Physics 115: Contemporary Physics III

Spring 2013

Programming Assignment 3

Due May 22, 2013

Moving Charges in Electric and Magnetic Fields

We have seen earlier how to model the dynamics (motion) of a particle subject to an applied force $\mathbf{F}$. The force in this case will be the Lorentz force law:

$$\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

For convenience, we will set the particle's mass m and charge q to be 1 kg and 1 C, respectively, but retain these quantities as variables in your program.

For velocity-dependent forces such as this, the simple Leapfrog method (e.g. as discussed in PHYS 105) becomes harder to use, since the acceleration needed to advance the velocity requires knowledge of the velocity at a future time. A better method, which is completely equivalent to Leapfrog in the time-independent case and doesn't require the positions and velocities to be defined at different times, is the Velocity Verlet (or second-order predictor-corrector) scheme. Here the position and velocity of the particle at time t_n are x_n and v_n (so $n = 0$ represents the initial conditions), and we advance the system as follows:

$$\begin{aligned}
a_n &= a(x_n, v_n) \\
x_{n+1} &= x_n + v_n \Delta t + \tfrac{1}{2} a_n \Delta t^2 \\
v_p &= v_n + a_n \Delta t \\
a_p &= a(x_{n+1}, v_p) \\
v_{n+1} &= v_n + \tfrac{1}{2}(a_n + a_p)\Delta t
\end{aligned}$$

Note that we use the average of the accelerations at the start (a_n) and end (a_p) of the step to advance the velocity and an approximate, "predicted" velocity v_p in the computation of the acceleration a_p.

Use this scheme to answer the following problems.

1. For initial position and velocity

$$\begin{aligned}
\mathbf{r}_0 &= (0,0,0)\,\mathrm{m} \\
\mathbf{v}_0 &= (3,0,0)\,\mathrm{m/s},
\end{aligned}$$

model the following scenarios, restarting the particle at its initial position and velocity after time $t = 10\,\mathrm{s}$. Use a time step of $\Delta t = 0.05$. Do not clear the scene, but overlay each scenario with a trail of a different color.

   (a) No electric or magnetic field (boring!):

$$\mathbf{E} = (0,0,0)\,\mathrm{N/m}, \quad \mathbf{B} = (0,0,0)\,\mathrm{T}.$$

   (b) An electric field in the y direction:

$$\mathbf{E} = (0,1,0)\,\mathrm{N/m}, \quad \mathbf{B} = (0,0,0)\,\mathrm{T}.$$

(c) A magnetic field in the z direction:

$$\mathbf{E} = (0, 0, 0)\,\mathrm{N/m}, \quad \mathbf{B} = (0, 0, 1)\,\mathrm{T}.$$

(d) An oblique magnetic field:

$$\mathbf{E} = (0, 0, 0)\,\mathrm{N/m}, \quad \mathbf{B} = (1, 1, 1)\,\mathrm{T}.$$

(e) Crossed electric and magnetic fields:

$$\mathbf{E} = (0, 1, 0)\,\mathrm{N/m}, \quad \mathbf{B} = (0, 0, 1)\,\mathrm{T}.$$

In each case, predict and sketch what you think the motion will be before you run your program. After you have computed them all, rotate the camera to better see each interaction.

2. Now calculate the motion of the particle in the magnetic field of a *dipole* at the origin, oriented in the z direction:

$$\mathbf{B} = \frac{3z\mathbf{r}}{r^5} - \frac{\hat{z}}{r^3},$$

where $\mathbf{r} = (x, y, z)$. In this case, start the particle at $\mathbf{r}_0 = (1, 0, 0)$ m with velocity $\mathbf{v}_0 = (0.01, 0, 0.03)$ m/s and follow its motion until time $t = 100$ s using a time step $\Delta t = 0.01$ s. Can you explain the motion?