Physics 115: Contemporary Physics III

Spring 2013

Homework 7

Due May 31, 2013

1. Chabay & Sherwood, Problem 24.P.21, adding a final part:

   (c) For $N = 1000$, R = 10 cm, and $I = 5$ A, calculate the magnitude and direction of $d\mathbf{E}/dt$ needed to cancel the magnetic field at location **X**.

2. *[Not nearly as scary as it looks!]* Rederive the main results of Chabay & Sherwood, Section 24.2 using the differential form of Maxwell's equations, as follows. Start by assuming that the wave is sinusoidal and that the electric and magnetic fields oscillate in phase as the wave propagates in the x direction, according to the equations:

$$\begin{aligned}
E_x &= E_{x0}\cos(x - vt)\\
E_y &= E_{y0}\cos(x - vt)\\
E_z &= E_{z0}\cos(x - vt)\\
B_x &= B_{x0}\cos(x - vt)\\
B_y &= B_{y0}\cos(x - vt)\\
B_z &= B_{z0}\cos(x - vt),
\end{aligned}$$

   where $\mathbf{E}_0 = (E_{x0}, E_{y0}, E_{z0})$ and $\mathbf{B}_0 = (B_{x0}, B_{y0}, B_{z0})$ are constant vectors. Note that $\mathbf{E}$ and $\mathbf{B}$ are *independent* of y and z, so all derivatives below with respect to y and z are zero.

   (a) As in class, show that

$$\nabla\cdot\mathbf{E} \;=\; \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \;=\; \frac{\partial E_x}{\partial x},$$

   and similarly for $\mathbf{B}$, and hence use the Gauss relations

$$\begin{aligned}
\nabla\cdot\mathbf{E} &= 0\\
\nabla\cdot\mathbf{B} &= 0
\end{aligned}$$

   to show that $E_{x0} = 0$ and $B_{x0} = 0$. That is, the wave is *transverse*—the fields are perpendicular to the direction of motion.

   (b) With this simplification, let's now choose the y axis to be in the direction of $\mathbf{E}_0$, so we can write

$$\begin{aligned}
\mathbf{E} &= (0,\ E,\ 0)\cos(x - vt)\\
\mathbf{B} &= (0,\ B_{y0},\ B_{z0})\cos(x - vt).
\end{aligned}$$

   Show that the rather complicated general expression for $\nabla\times\mathbf{E}$,

$$\nabla\times\mathbf{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z},\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x},\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

simplifies to

$$\nabla \times \mathbf{E} = (0,\ 0,\ \ - E\sin(x - vt)).$$

Also show that

$$\frac{\partial \mathbf{B}}{\partial t} = \left(0,\ vB_{y0}\sin(x - vt),\ vB_{z0}\sin(x - vt)\right),$$

and hence from Faraday's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

show that $B_{0y} = 0$ and $E = vB_{0z}$.

(c) Based on parts (a) and (b) we can now write

$$\begin{aligned} \mathbf{E} &= (0,\ E,\ 0)\cos(x - vt) \\ \mathbf{B} &= (0,\ 0,\ B)\cos(x - vt), \end{aligned}$$

with $E = vB$. Use similar reasoning as in part (b), applied to the Ampère–Maxwell law

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t},$$

to show that $v = c$.

3. Consider a classical electron orbiting a positively charged nucleus in a circular orbit of radius R with speed V.

(a) What is the electron's acceleration?

(b) Use this acceleration in the equation at the bottom of Chabay & Sherwood, p. 996, to write down an *estimate* of the radiative electric field at distance r from the atom due to the electron's circular motion. For sake of argument, suppose that we are looking in a direction instantaneously perpendicular to the acceleration.

(c) What is the radiative magnetic field associated with this electric field?

(d) Hence estimate the total *power* instantaneously radiated by the atom.

(e) Estimate the atom's *lifetime* as the time taken to radiate away the electron's initial kinetic energy $\frac{1}{2}mV^2$. Write down an expression for this lifetime, and evaluate it for $R = 10^{-10}$ m, $V = 1.6 \times 10^6$ m/s?

4. Chabay & Sherwood, Problem 24.X.48.

5. Suppose that you lie in the Sun for 2.5 hours, exposing an area of $1.3\,\mathrm{m}^2$ to the Sun's rays of intensity $1.1\,\mathrm{kWm}^{-2}$. Assuming complete absorption of the rays, how much momentum is delivered to your body? What is the Sun's radiation pressure on you, in atmospheres? (1 atmosphere $= 1.0 \times 10^5$ N/m^2.)

6. A sinusoidal electromagnetic wave is emitted by an electric dipole antenna. The radiation has wavelength $\lambda = 0.01$ m. Answer all of the following questions in SI units (kg, m, s).

(a) What is the speed of the wave?

(b) What is the frequency in Hz of the radiation?

Suppose we observe the wave a distance of 10 m from the antenna. At that distance, the magnetic part of the wave has an amplitude of $B = 1.1 \times 10^{-4}$ T.

(c) What is the amplitude of the electric part of the wave?

(d) What is the energy per unit time per unit area of the radiation moving past this point?

(e) Now measure the amplitudes of the electric and magnetic fields at a distance of 20 m. By what factor are they smaller than at 10 m?