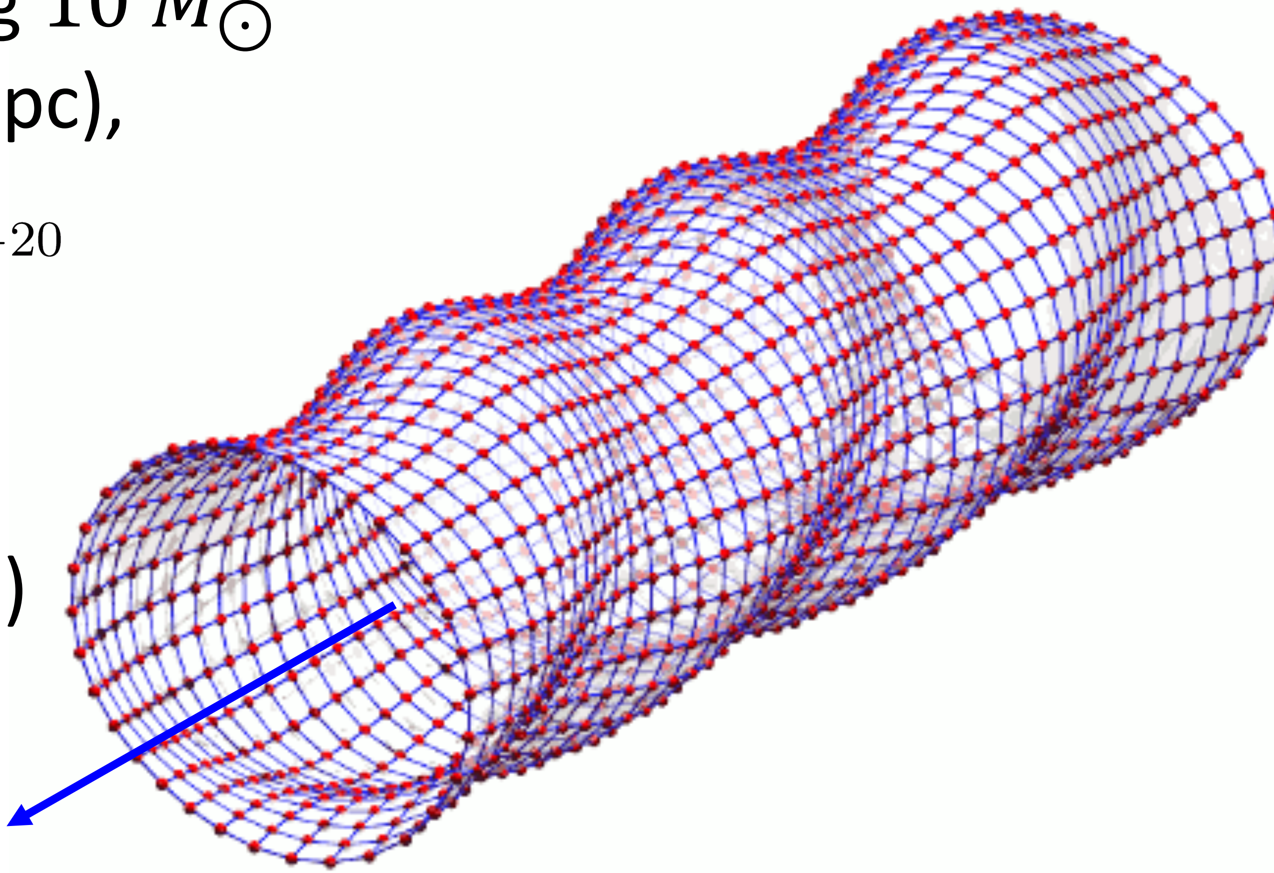
$$\frac{\Delta h}{h} \approx \frac{GM}{c^2}\left(\frac{v}{c}\right)^2 \times \frac{1}{r}$$

in practice (merging 10 $M_\odot$ black holes at 10 Mpc),

$$\frac{\Delta h}{h} \approx 3 \times 10^{-20}$$

- tiny!

(proton = 10^{-15} m)



