Physics 115: Contemporary Physics III

Spring 2013

Homework 7 Solutions

1. (a) Measuring $B = |\mathbf{B}|$ positive in the clockwise direction in the diagram, and applying Ampère's Law to a clockwise circular loop C of radius R along the mid-line of the torus, then the current threads C in the positive (inward) sense and we have

$$\begin{aligned} \oint_C \mathbf{B} \cdot d\mathbf{l} &= \mu_0 NI \\ 2\pi RB &= \mu_0 NI \\ B &= \frac{\mu_0}{4\pi} \frac{2NI}{R}. \end{aligned}$$

(b) With C as defined above, the normal to the surface spanning C is into the page. Let $E > 0$ be the component of the electric field into the page. As E varies, the Ampère–Maxwell law implies

$$\begin{aligned} \oint_C \mathbf{B} \cdot d\mathbf{l} &= \mu_0 NI + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \\ B &= \frac{\mu_0}{4\pi} \frac{2NI}{R} + \frac{\mu_0 \epsilon_0 \pi R^2}{2\pi R} \frac{dE}{dt} \\ &= \frac{\mu_0}{4\pi} \frac{2NI}{R} + \frac{R}{2c^2} \frac{dE}{dt}. \end{aligned}$$

The additional magnetic field is parallel to the original one, pointing at 45° toward the top left at location **X**.

(c) In order to cancel the original magnetic field, we require dE/dt negative, with

$$\frac{dE}{dt} = -\frac{NI}{\pi \epsilon_0 R^2}.$$

For the given values, $dE/dt = -1.8 \times 10^{16}\,\mathrm{N/Cs}$.

2. (a) Since all components depend only on x and t, all partial derivatives with respect to y or z are zero. Hence

$$\nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} = -E_{x0} \sin(x - vt),$$

so Gauss's law implies that $E_{x0} = 0$. Similarly, $B_{x0} = 0$.

(b) For

$$\begin{aligned} \mathbf{E} &= (0,\ E,\ 0) \cos(x - vt) \\ \mathbf{B} &= (0,\ B_{y0},\ B_{z0}) \cos(x - vt), \end{aligned}$$

and setting all derivatives with respect to y or z to zero, we find

$$\nabla \times \mathbf{E} = \left(0,\ 0,\ \frac{\partial E_y}{\partial x}\right) = (0,\ 0,\ -E \sin(x - vt))$$

and

$$\begin{aligned}\frac{\partial \mathbf{B}}{\partial t} &= \left(0,\ \frac{\partial B_y}{\partial t},\ \frac{\partial B_z}{\partial t},\right) \\ &= (0,\ vB_{y0}\sin(x-vt),\ vB_{z0}\sin(x-vt))\,.\end{aligned}$$

Hence Faraday's Law implies

$$\begin{aligned}B_{y0} &= 0 \\ vB_{z0} &= E.\end{aligned}$$

(d) Similarly, writing $B_{z0} = B$, we have

$$\nabla \times \mathbf{B} = \left(0,\ -\frac{\partial B_z}{\partial x},\ \frac{\partial B_y}{\partial x}\right) = (0,\ B\sin(x-vt),\ 0)$$

and

$$\frac{\partial \mathbf{E}}{\partial t} = (0,\ vE\sin(x-vt),\ 0),$$

so the Ampère–Maxwell Law implies $B = \mu_0\epsilon_0 vE$. Combining this with $E = vB$ from part (c) we find $\mu_0\epsilon_0 v^2 = 1$, so $v = c$.

3. (a) The electron's acceleration is

$$a = \frac{V^2}{R} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mR^2},$$

where the second term is just the Coulomb acceleration of an electron moving in a circular orbit.

(b) The radiative electric field at distance r, neglecting the angular dependence, is

$$E_r \approx \frac{1}{4\pi\epsilon_0}\frac{ea}{c^2 r}.$$

(c) The radiative magnetic field is $B_r = E_r/c$.

(d) The total instantaneous energy flux (power per unit area) is

$$f = E_r B_r/\mu_0 = E_r^2\, c\epsilon_0.$$

Again neglecting the angular distribution (or averaging over all angles and electron orientations), the total radiated power is

$$\begin{aligned}P &= 4\pi r^2 f \approx 4\pi r^2\, E_r^2 c\epsilon_0 \\ &= 4\pi r^2 \frac{1}{(4\pi\epsilon_0)^2}\frac{e^2V^4}{c^4r^2R^2}\, c\epsilon_0 \\ &= \left[\frac{1}{4\pi\epsilon_0}\frac{e^2}{R}\right]\left(\frac{V}{c}\right)^3 \frac{V}{R}.\end{aligned}$$

From part (a), the first factor (in square brackets) is mV^2, or twice the electron kinetic energy $K = \frac{1}{2}mV^2$. Thus we can estimate the lifetime as

$$T \sim \frac{K}{P} = \left(\frac{c}{V}\right)^3 \frac{R}{2V}.$$

For $R = 10^{-10}$ m, $V = 1.6\times 10^6$ m/s, we find $T \sim 2\times 10^{-10}$ s.

