21.X.35

$\vec{\mathrm{F}}$ is toward the center of the circle. The right-hand rule shows that $\vec{\mathrm{B}}$ is outward, in the +z direction, so $\hat{\mathrm{B}} =< 0, 0, 1 >$.

$$
\begin{aligned}
\left|\vec{\mathrm{F}}_{\mathrm{net}}\right| &= \left|\frac{d\vec{\mathrm{p}}}{dt}\right| \\
\left|\vec{\mathrm{F}}_{\mathrm{mag}}\right| &= \left|\frac{d\vec{\mathrm{p}}}{dt}\right| \\
qvB &= \frac{mv^2}{R} \\
qB &= \frac{mv}{R} \\
B &= \frac{mv}{qR} \\
&= \frac{m\left(\frac{2\pi R}{T}\right)}{qR} \\
&= \frac{m2\pi}{qT} \\
&= \frac{4(1.67 \times 10^{-27}\ \mathrm{kg})2\pi}{2(1.6 \times 10^{-19}\ \mathrm{C})(80 \times 10^{-9}\ \mathrm{s})} \\
&= 1.64\ \mathrm{T}
\end{aligned}
$$

21.X.36

$\vec{\mathrm{F}} = q\vec{\mathrm{v}} \times \vec{\mathrm{B}}$. The right-hand rule shows that at this location, the force by the magnetic field on the electron is downward and to the right. Therefore, the electron will travel in a circle, clockwise.

21.P.37

(a) The proton's $\vec{\mathrm{E}}$ is to the right. There is no magnetic field.

(b) The electric force on the electron is to the left. There is no magnetic force.

(c) The electron's $\vec{\mathrm{E}}$ is to the right. The electron's magnetic field is into the page.

(d) The electric force is to the right. The magnetic force is up.

(e) Reciprocity apparently does not apply to magnetic forces.

(f) If reciprocity does not apply, then the total system momentum will change. The momentum principle is violated **unless** as assign momentum to the fields.

21.P.38

(a)

$$\left|\vec{\mathrm{B}}_{\mathrm{p}}\right| = \frac{\mu_o}{4\pi}\frac{e\left|\vec{\mathrm{v}}_{\mathrm{p}}\right|\sin\theta}{|\vec{\mathrm{r}}|^2} \text{ into page}$$

$$\left|\vec{\mathrm{E}}_{\mathrm{p}}\right| = \frac{1}{4\pi\varepsilon_o}\frac{e}{|\vec{\mathrm{r}}|^2}\text{to right}$$

$$\left|\vec{\mathrm{F}}_{\mathrm{mag}}\right| = e\left|\vec{\mathrm{v}}_{\mathrm{e}}\right|\frac{\mu_o}{4\pi}\frac{e\left|\vec{\mathrm{v}}_{\mathrm{p}}\right|\sin\theta}{|\vec{\mathrm{r}}|^2} \text{ to right}$$

$$= \frac{\mu_o}{4\pi}\frac{e^2\left|\vec{\mathrm{v}}_{\mathrm{p}}\right|\left|\vec{\mathrm{v}}_{\mathrm{e}}\right|\sin\theta}{|\vec{\mathrm{r}}|^2} \text{ to right}$$

$$\left|\vec{\mathrm{E}}_{\mathrm{el}}\right| = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{|\vec{\mathrm{r}}|^2} \text{ to left}$$

$$\mathrm{F}_{\mathrm{net},x} = -\frac{1}{4\pi\varepsilon_o}\frac{e^2}{|\vec{\mathrm{r}}|^2}\left(1-\frac{\left|\vec{\mathrm{v}}_{\mathrm{p}}\right|\left|\vec{\mathrm{v}}_{\mathrm{e}}\right|\sin\theta}{c^2}\right)$$

$$\mathrm{F}_{\mathrm{net},y} = 0$$

(b)

$$\left|\vec{\mathrm{B}}_{\mathrm{e}}\right| = \frac{\mu_o}{4\pi}\frac{e\left|\vec{\mathrm{v}}_{\mathrm{e}}\right|}{|\vec{\mathrm{r}}|^2} \text{ into page}$$

$$\left|\vec{\mathrm{E}}_{\mathrm{e}}\right| = \frac{1}{4\pi\varepsilon_o}\frac{e}{|\vec{\mathrm{r}}|^2} \text{ to right}$$

$$\left|\vec{\mathrm{F}}_{\mathrm{mag}}\right| = e\left|\vec{\mathrm{v}}_{\mathrm{p}}\right|\frac{\mu_o}{4\pi}\frac{e\left|\vec{\mathrm{v}}_{\mathrm{e}}\right|}{|\vec{\mathrm{r}}|^2} \text{ to upper left}$$