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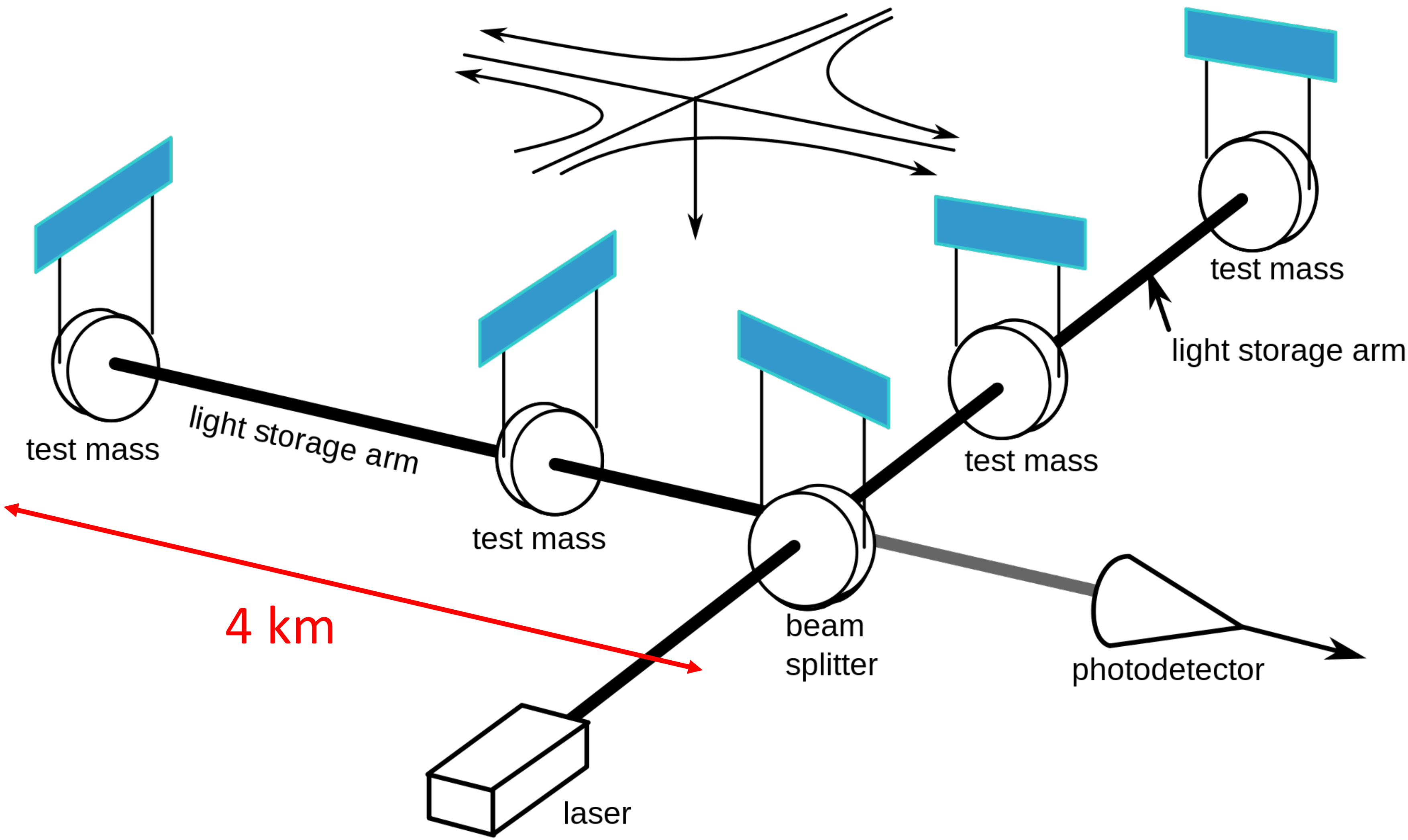
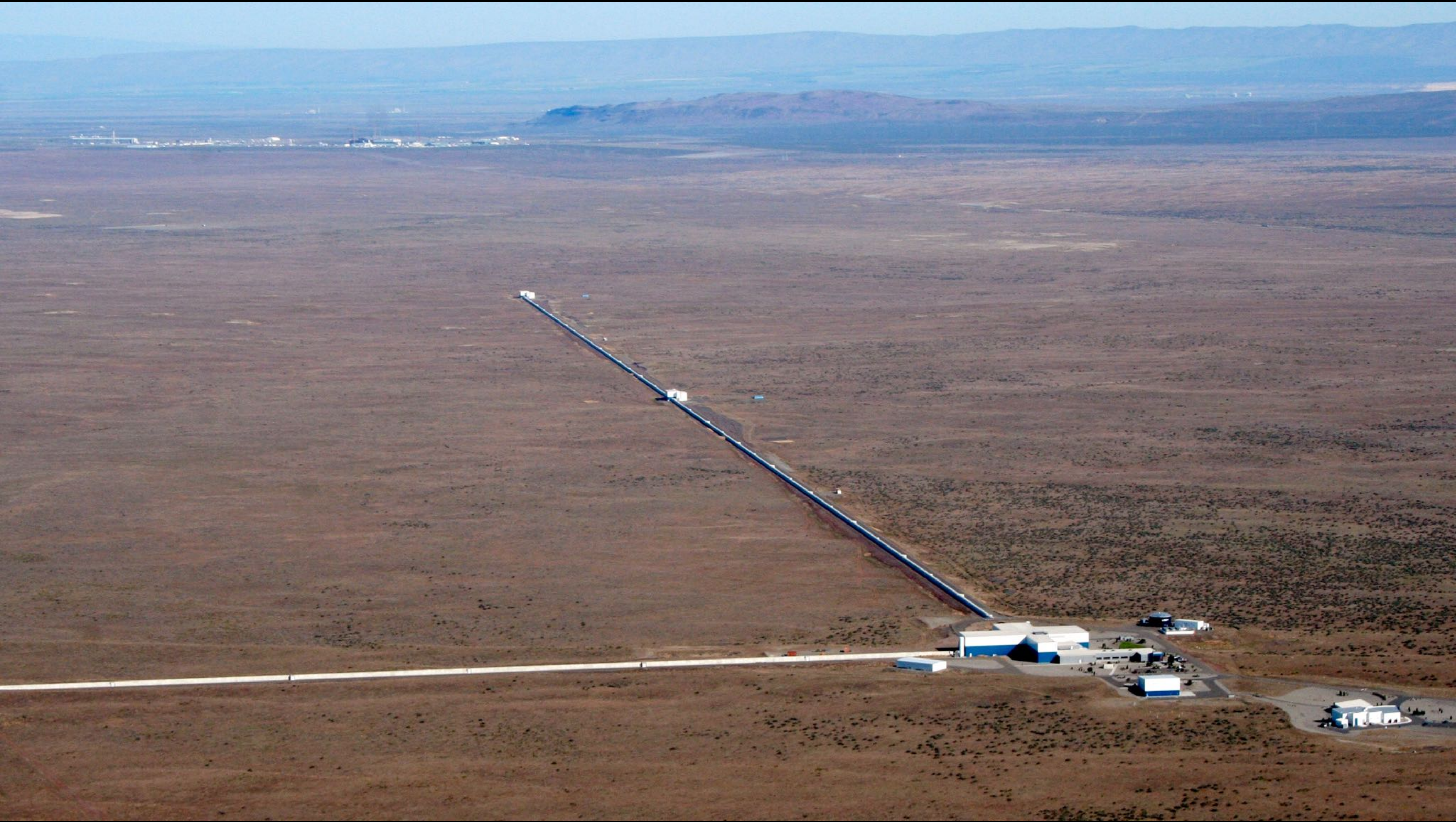
