Physics 115: Contemporary Physics III

Spring 2013

Homework 6 Solutions

There will be no induced current in the ring (zero!).

The magnetic field around the wire is tangent to the ring. Therefore, the magnetic flux is $\int BdA\cos(\theta) = 0$, since the angle between the magnetic field and unit vector normal to the plane of the ring is 90° and $\cos(90°) = 0$. Then, Faraday's law predicts that $\vec{E}_{NC} = 0$ at points inside the metal ring, and therefore no current will be "pushed" through the ring.

23.P.26

Assume the magnet's north pole is falling from above the tube. Assume the tube is composed of many thin concentric copper rings. As the falling north pole approaches the topmost ring, the magnetic flux through that ring changes with the bar's magnetic field pointing down and increasing in magnitude. There is a curly electric field in the conducting ring that curls counterclockwise as seen from above the tube. The curly electric field drives a counterclockwise conventional current in the ring. This current creates a brand new magnetic field at the ring's center with an induced north pole repelling the falling north pole. Therefore, the bar's motion will be retarded as it falls through the topmost ring. This argument holds for all successive rings as the north pole falls through them.

Now consider the falling south pole. As it falls through the topmost ring, the magnetic flux through that ring changes with the bar's magnetic field pointing down and decreasing in magnitude. There is a curly electric field in the conducting ring that curls clockwise as seen from above the tube. The curly electric field drives a clockwise conventional current in the ring. This current creates a brand new magnetic field at the ring's center with an induced north pole pointing down, attracting the falling south pole. Therefore, as above, the bar's motion will be retarded as it falls. This argument holds for all successive rings as the south pole falls through them.

As an additional exercise, students should write out an explanation for reversing the magnet and dropping it.

23.P.27

(a) $\vec{B}$ points away from the loop toward the coil

(b) There is no field in the copper loop.

(c) There is no location Q in the copper loop, but there is still no field anywhere in the loop.

(d) $\frac{dI}{dt} < 0$ therefore $\left|\vec{B}\right|$ decreases and $\frac{d\vec{B}}{dt}$ points toward the loop away from the coil.

(e) $-\frac{d\vec{B}}{dt}$ points away from the loop toward the coil.

(f) At P, $\vec{E}_{NC}$ is in the $-y$ direction.

(g) $\left|\Phi_{mag}\right|$ decreases with time.

23.P.28

(a)

$$\begin{aligned}\left|\Phi_{mag}\right| &= \left|\vec{B}\right| A = \left(\frac{\mu_o}{4\pi}\right) A \approx \left(\frac{\mu_o}{4\pi}\frac{2NI\pi r_c^3}{d_{cl}^3}\right)\left(\pi r_l^2\right) \\ &\approx \left(1\times 10^{-7}\ \frac{\mathrm{T\cdot m^2}}{\mathrm{C\cdot m/s}}\right)\frac{2(300)(5\ \mathrm{A}\pi^2\,(0.09\ \mathrm{m})^2\,(0.04\ \mathrm{m})^2}{(0.22\ \mathrm{m})^3} \\ &\approx 2.6\times 10^{-6}\ \mathrm{T\cdot m^2}\end{aligned}$$

(b) Treat the coil as a magnetic dipole.

(c) There is no electric field in the loop.

(d) $\frac{dI}{dt} = -0.3$ A/s and $\vec{\mathrm{E}}_{\mathrm{NC}}$ is in the $-y$ direction.

(e)

$$\begin{aligned}\left|\frac{d\Phi_{\mathrm{mag}}}{dt}\right| &= \frac{\mu_o}{4\pi}\frac{2N\pi^2 r_c^2 r_l^2}{d_{cl}^3}\left|\frac{dI}{dt}\right| \\ &\approx \left(1\times 10^{-7}\ \frac{\mathrm{T\cdot m^2}}{\mathrm{C\cdot m/s}}\right)\frac{2(300)\pi^2\,(0.09\ \mathrm{m})^2\,(0.04\ \mathrm{m})^2}{(0.22\ \mathrm{m})^3}\,(0.3\ \mathrm{A/s}) \\ &\approx 2.16\times 10^{-7}\ \mathrm{V}\end{aligned}$$