$$= \frac{\mu_o}{4\pi}\frac{e^2\left|\vec{\mathrm{v}}_{\mathrm{p}}\right|\left|\vec{\mathrm{v}}_{\mathrm{e}}\right|}{|\vec{\mathrm{r}}|^2} \text{ to upper left, perpendicular to proton's velocity}$$

$$\left|\vec{\mathrm{F}}_{\mathrm{el}}\right| = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{|\vec{\mathrm{r}}|^2} \text{ to right}$$

$$\mathrm{F}_{\mathrm{net},x} = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{|\vec{\mathrm{r}}|^2}\left(1-\frac{\left|\vec{\mathrm{v}}_{\mathrm{p}}\right|\left|\vec{\mathrm{v}}_{\mathrm{e}}\right|\sin\theta}{c^2}\right)$$

$$\mathrm{F}_{\mathrm{net},y} = \frac{\mu_o}{4\pi}\frac{e^2\left|\vec{\mathrm{v}}_{\mathrm{p}}\right|\left|\vec{\mathrm{v}}_{\mathrm{e}}\right|\cos\theta}{|\vec{\mathrm{r}}|^2}$$

(c) Reciprocity apparently does not apply to magnetic forces.

(d) If reciprocity does not apply, then the total system momentum will change. The momentum principle

21.P.39

(a) The carbon ions are positively charged. If they accelerate to the right, then the left plate must be positively charged.

To null out the upward magnetic deflection, there must be a downward electric deflection. $\vec{\mathrm{E}}$ must point down so the top plate must be positively charged.

$\vec{\mathrm{B}}$ must be into the page to produce a centripetal force to the left.

21.P.50

At all locations to the right of the long wire, the magnetic field $\vec{B}_1$ due to the long wire is in the -z direction with magnitude $B_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{r}$.

The net force on the loop is the sum of the forces on the sides of the loop. The force on the top side is in the +y direction, and the force on the bottom side is in the -y direction. These forces vary along the length of the sides but are equal in magnitude, so they cancel.

The force on the left side is given by $\vec{F} = I_2\vec{L} \times \vec{B}$, and is in the -x direction. Thus,

$$\begin{aligned} F_{left,x} &= -I_2 h B_1 \\ &= -I_2 h \frac{\mu_0}{4\pi}\frac{2I_1}{d} \\ &= -\frac{\mu_0}{4\pi}\frac{2I_1 I_2 h}{d} \end{aligned}$$

The force on the right side is in the +x direction and is

$$\begin{aligned} F_{right,x} &= I_2 h B_1 \\ &= I_2 h \frac{\mu_0}{4\pi}\frac{2I_1}{(d+w)} \\ &= \frac{\mu_0}{4\pi}\frac{2I_1 I_2 h}{d+w} \end{aligned}$$

The net force on the loop is

$$\begin{aligned} \vec{F}_{net} &= < \frac{\mu_0}{4\pi} 2I_1 I_2 h (\frac{1}{d+w} - \frac{1}{d}), 0, 0 > \\ \vec{F}_{net} &= < \frac{\mu_0}{4\pi} 2I_1 I_2 h \left(\frac{d-(d+w)}{d(d+w)}\right), 0, 0 > \\ &= < \frac{\mu_0}{4\pi}\frac{2I_1 I_2 h w}{d(d+w)}, 0, 0 > \end{aligned}$$

21.P.51

The metal rod is in equilibrium, so $\vec{F}_{net} = 0$. The gravitational force by Earth on the rod is downward, so the magnetic force by the magnetic field on the rod must be upward. Since $\vec{E}_{mag} = I\vec{L} \times \vec{B}$ and I flows in the +x direction, $\vec{B}$ must be in the -z direction. Applying the Momentum Principle gives

$$\begin{aligned}
\vec{F}_{net} &= 0 \\
\vec{F}_{mag} + \vec{F}_{grav} &= 0 \\
\left|\vec{F}_{mag}\right| &= \left|\vec{F}_{grav}\right| \\
ILB &= mg \\
B &= \frac{mg}{IL} \\
&= \frac{(0.07 \text{ kg})(9.8 \frac{\text{N}}{\text{kg}})}{(5 \text{ A})(0.12 \text{ m})} \\
&= 1.14 \text{ T}
\end{aligned}$$

So, $\vec{B} = \langle 0, 0, -1.14 \rangle$ T.