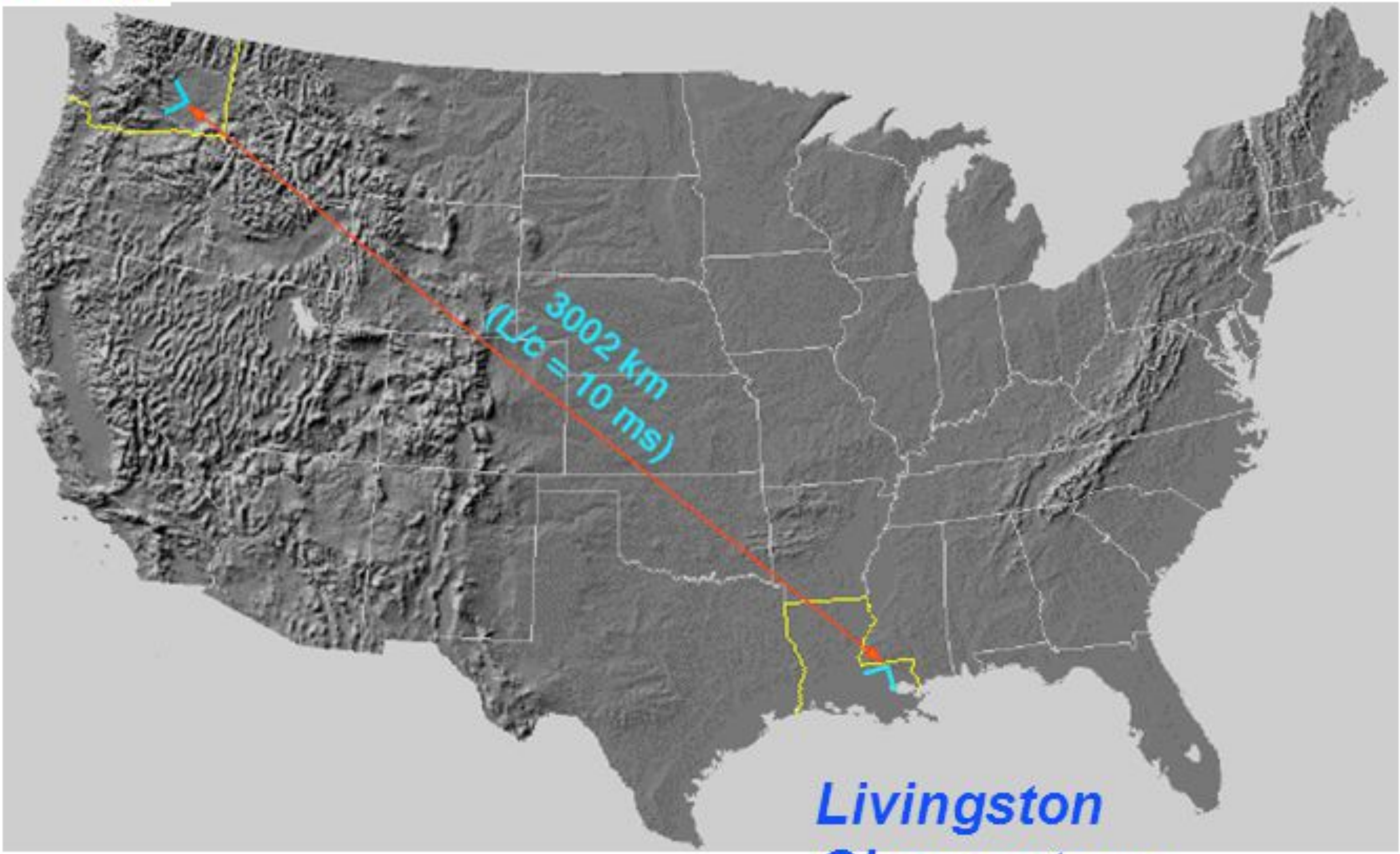
Laser Interferometer Gravitational-Wave Observatory