(f)

$$\begin{aligned}|\mathrm{emf}| &= \left|\frac{d\Phi_{\mathrm{mag}}}{dt}\right| \\ &\approx 2.16\times 10^{-7}\ \mathrm{V}\end{aligned}$$

(g)

$$\begin{aligned}|\mathrm{emf}| &= \oint_C \vec{\mathrm{E}}_{\mathrm{NC}}\bullet d\vec{\mathrm{l}} = 2\pi r_l\left|\vec{\mathrm{E}}_{\mathrm{NC}}\right| \\ \left|\vec{\mathrm{E}}_{\mathrm{NC}}\right| &= \frac{|\mathrm{emf}|}{2\pi r_l} \\ &\approx \frac{2.16\times 10^{-7}\ \mathrm{V}}{2\pi\,(0.04\ \mathrm{m})} \\ &\approx 8.59\times 10^{-7}\ \mathrm{V/m}\end{aligned}$$

(h) Removing the loop doesn't change the curly electric field.

23.P.29

(a) -dervectBt is into the page, so $\vec{\mathrm{E}}_{\mathrm{NC}}$ is clockwise around the loop. So at P, $\vec{\mathrm{E}}_{\mathrm{NC}}$ is up.

(b)

$$\begin{aligned}\frac{d\left|\vec{\mathrm{B}}\right|}{dt} &= 3bt^2 \\ |\mathrm{emf}| &= \frac{d\left|\vec{\mathrm{B}}\right|}{dt}\pi r_1^2 = \oint_C \vec{\mathrm{E}}_{\mathrm{NC}}\bullet d\vec{\mathrm{l}} = 2\pi r_1\left|\vec{\mathrm{E}}_{\mathrm{NC}}\right| \\ \left|\vec{\mathrm{E}}_{\mathrm{NC}}\right| &= \frac{3}{2}bt^2 r_1 \\ &\approx \frac{3}{2}\left(1.4\ \mathrm{T/s^3}\right)(1.3\ \mathrm{s})^2\,(0.036\ \mathrm{m}) \\ &\approx 0.128\ \mathrm{N/C}\end{aligned}$$

(c) At Q, $\vec{\mathrm{E}}_{\mathrm{NC}}$ is down.

(d)

$$\begin{aligned}
|\text{emf}| &= 3bt^2\pi R^2 = \oint C\vec{\mathrm{E}}_{\text{NC}} \bullet d\vec{\mathrm{l}} = 2\pi r_2 \left|\vec{\mathrm{E}}_{\text{NC}}\right| \\
\left|\vec{\mathrm{E}}_{\text{NC}}\right| &= \frac{3bt^2R^2}{2r_2} \\
&\approx \frac{3\left(1.4\ \text{T/s}^3\right)(1.3\ \text{s})^2(0.17\ \text{m})^2}{2\,(0.51\ \text{m})} \\
&\approx 0.2\ \text{N/C}
\end{aligned}$$

23.P.30

(a)

$$\begin{aligned}
\left|\vec{\mathrm{B}}\right| &= \frac{\mu_o}{4\pi}\frac{2I}{x} \\
\left|\frac{d\vec{\mathrm{B}}}{dt}\right| &= \frac{\mu_o}{4\pi}\frac{2I\,|\vec{\mathrm{v}}|}{x^2} \\
&\approx \left(1\times 10^{-7}\ \frac{\text{T}\cdot\text{m}^2}{\text{C}\cdot\text{m/s}}\right)\frac{2\,(3\ \text{A})\,(3.2\ \text{m/s})}{(0.13\ \text{m})^2} \\
&\approx 1.14\times 10^{-4}\ \text{T/s}
\end{aligned}$$

(b)

$$\begin{aligned}
|\text{emf}| &= N\left|\frac{d\Phi_{\text{mag}}}{dt}\right| = N\left|\frac{d\vec{\mathrm{B}}}{dt}\right|\pi r^2 \\
&\approx (11)\left(1.14\times 10^{-4}\ \text{T/s}\right)\pi\,(0.02\ \text{m})^2 \\
&\approx 1.58\times 10^{-6}\ \text{V}
\end{aligned}$$

(c) $\frac{d\vec{\mathrm{B}}}{dt}$ is into the page.