Simply put, the direction of the force due to radiation pressure on a positively charged particle is in the same direction as for a negatively charged particle. As a result, the net radiation pressure on the neutral dust grain will be to the right.

Model a neutral atom in the particle of dust as a positively charged nucleus with a negatively charged electron cloud surrounding the nucleus, so it can be treated as an electric dipole. Suppose that a pulse of electromagnetic radiation travels to the right in the $+x$ direction, with $\vec{E}$ upward in the $+y$ direction and $\vec{B}$ outward in the $+z$ direction. When this pulse reaches the neutral atom, the electric force on the positively charged nucleus is upward, causing the nucleus to accelerate upward. With an upward velocity, the magnetic field exerts a force on the nucleus that is to the right, in the $+x$ direction. Now consider the electron cloud. The electric field exerts a downward force on the electron cloud accelerating it downward and giving it a downward velocity. The magnetic force by the magnetic field on the downward moving electron cloud is also to the right. Thus, the radiation pressure on the neutral atom is to the right.

24.X.47

It tells you that the transmitting antennas must also be mounted horizontally. As seen in the textbook in Figure 24.42, the maximum radiative electric field (which is what drives electrons in the receiving antenna) occurs along an axis parallel to the transmitting antenna. The reason is that $\vec{E}_{radiative}$ is proportional to $\vec{a}_\perp$.

24.X.48

Suppose we define East to be in the $+x$ direction (to the right), so sunlight travels west. When you look overhead, you mostly see light traveling in the $-y$ direction (with $+y$ defined as upward, perpendicular Earth's surface) that is reradiated from air molecules. This reradiation is due to charges accelerating in the $\pm z$ direction (which in this case is the north-south direction). As a result, the reradiated light that is traveling downward toward you is polarized parallel to the accelerating charges, in the north-south direction.

24.X.49

The reason that on Earth you see light coming from other directions besides the Sun, but you do not see this on the Moon, is that Earth has an atmosphere and the Moon does not. On Earth, when you look away from the Sun, air molecules in this direction are being accelerated by radiation from the Sun, and they reradiate light toward you. Thus, you see a bright sky. The reradiated sunlight from the sky has a much greater energy than the light from the stars, so the starlight cannot be seen during the day on Earth.

On the Moon, there is no atmosphere. Thus, if you look away from the Sun, the only radiation interacting with your eye comes from stars.

24.X.50

When the radiative electric field interacts with electrons in the metal rod, it accelerates the accelerates the electrons in the $\pm z$ direction. The re-radiation from these electrons at a given location in space is proportional to the perpendicular component of the acceleration of the electrons. Use this to answer the given questions.

(a) **Detector A:**

  (1) The radiation propagates in the $-y$ direction.

  (1) $\vec{E}$ at detector A oscillates in the $\pm z$ direction.

  (1) Since the wave propagates in the direction of $\vec{E} \times \vec{B}$, then $\vec{B}$ oscillates in the $\pm x$ direction at detector A.

(b) **Detector B:**

  Because detector B is along the same axis as the acceleration of the electrons in the wire, then there will be no radiation detected at B. Thus, $\vec{E}$ and $\vec{B}$ are zero.

5 The Sun's energy flux is $f = E_r B_r/\mu_0 = 1.1\,\mathrm{kW\,m^{-2}}$. Thus, in 2.5 hours, the total energy absorbed by an area of $1.3\,\mathrm{m}^2$ is $E = 1.1 \times 10^3 \times 1.3 \times 2.5 \times 3600 = 1.29 \times 10^7\,\mathrm{J}$. The total momentum is $p = E/c = 0.043$ kg m/s^2. The radiation pressure is $f/c = 3.7 \times 10^{-6}\,\mathrm{N/m^2} = 3.7 \times 10^{-11}$ atmospheres.

6 (a) The speed is $c = 3 \times 10^8\,\mathrm{m/s}$.

(b) The frequency is $f = c/\lambda = 3 \times 10^{10}\,\mathrm{Hz}$.

(c) The electric amplitude is $E = cB = 3.3 \times 10^4\,\mathrm{V/m}$.

(d) The radiation flux is $EB/\mu_0 = 2.9 \times 10^6\,\mathrm{W/m^2}$ (huge!).

(e) At double the distance the fields will be smaller by a factor of two: $E = 1.7 \times 10^4\,\mathrm{V/m}$ and $B = 5.5 \times 10^{-5}\,\mathrm{T}$.