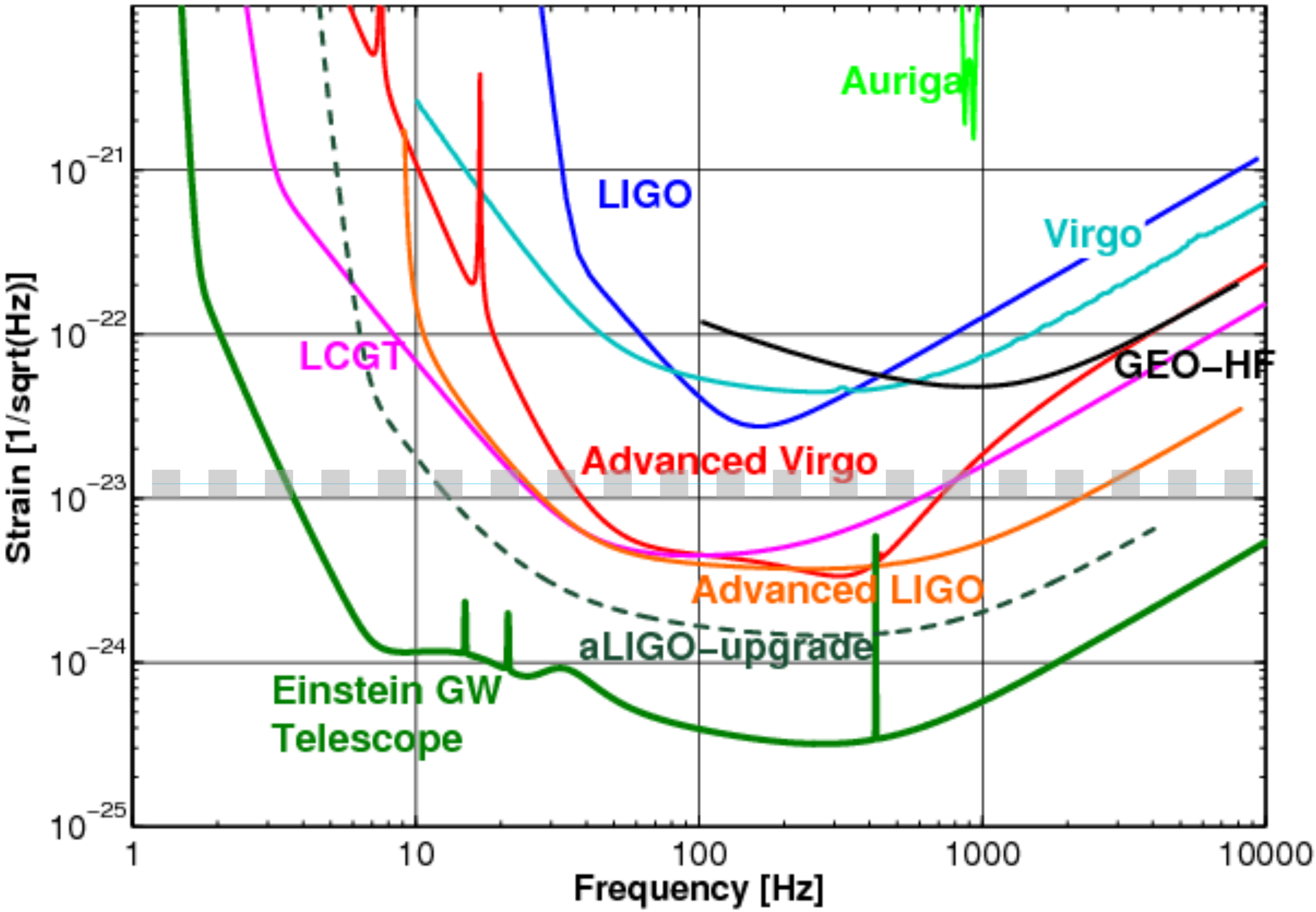


LIGO Sites

Hanford Observatory



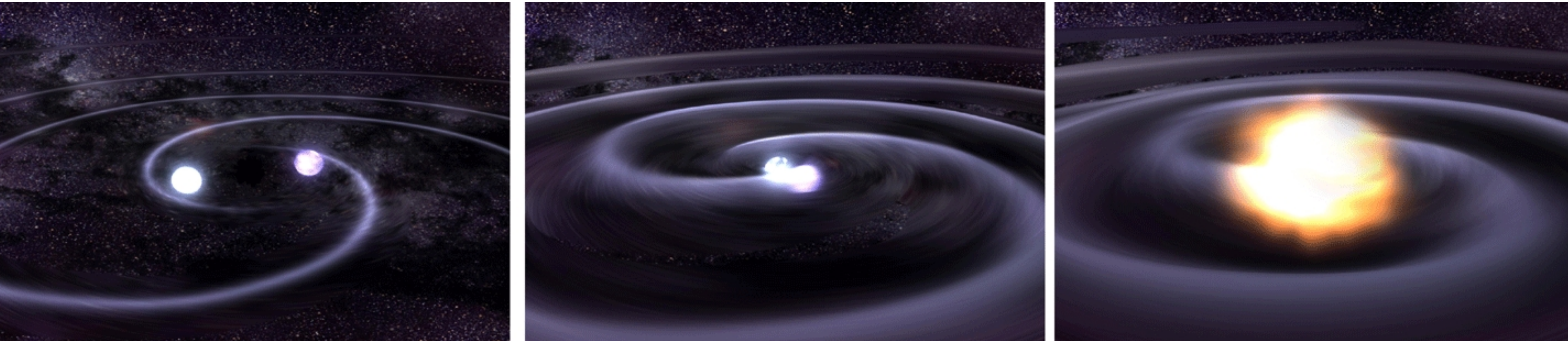
Livingston Observatory

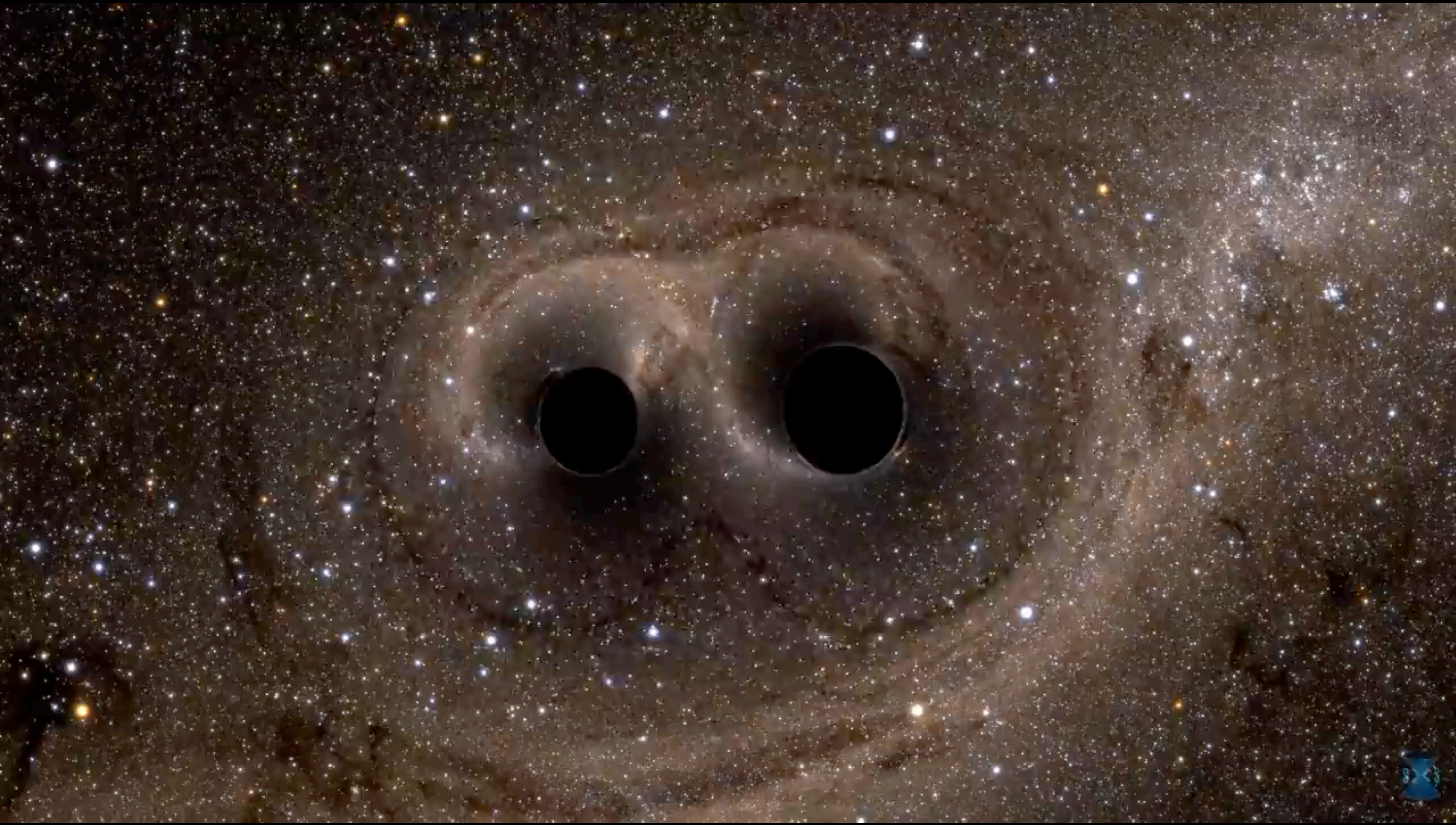


Auriga
LIGO
Virgo
GEO-HF
LCGT
Advanced Virgo
Advanced LIGO
aLIGO-upgrade
Einstein GW
Telescope
Strain [1/sqrt(Hz)]
Frequency [Hz]
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-25}
1
10
100
1000
10000

