Physics 115: Contemporary Physics III

Spring 2013

Programming Assignment 2

Due May 3, 2013

Resistive and Capacitive Circuits

1. Solve the complex resistive circuit shown in Figure 20.46 (Section 20.10, p. 819) of Chabay & Sherwood. Use the following numerical values for the components:

$$\begin{array}{llll} \mathrm{emf}_1 = 6V, & r_1 = 0.5\Omega & & \\ \mathrm{emf}_2 = 9V, & r_2 = 0.25\Omega & & \\ R_1 = 1\Omega, & R_2 = 2\Omega, & R_3 = 2\Omega & R_4 = 5\Omega \\ R_5 = 10\Omega, & R_6 = 3\Omega & R_7 = 6\Omega & \end{array}$$

   Following the text, identify the unknown currents in the problem and write down, using the Kirchhoff Node and Loop rules, a sufficient number of equations to solve for these currents. Solve these equations numerically using the `solve` function in the `numpy.linalg` package. Verify that the currents you obtain do indeed satisfy the equations you derived.

2. Solve numerically the discharging capacitor problem. In the RC circuit shown in Chabay & Sherwood Figure 20.39 (p. 813), write

$$RI + \frac{Q}{C} = 0,$$

   with $I = dQ/dt$, so

$$\frac{dQ}{dt} + \frac{Q}{RC} = 0$$

   (the same differential equation as for the current in a charging capacitor!). Set $RC = 1$ (that is, take RC as the unit of time), so

$$\frac{dQ}{dt} + Q = 0,$$

   and imagine that the capacitor discharges not continuously, but in small steps, each of time duration Δt. Then in time Δt, we have

$$\frac{\Delta Q}{\Delta t} = -Q,$$

   so the change in the capacitor charge is

$$\Delta Q = -Q\Delta t.$$

   Starting from an initial charge $Q_0 = Q(0) = 1$, and taking steps $\Delta t = 0.01$, calculate approximately how the charge on the capacitor changes in time, for $t = 0, \Delta t, 2\Delta t, ..., 10$, using the rule

$$Q(t + \Delta t) = Q(t) - Q(t)\Delta t.$$

   Compare your solution with the analytic solution to the differential equation

$$Q = Q_0 e^{-t/RC} \ (= e^{-t} \ \text{ here})$$

   by plotting them on the same graph.