(d) $\vec{\mathrm{E}}_{\text{NC}}$ is counterclockwise in the loop.

23.P.31

(a)

$$\begin{aligned}
\left|\vec{\mathrm{B}}_1\right| &= \frac{\mu_o}{4\pi}\frac{2\,|\vec{\mu}|}{x^3} = \frac{\mu_o}{4\pi}\frac{2N_1I_1\pi r_1^2}{x^3} \\
\left|\Phi_2\right| &= \left|\vec{\mathrm{B}}_1\right| A_2 = \frac{\mu_o}{4\pi}\frac{2N_1N_2\pi^2r_1^2r_2^2I_1}{x^3} \\
|\text{emf}| &= \left|\frac{d\Phi_{\text{mag}}}{dt}\right| = \frac{\mu_o}{4\pi}\frac{2N_1N_2\pi^2r_1^2r_2^2}{x^3}\left|\frac{dI_1}{dt}\right| \\
&= \frac{\mu_o}{4\pi}\frac{2N_1N_2\pi^2r_1^2r_2^2}{x^3}\,(b+2ct)
\end{aligned}$$

(b) $dI_1/dt > 0$ so $\frac{d\vec{\mathrm{B}}}{dt}$ is toward the first coil so -vectderBt is toward the second coil. So at P, $\vec{\mathrm{E}}_{\text{NC}}$ is down.

$$
\begin{aligned}
E &= \frac{\Delta V}{2\pi r} \\
&= \frac{3.95 \times 10^{-9}\ \mathrm{V}}{2\pi(0.005\ \mathrm{m})} \\
&= 1.26 \times 10^{-7}\ \frac{\mathrm{V}}{\mathrm{m}}
\end{aligned}
$$

23.P.38

(a) $\vec{\mathrm{B}}$ is out of the page and increasing. So $-\frac{d\vec{\mathrm{B}}}{dt}$ is into the page and the induced current in the rectangular coil flows clockwise.

(b) Apply Faraday's Law to the rectangular coil.

$$
\begin{aligned}
|\mathsf{emf}| &= \left|\frac{d\Phi}{dt}\right| \\
&= NA_{\mathrm{solenoid}}\left|\frac{dB}{dt}\right| \\
&= N\pi r^2 \frac{d}{dt}(0.07 + 0.03t^2) \\
&= N\pi r^2 2(0.03\ \frac{\mathrm{T}}{\mathrm{s}^2})t \\
&= 4\pi(0.03\ \mathrm{m})^2 2(0.03\ \frac{\mathrm{T}}{\mathrm{s}^2})(2\ \mathrm{s}) \\
&= 0.00135\ \mathrm{V} \\
&= 1.35\ \mathrm{mV}
\end{aligned}
$$

Apply Ohm's Law to the coil.

$$
\begin{aligned}
\Delta V &= IR \\
I &= \frac{\Delta V}{R} \\
&= \frac{0.00136\ \mathrm{V}}{0.1\ \Omega} \\
&= 0.0136\ \mathrm{A} \\
&= 13.6\ \mathrm{mA}
\end{aligned}
$$

23.P.39

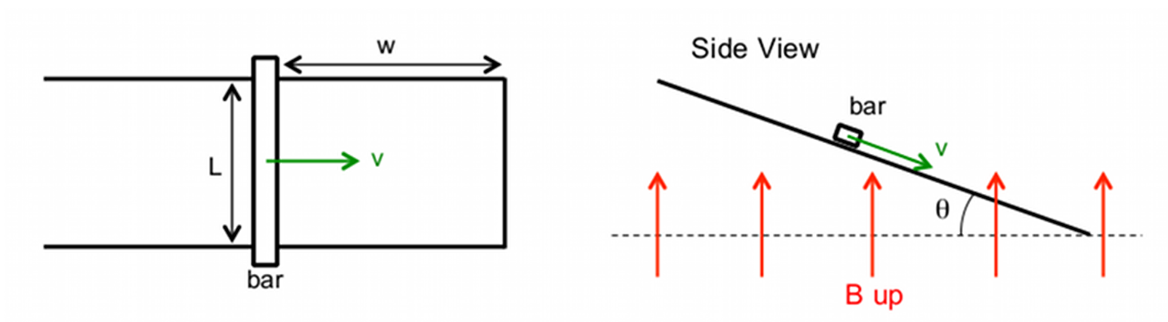
(a) Fundamental principles: Changing magnetic field in solenoid creates curly non-Coulomb electric field around itself, which drives current in the metal ring. This new current creates a magnetic field, and at the center of the ring this field plus the magnetic field of the solenoid makes the net magnetic field at that location. (See Figure 3.)