Numerical Relativity

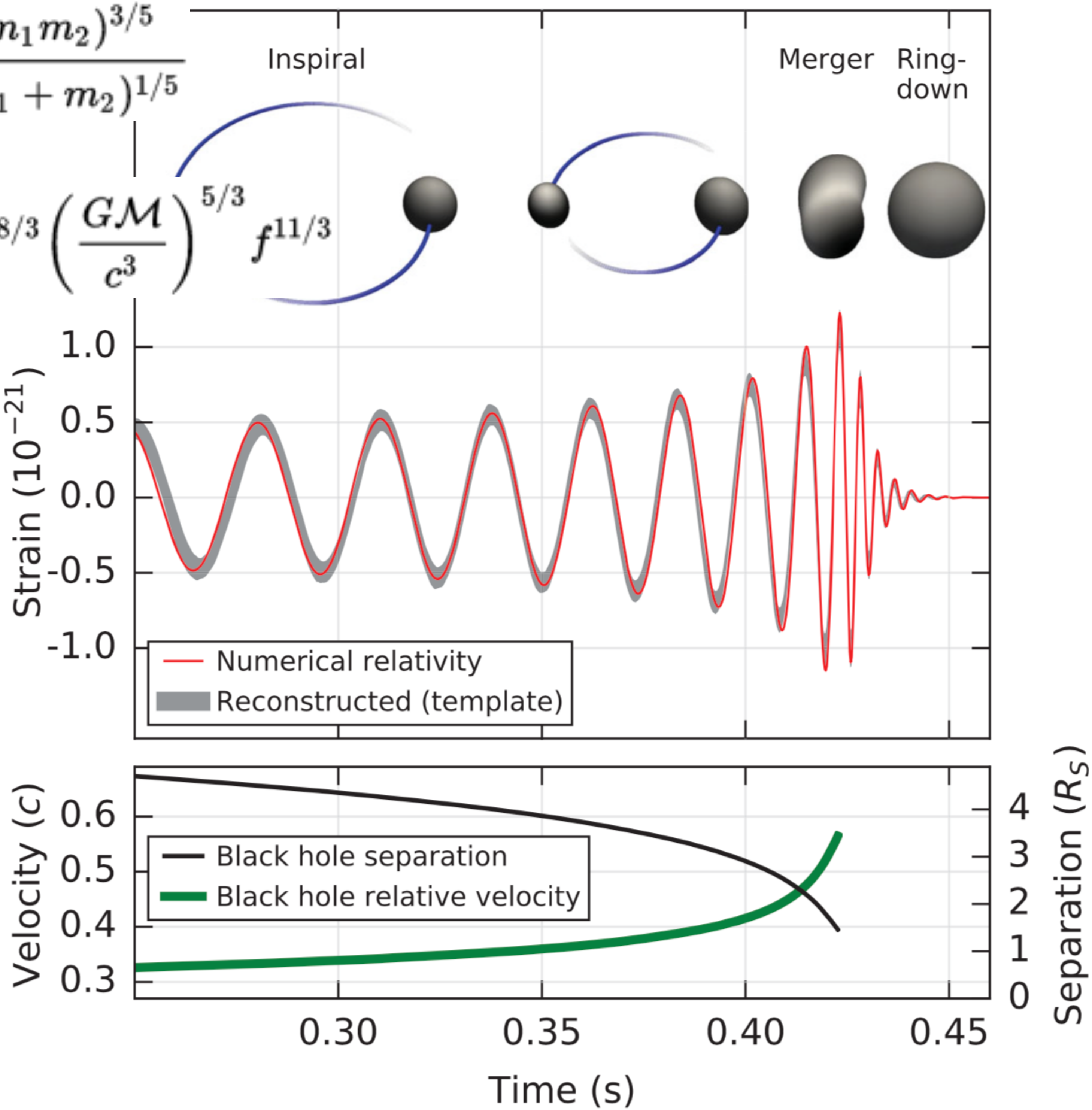
- Einstein equations very hard to solve analytically
- generally must approach the problem numerically
- efforts underway since 1970s – many numerical difficulties
- BH problem solved in 2005



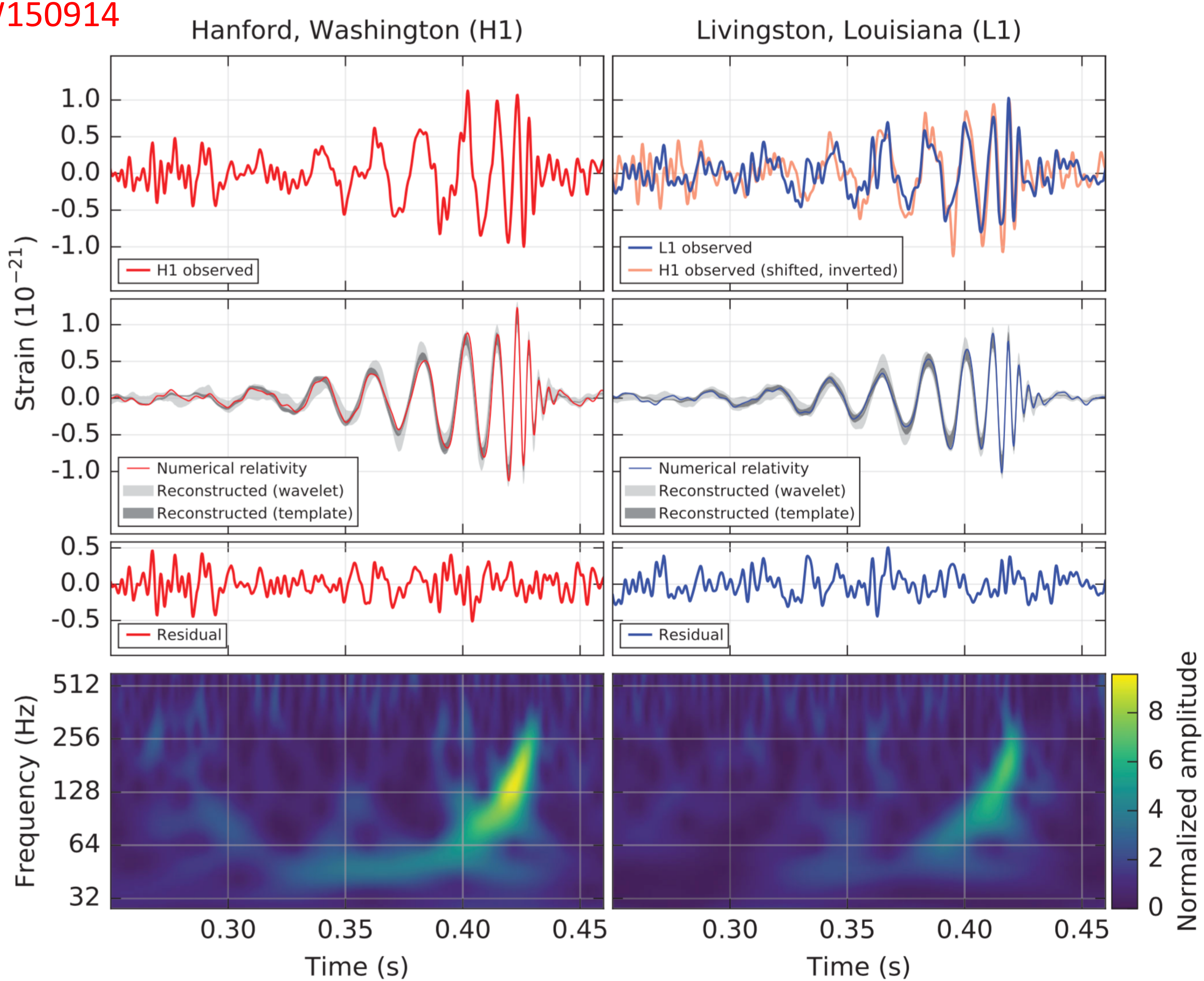


$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$\dot{f} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} f^{11/3}$$



