(2) Quasi-steady state: magnitude of $\vec{\mathrm{E}}$ is uniform throughout the wire, drives current in the direction to reduce charge on capacitor plates (top plate gets less positive, bottom plate gets less negative). There may be some excess charge on the bends, but the main feature is that there is a gradient of surface charge all along the wire (see Figure 9). As capacitor charge decreases, polarization and surface charge also decrease (proportionally). Hence $\vec{\mathrm{E}}$ decreases and electron current $i = nAu\left|\vec{\mathrm{E}}\right|$ decreases with time (may take many seconds).

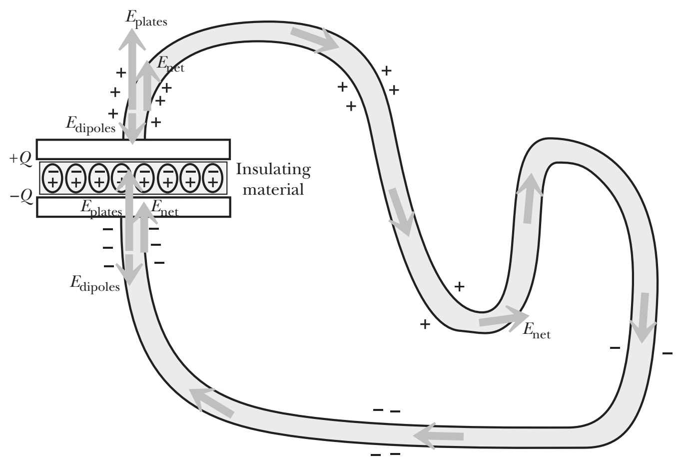

Figure 9: Sketch the quasi-steady state for the circuit in Question 10.P.56.

(3) Static equilibrium: eventually charge on the plates is reduced to zero, there is no polarization in the plastic, and there is no surface charge on the wire.

20.P.57

(a) Apply the loop rule to this circuit when the capacitor is fully charged. Then, emf $= \Delta V_C = Q/C$. Thus, the charge on the capacitor when fully charged is $Q = C(\text{emf})$.

(b) The potential difference across the plates remains constant, and the net electric field within the capacitor remains constant since $E_{net} = \Delta V_C/s$. However, there is a component of $\vec{E}_{net}$ that is due to polarized molecules in the dielectric. In fact, $E_{net,x} = E_{vacuum,x} - E_{dipoles,x}$. (I've assumed that the capacitor is aligned with the x-axis with $\vec{E}_{net}$ in the +x direction.) Since E_{net} remains constant, then E_{vacuum} must have increased. Since $E_{vacuum} = (Q/A)/\varepsilon$, then Q must increase, and current runs through the bulb until the capacitor is again fully charged.

From the loop rule applied to this circuit, $Q = C(\text{emf})$ with a capacitance that is now $C = KC_0$, where C_0 is the capacitance with air between the plates. Since C is larger by a factor K, then Q is also larger by a factor of K. If Q_0 is the charge on the capacitor before the dielectric is inserted, then $Q = KQ_0$ is the charge after the dielectric is inserted.

20.P.58

First sketch a circuit, like the one shown in Figure 10.

Now, sketch the capacitor with the dielectric and the capacitor without the dielectric, as shown in Figure 11.

The net electric field within the plates is related to the potential difference across the plates by $E_{net} = \Delta V/s$. Applying the loop rule to the circuit shows that emf $= \Delta V_C$, so $E_{net} = \text{emf}/s$.

Since the potential difference across the plates doesn't change when you remove the dielectric, then the net electric field must remain the same. However, when the dielectric is within the plates, the net electric field is due to charge on the plates as well

20.P.59

(a) The electric potential outside a uniformly charged sphere is that of a point particle at the center of the sphere, $V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$. Inside the sphere, the electric field is zero. Thus, at $r = r$, $V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$.

At $r \geq r_2$, $V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} + \frac{1}{4\pi\varepsilon_o}\frac{(-Q)}{r} = 0$. Just inside r_2, $V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r_2}$. Thus,

$$\begin{aligned}\Delta V &= V_2 - V_1 \\ &= \frac{1}{4\pi\varepsilon_o}\frac{Q}{r_2} - \frac{1}{4\pi\varepsilon_o}\frac{Q}{r_1} \\ &= \frac{1}{4\pi\varepsilon_o}Q(\frac{1}{r_2} - \frac{1}{r_1})\end{aligned}$$

Solve for Q to get

$$Q = \frac{4\pi\epsilon_0}{(\frac{1}{r_2} - \frac{1}{r_2})}\Delta V$$

Comparing to $Q = C\Delta V$ gives

$$C = \frac{4\pi\epsilon_0}{(\frac{1}{r_2} - \frac{1}{r_1})}$$

(b)

$$\begin{aligned}\frac{1}{r_2} - \frac{1}{r_1} &= \frac{r_2 - r_1}{r_1 r_2} = \frac{s}{r_1 r_2} \\ C &= 4\pi\epsilon_0\left(\frac{r_1 r_2}{s}\right)\end{aligned}$$

Since $r_1 \approx r_2 = R$, then $r_1 r_2 \approx R^2$ and

$$\begin{aligned}C &\approx \frac{\epsilon_0 4\pi R^2}{s} \\ C &\approx \frac{\epsilon_0 A}{s}\end{aligned}$$

20.P.60

According to the loop rule for the circuit, $\mathsf{emf} = \Delta V_C = \frac{E_{net}}{s}$. If s is suddenly increased, then E_{net} must decrease. Since $E_{net} = \frac{\frac{Q}{A}}{K\epsilon_0}$, then Q must decrease. There will be a current as positive charge (conventional current) leaves the positively charged plate, until the capacitor is in static equilibrium once again. Then the new charge on the capacitor will be $Q_{new} = C_{new}(\mathsf{emf}) = \frac{K\epsilon_0 A}{s_{new}}$.