21.X.52

$\vec{F}_{elec} = q\vec{E}$ is in the +y direction, so $\vec{F}_{mag}$ must be in the -y direction so that $\vec{F}_{net} = 0$. Thus, $\vec{B}$ is in the +z direction. Since $\vec{F}_{net} = 0$,

$$\begin{aligned}
\left|\vec{F}\right| elec &= \left|\vec{F}\right| mag \\
qE &= qvB \\
B &= \frac{E}{v}
\end{aligned}$$

21.X.53

Since $\vec{F}_{net} = 0$, then

$$\begin{aligned}
\left|\vec{F}_{elec}\right| &= \left|\vec{F}_{mag}\right| \\
qE &= qvB \\
v &= \frac{E}{B} \\
&= \frac{3800 \frac{\text{V}}{\text{m}}}{0.4 \text{ T}} \\
&= 9500 \text{ m/s}
\end{aligned}$$

21.X.54

(a)

$$\begin{aligned}
\vec{F} &= q\vec{v} \times \vec{B} \\
&= (-1.6 \times 10^{-19} \text{ C})(\left\langle 5 \times 10^5, 0, 0 \right\rangle \text{ m/s}) \times \langle 0, 0.9, 0 \rangle \text{ T} \\
&= \langle 0, 0, -7.2 \ -14 \rangle \text{ N}
\end{aligned}$$

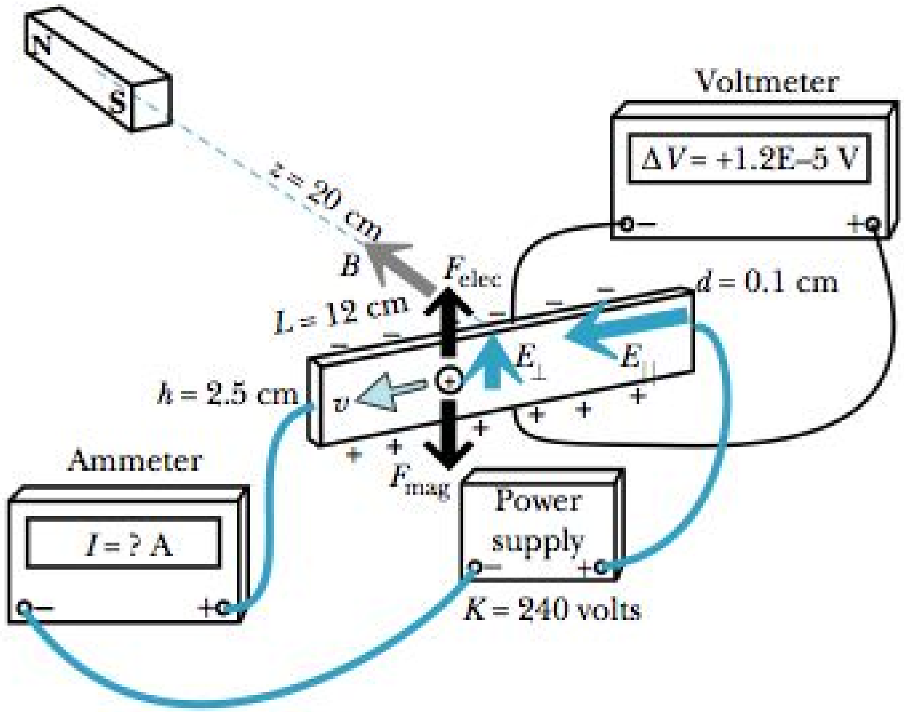

Figure 7: A sketch used to answer Question 21.P.71.

(b) We can find the magnetic field of the bar magnet from $E_\perp = vB$, since $E_\perp = \Delta V/h$: $B = \Delta V/(hv)$. But the field made by the bar magnet is $B = \frac{\mu_o}{4\pi}2\mu/z^3$ (assuming that z is large compared to the size of the bar magnet). Therefore we can determine μ, the magnetic dipole moment of the bar magnet from the relation $\frac{\Delta V}{hv} = \frac{\mu_o}{4\pi}\frac{2\mu}{z^3}$, where μ is the only unknown quantity.

(c) The ammeter reads $I = enAuE_\parallel$, where the cross-sectional area is $A = hd$ and all other quantities are known.

21.P.72

Since the mobile charge carriers are positive, then they move to the right (+x) through the bar with a speed v as shown in Figure 8. Because they enter the positive terminal of the ammeter, the ammeter reading will be positive.

The magnetic force on these carriers is downward, causing the bottom side of the bar to become positively charged and the top side of the bar to become negatively charged. Because the − terminal is connected to the bottom of the bar which is at a *higher* potential than the + terminal, the voltmeter reading will be negative.