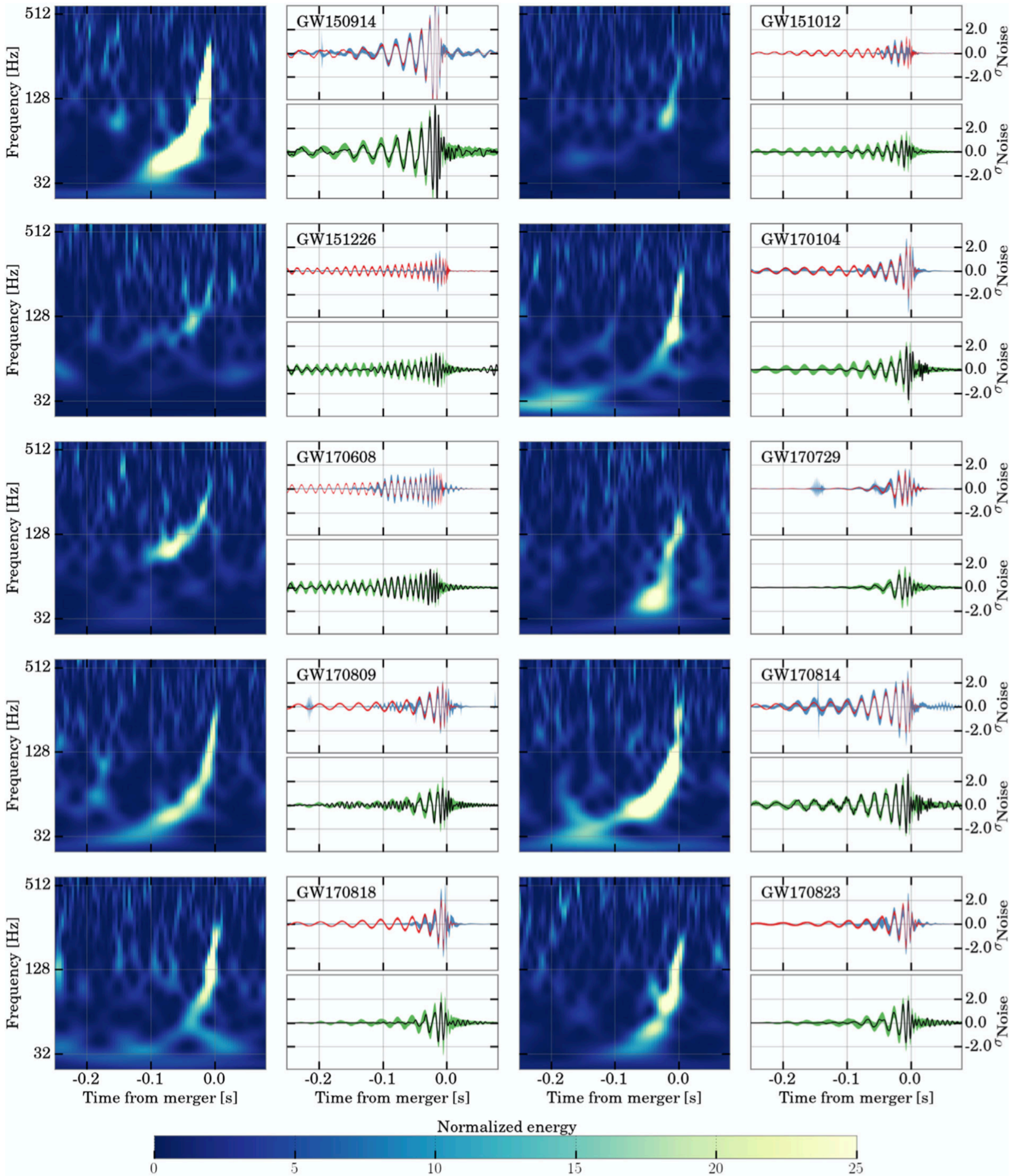
GW150914



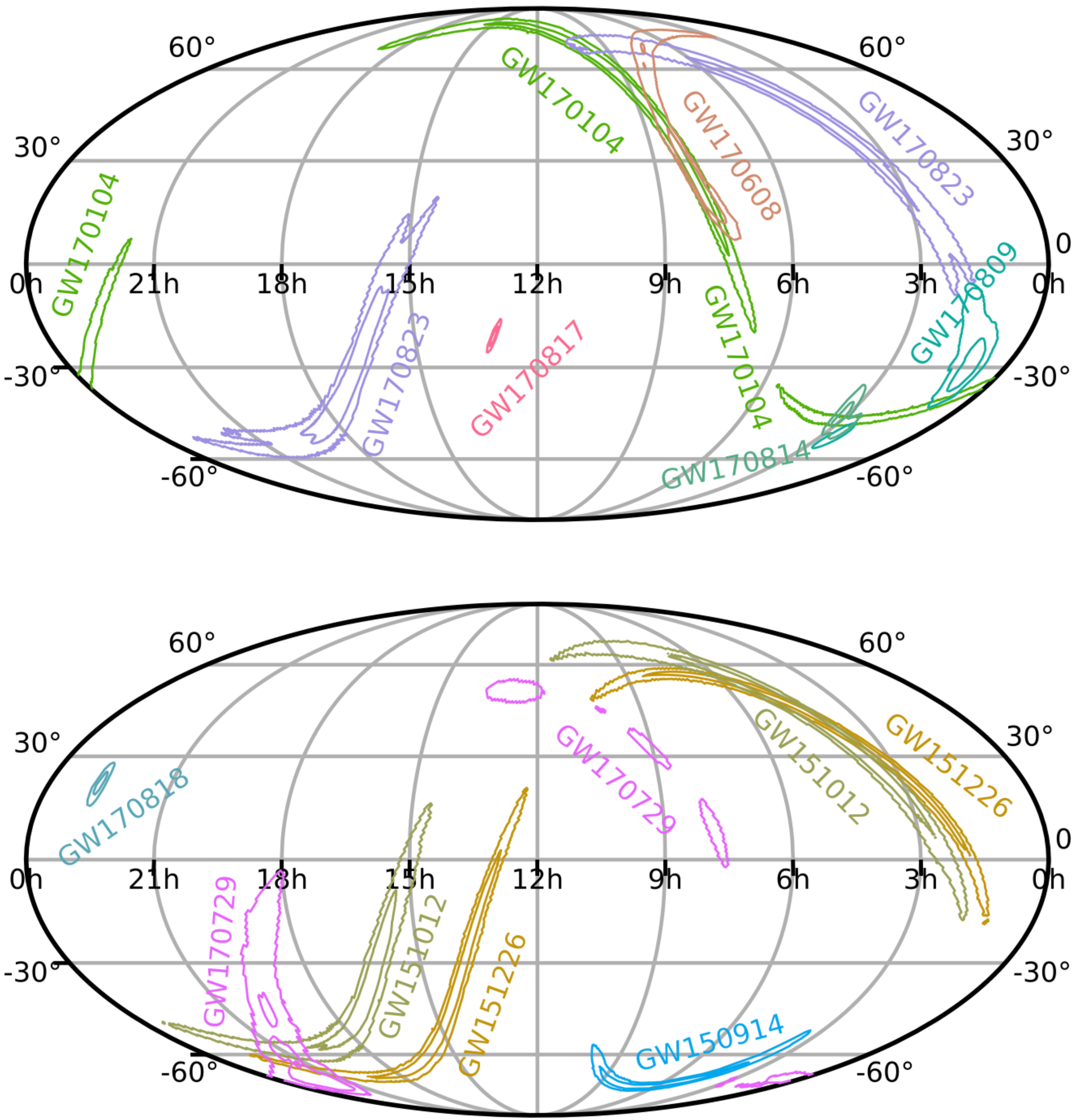
Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
Time (s)
Normalized amplitude