4. The magnetic flux through the square (of side $L = 2\,\mathrm{cm}$) is

$$\begin{aligned}\Phi_M(t) &= \int_0^L \int_0^L B_z(x, y, y)\, dx dy \\ &= 4t^2 \int_0^L dx \int_0^L y\, dy \\ &= 2t^2 L^3 .\end{aligned}$$

Hence $d\Phi_M/dt = 4tL^3 = 9.6 \times 10^{-5}\,\mathrm{V}$ at time $t = 3\,\mathrm{s}$. This is the induced emf around the wire. Its sense, by Lenz's law, opposes the increase of the flux, so its associated magnetic field must be in the negative z direction, and the induced electric field is in the clockwise direction.

5. In the diagram below, in steady state, the bar slides downward with speed V at an angle θ to the horizontal. The magnetic flux through the $L \times w$ rectangle is $\Phi_M = BLw \cos\theta$.



(a) As the bar moves,

$$\frac{d\Phi_M}{dt} = BL\cos\theta\,\frac{dw}{dt} = BL\cos\theta\, v,$$

so the induced emf is $\mathcal{E} = BLv\cos\theta$ and the induced current is $I = BLv\cos\theta/R$. The magnetic force on the bar is ILB to the left. Its component up the slope is $ILB\cos\theta$. Equating this (in steady state) to the gravitational force down the ramp, $mg\sin\theta$, we find

$$(BLv\cos\theta/R)\,(LB\cos\theta) = mg\sin\theta,$$

so

$$v = \frac{mgR}{B^2L^2}\,\frac{\sin\theta}{\cos^2\theta}. \qquad (\dagger)$$

(b) As the bar slides, the rate of change of gravitational potential energy is

$$-\frac{d}{dt}(mgw\sin\theta) = -mgv\sin\theta.$$

The power induced in the bar is

$$\begin{aligned}P = I \times \mathrm{emf} &= \frac{(BLv\cos\theta)^2}{R} \\ &= mgv\sin\theta \qquad [\text{from } (\dagger)].\end{aligned}$$

(c) If $\mathbf{B}$ pointed down, rather than up, the emf in the bar would reverse direction, but the steady-state velocity would be the same.

6. For an R–L circuit, we have

$$I(t) = \frac{\mathcal{E}}{R}\left[1 - e^{-(R/L)t}\right].$$

(a) Before the fuse blows, $R = 0$, since the fuse is in parallel with the resistor, so the above equation is indeterminate. Going back to first principles, the loop rule around the circuit in that case (with zero resistance) gives

$$\mathcal{E} - L\frac{dI}{dt} = 0,$$

so

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L},$$

the solution to which [with $I(0) = 0$] is

$$I = \frac{\mathcal{E}}{L}\,t = 2t \quad \text{here.}$$

Thus the current reaches 3 A at time $t = 1.5$ s.

(b) At that point the fuse blows and the resistance in the circuit jumps to $R = 15\,\Omega$. The steady-state current in the new circuit is $\mathcal{E}/R = \frac{2}{3}$ A.

(c) Because of the inductance in the circuit, the current through the inductor cannot immediately drop to the new steady-state value. Any attempt to reduce the current too rapidly will result in a large emf in the inductor opposing it. To account for the instantaneous current of $I = 3$ A immediately after the fuse blows, we must have

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0,$$

or

$$\frac{dI}{dt} = \frac{\mathcal{E} - IR}{L} = -7\text{A/s}.$$

Subsequently, the current drops exponentially toward the steady-state value, with a time scale $\tau = L/R = \frac{1}{2}$ s.