Once equilibrium is reached, mobile charges move with constant velocity through the bar. The net force on the charge carriers is zero, so in the y direction

$$\begin{aligned} F_{net,y} &= 0 \\ F_{mag} &= F_{elec} \\ evB &= eE_\perp \\ vB &= E_\perp \end{aligned}$$

Since the electric field is uniform in the y-direction, $\Delta V_\perp = E_\perp h$ where $\Delta V_\perp$ is the Hall voltage measured with the voltmeter across the bottom and top of the bar and h is the height of the bar in the y-direction. The conventional current in the bar

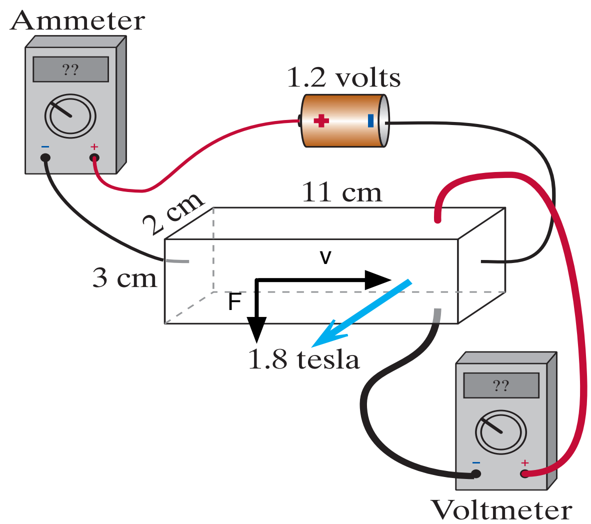

Figure 8: Velocity and magnetic force on positive mobile charges in the bar in Question 21.P.72.

is $I = neAv$ where $A = hd$ is the cross-sectional area of the bar and d is the depth of the bar in the z-direction. Using the Hall voltage and the conventional current, the magnetic field due to the long wire can be calculated, as

$$\begin{aligned}\Delta V_\perp &= E_\perp h \\ &= vBw \\ &= \frac{I}{neA}Bh \\ &= \frac{IB\cancel{h}}{ne\cancel{h}d} \\ &= \frac{IB}{ned}\end{aligned}$$

The conventional current through the bar is related to the properties of the bar and the emf of the battery by, $I = neAv = neA(uE_\parallel) = neAu\frac{\text{emf}}{L}$ where L is the length of the bar in the x-direction. Substituting $A = hd$ along with I into the equation for the Hall voltage gives:

$$\begin{aligned}\Delta V_\perp &= \frac{IB}{ned} \\ &= \frac{nehduB\frac{\text{emf}}{L}}{ned} \\ &= \frac{huB\text{emf}}{L} \\ &= \frac{(0.03\text{ m})(3\times10^{-5}\text{ (m/s)/(V/m)})(1.8\text{ T})}{0.11\text{ m}} \\ &= 1.47\times10^{-5}\text{ V}\end{aligned}$$

Because the − terminal is connected to the positively charged side of the bar, the voltmeter will read -1.47×10^{-5} V. This is a very small voltage, as expected for a Hall voltage.

The conventional current through the bar is

$$\begin{aligned} I &= neAu\frac{\text{emf}}{L} \\ &= (7\times 10^{23}\ \text{m}^{-3})(1.602\times 10^{-19}\ \text{C})(0.03\ \text{m})(0.02\ \text{m})(3\times 10^{-5}\ (\text{m/s})/(\text{V/m}))(1.2\ \text{V})/(0.11\ \text{m}) \\ &= 0.0220\ \text{A} \end{aligned}$$

As described earlier, since positive charge flows into the positive terminal of the ammeter, it will read a positive 0.022 A.

21.X.73

(3). The magnetic force on mobile electrons is in the $-y$ direction which causes the electron "sea" to shift downward, leaving the top end positively charged and the bottom end negatively charged.

21.X.74

(4). The magnetic force on mobile electrons is in the $+y$ direction which causes the electron "sea" to shift upward, leaving the top end negatively charged and the bottom end positively charged.

21.X.75

Conventional current flows in the $-y$ direction through the resistor. The magnetic force on mobile electrons in the bar is in the $-y$ direction which causes the electron "sea" to shift downward, leaving the top end of the bar positively charged and the bottom end of the bar negatively charged. However, because the ends are connected by a conductor, positive charge (conventional current) will flow from the top end, through the resistor, and to the bottom end of the bar. As a result, conventional current flows counterclockwise through the circuit.