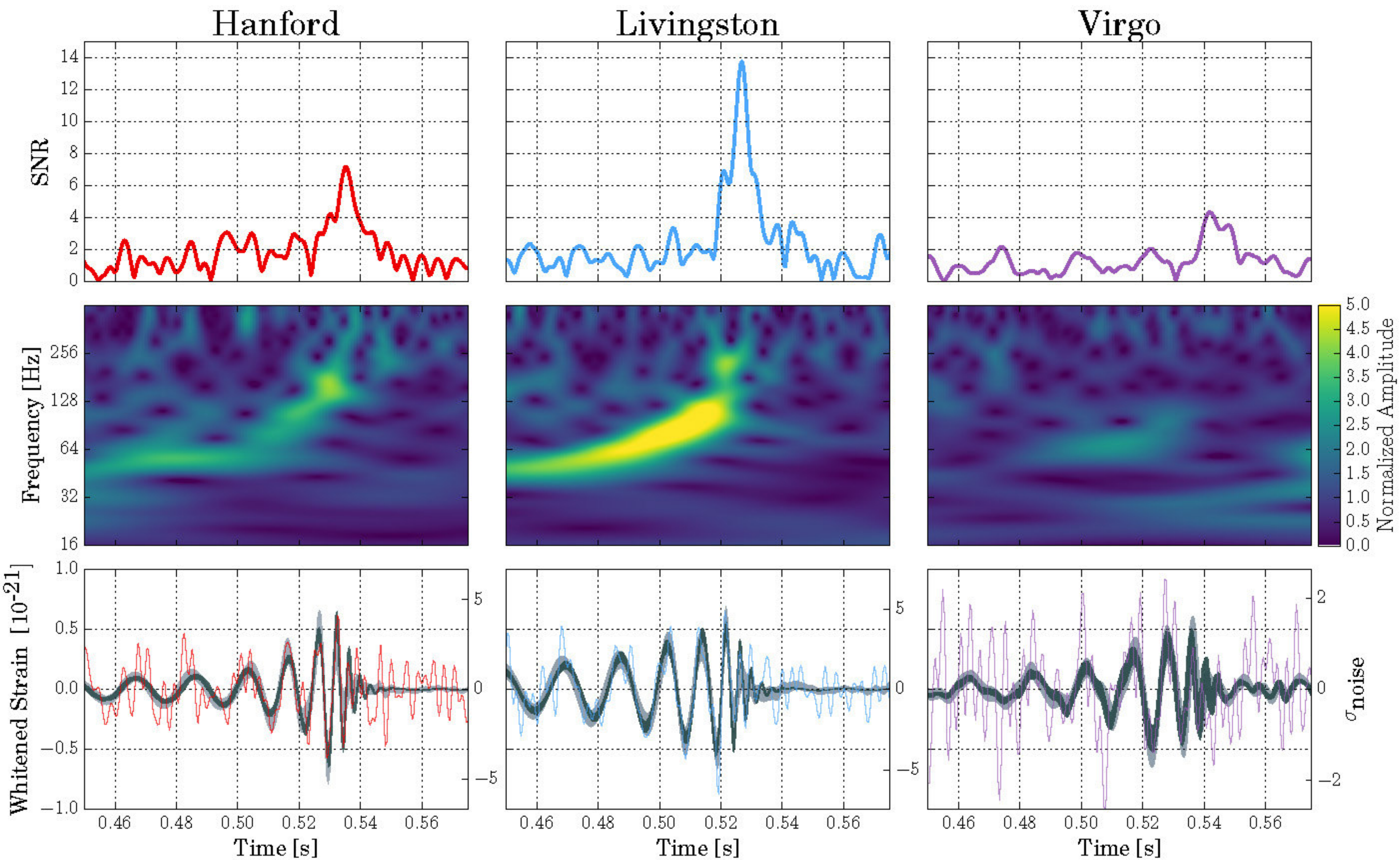
GW150914

- black hole masses 36 $M_\odot$ and 29 $M_\odot$
- $36 + 29 = 62!$
- $3\, M_\odot c^2$ energy $= 5.4 \times 10^{47}$ J emitted as gravitational waves (Earth annual consumption $= 4 \times 10^{20}$ J)
- 400 Mpc away



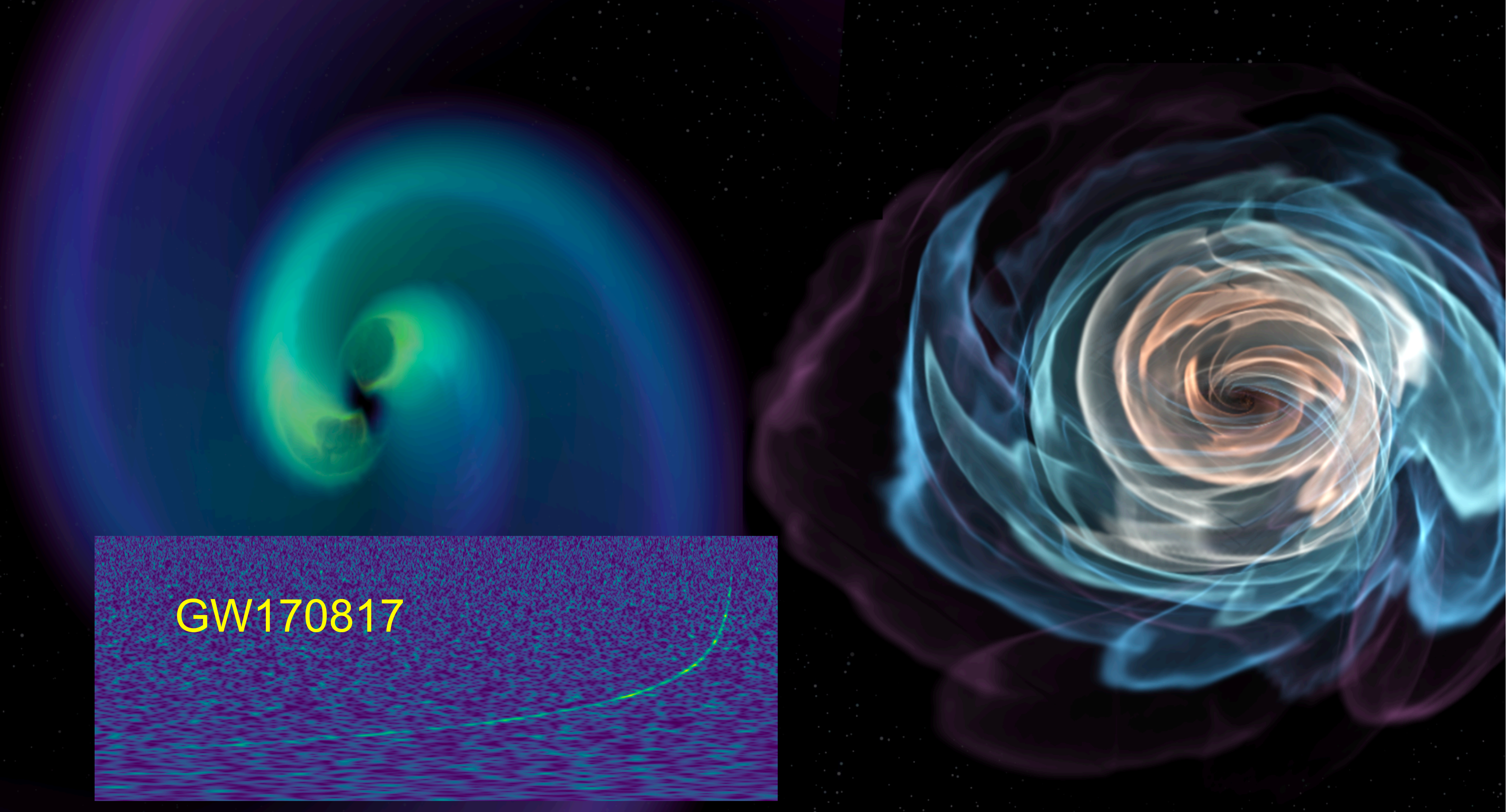
GW150914
GW151012
GW151226
GW170104
GW170608
GW170729
GW170809
GW170814
GW170818
GW170823
Frequency [Hz]
512
128
32
σ_{Noise}
2.0
0.0
-2.0
-0.2
-0.1
0.0
Time from merger [s]
Normalized energy
0
5
10
15
20
25


60°
30°
0°
-30°
-60°
0h
21h
18h
15h
12h
9h
6h
3h
GW170104
GW170608
GW170823
GW170809
GW170817
GW170814
GW170818
GW170729
GW151012
GW151226
GW150914



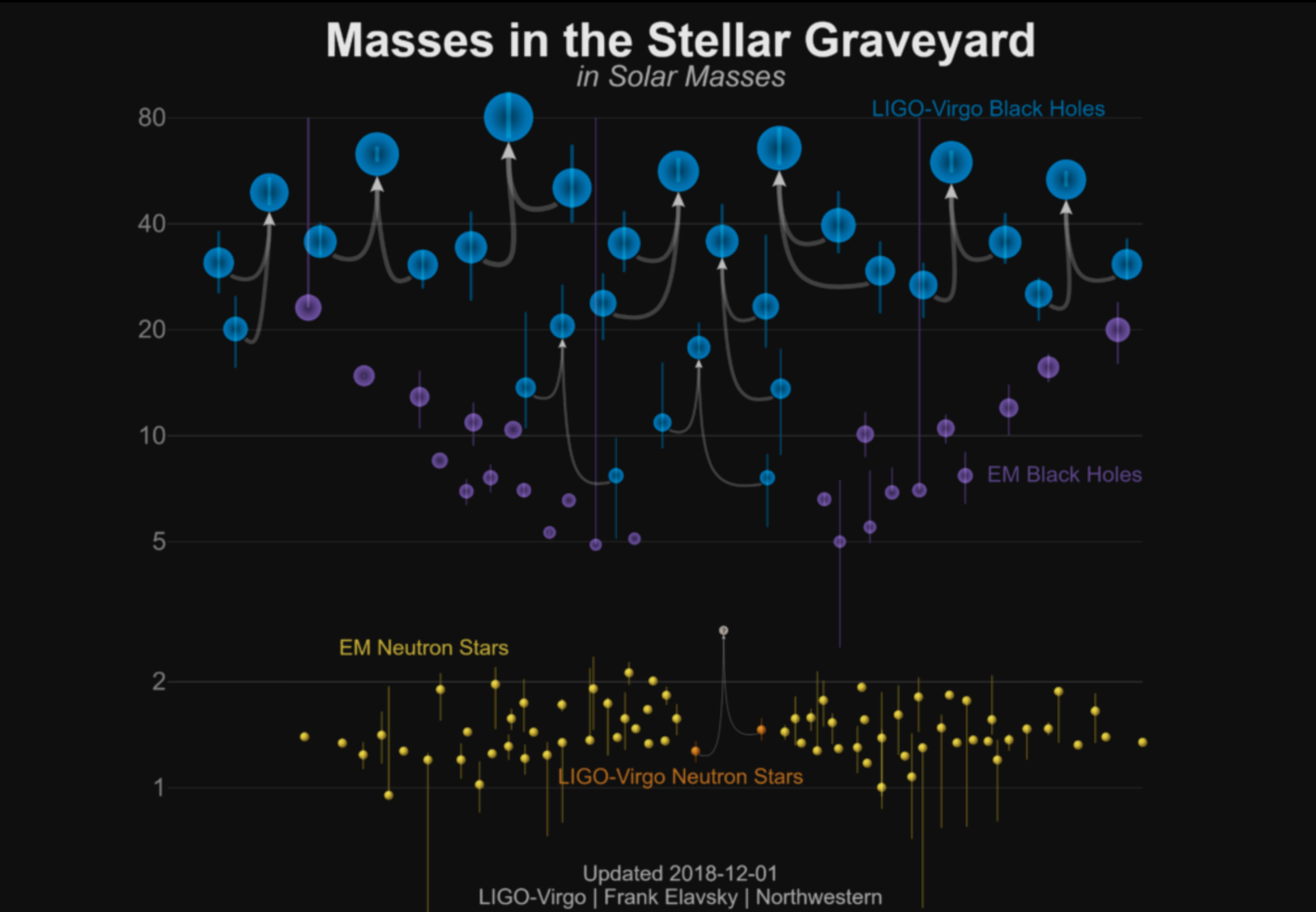
Hanford
Livingston
Virgo
SNR
Frequency [Hz]
Normalized Amplitude
Whitened Strain [10^-21]
σ_noise
Time [s]

First Cosmic Event Observed in Gravitational Waves and Light



LIGO

Georgia Tech Center for Relativistic Astrophysics College of Sciences


Masses in the Stellar Graveyard
in Solar Masses
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
EM Black Holes
EM Neutron Stars
LIGO-Virgo Neutron Stars
?
Updated 2018-12-01
LIGO-Virgo | Frank Elavsky | Northwestern