Since I flows in the $+y$ direction and $\vec{B}$ is in the $-z$ direction, the magnetic force on the wire, $\vec{\text{F}}_{\text{mag}} = I\vec{\text{L}}\times\vec{\text{B}}$, is in the $-x$ direction. If you pull the bar at a constant speed v in the $+x$ direction, you must exert a force $\vec{F}$ in the +x direction that is equal in magnitude to the magnetic force in the $-x$ direction so that the net force on the bar is zero.

21.X.76

(2) According to the right-hand rule, the magnetic force on the negatively charged mobile electrons is to the left, causing negative charge to pile up on the left end of the bar and positive charge to pile up on the right end of the bar.

21.X.77

(a) The velocity of each wire is tangent to its path. As a result, the magnetic force on a mobile electron in the wire of length h is perpendicular to the wire and does not "push" the charge through the wire. However, for the wires of length w, the magnetic force on the right wire causes positive charge to pile up on the front end (+z) and negative charge to pile up on the back end (-z). The magnetic force does work a mobile electron, moving it from the front (+z) to the back (-z) in the right wire. The right wire thus acts like a battery. Its emf is the work done by the magnetic force per unit charge in moving a charged particle a distance w.

$$\begin{aligned} \Sigma_{\text{right / wire}} &= \frac{W}{q} \\ &= \frac{(E_{\text{mag}})(w)}{q} \\ &= \frac{qvB\sin\theta w}{q} \\ &= vB\sin\theta w \end{aligned}$$

The magnetic force in the left wire pushes an electron from the back (-z) to the front (+z) creating an identical emf as in the right wire and they are in series. Thus, the total emf of the two wires is,

$$\Sigma = 2vBw\sin\theta$$

The current through the four wires is $I = \frac{\Sigma}{R} = \frac{2vB\sin\theta}{R}$. Since the right and left wires have a tangential speed v that is $v = r\omega$ where r is the distance from the asle, $\frac{h}{2}$. Thus, $v = \left(\frac{h}{2}\right)\omega$. As a result,

$$\begin{aligned}\Sigma &= 2\omega\left(\frac{h}{2}\right)Bw\sin\theta\\ &= \omega B(hw)sin\theta\end{aligned}$$

where hw is the area of the loop.

The current is $I = \frac{\Sigma}{R} = \frac{\omega B(hw)\sin\theta}{R}$. Because of the "polarity" of the emf in the right and left wires, current flows to the left in the front wire of length h at the angle shown in the side view of the diagram. After θ exceeds 90°, the current will reverse direction.

(b) At this point, I flows clockwise around the loop creating a magnetic dipole moment $\vec{\mu}$ that is $\perp$ to the plane of the loop. The torque by the magnetic field on the loop is $\vec{\tau} = \vec{\mu} \times \vec{\mathrm{B}}$. At the instant shown, $\vec{\tau}_{\mathrm{mag}}$ is in the -z direction. Since $\vec{\omega}$ is in the +z direction, this torque would cause the loop to slow down. There must be an applied torque in the +z direction so that the net torque is zero and ω is constant. The applied external forces must act at an angle with respect to $\vec{\mathrm{r}}$ to give a torque in the +z direction.

21.X.78

$\vec{\mathrm{F}}_{\mathrm{mag}} = q\vec{\mathrm{v}} \times \vec{\mathrm{B}}$ exerts a force on mobile electrons in the rod that is in the +y direction. As a result, the top end becomes negatively charged and the bottom end becomes positively charged.

The Coulomb electric field points in the +y direction, toward the negatively charged end.

The net force on a mobile electron is zero since it is in equilibrium.

$$\begin{aligned}\vec{\mathrm{F}}_{\mathrm{net}} &= \vec{\mathrm{F}}_{\mathrm{C}} + \vec{\mathrm{F}}_{\mathrm{mag}} = 0\\ \vec{\mathrm{F}}_{\mathrm{C}} &= -\vec{\mathrm{F}}_{\mathrm{mag}}\\ \left|\vec{\mathrm{F}}_{\mathrm{C}}\right| &= \left|\vec{\mathrm{F}}_{\mathrm{mag}}\right|\\ qE_{\mathrm{C}} &= qvB\\ E_{\mathrm{C}} &= vB\end{aligned}$$

The emf across the rod is equal to $\Delta C = EL$, so $\mathsf{emf} = vBL$.

No current flows through the rod, so $\vec{\mathrm{F}}_{\mathrm{mag}}$ on the rod is zero. Since $\vec{\mathrm{F}}_{\mathrm{net}} = 0$ then no force must be applied to keep it moving at constant speed.

21.P.79