Note that the new capacitance is less since s_{new} is larger.

20.P.61

$$\begin{aligned}
\mathsf{emf} - I(R_1 + R_2) &= 0 \\
I &= \frac{\mathsf{emf}}{R_1 + R_2} \\
&= \frac{7.9\ \mathrm{V}}{23\ \Omega + 44\ \Omega} \\
&= 0.118\ \mathrm{A}
\end{aligned}$$

$$\begin{aligned}
\Delta V_1 &= IR_1 \\
&= (0.118\ \mathrm{A})(23\ \Omega) \\
&= 2.71\ \mathrm{V}
\end{aligned}$$

(b)

$$I_2 = I_1 = 0.118\ \mathrm{A}$$

20.X.69

A light bulb filament is non-ohmic. Its resistance increases with temperature. A greater current causes the filament to get hotter, which causes resistance to increase.

When 1 bulb was connected to the battery, it had a greater resistance than when two bulbs in series were connected to the battery.

20.X.70

Suppose $R_1 = 10\ \Omega$, $R_2 = 5\ \Omega$, and $R_3 = 20\ \Omega$.

$$\begin{aligned}
\frac{1}{R_{2,3}} &= \frac{1}{5} + \frac{1}{20} \\
R_{2,3} &= 4\ \Omega \\
R_{\mathrm{eq}} &= R_1 + R_{2,3} = 10\ \Omega + 4\ \Omega = 14\ \Omega
\end{aligned}$$

20.X.71

(a)

$$\begin{aligned}
\frac{1}{R_{1,2}} &= \frac{1}{R_1} + \frac{1}{R_2} \\
&= \frac{1}{31\ \Omega} + \frac{1}{47\ \Omega} \\
R_{1,2} &= 18.7\ \Omega
\end{aligned}$$

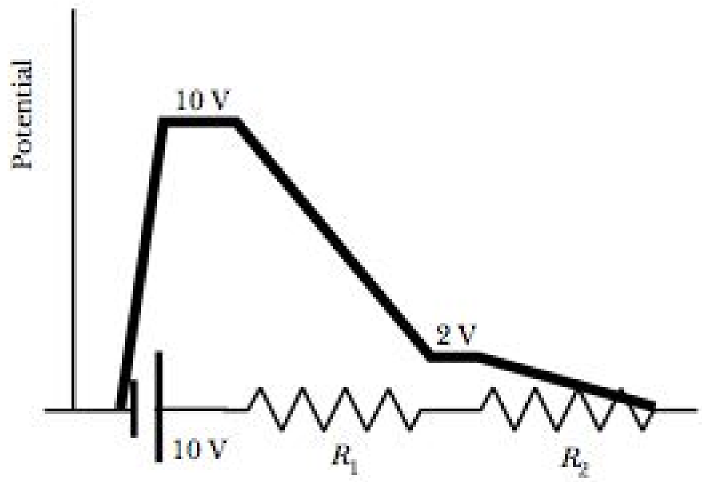

Figure 17: A graph of the electric potential V as a function of location along the circuit in Question 20.P.82.

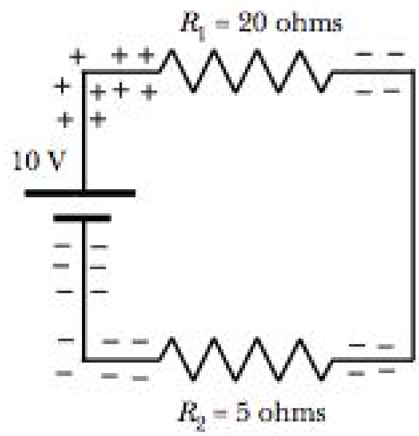

Figure 18: Surface charge for the circuit in Question 20.P.82.

20.P.83

(a) Apply the loop rule to ABCHA:

$$\mathsf{emf}_1 - I_1 R_1 - I_4 R_4 \quad = \quad 0$$

Apply the loop rule to FEDCF:

$$\mathsf{emf}_2 - I_2 R_2 - I_3 R_3 \quad = \quad 0$$

Apply the loop rule to CFGHC:

$$-I_3 R_3 - I_5 R_5 + I_4 R_4 = 0$$

(Note the positive $I_4 R_4$ term since the loop goes H-C and the current goes C-H. This means that C is at a higher potential than H and $V_C - V_H$ is positive, at least according to the direction of I_4 if it's positive.)

Apply the node rule at C:

$$I_1 + I_2 = I_3 + I_4$$

Apply the node rule at H:

$$I_4 + I_5 = I_1$$

Substitute emfs and resistances into the above equations and solve for I_1, I_2, I_3, I_4, and I_5 using linear algebra.

(b)

$$V_D - V_A = (V_H - V_A) + (V_C - V_H) + (V_D - V_C)$$

Since $V_H = V_A$, write this as

$$\begin{aligned} V_D - V_A &= (V_C - V_A) + (V_D - V_C) \\ &= I_4 R_4 + I_2 R_2 \end{aligned}$$

Since directions of current are presumed to be positive, then $V_D - V_A$ is positive.

(c) The current through battery 2 is I_2. So, $P = (\mathsf{emf}_2) I_2$.

20.P.84

(a) A sketch is shown in Figure 19. With the switch open, no current; surface charge + on left branch, – on right branch. There is an electric field in the gap between the two parts of the switch.

(b) No current flows through the wire, so all parts of a wire that are connected to one terminal of the battery are at the same potential V as that terminal of the battery. Thus, V_K and V_D are at the same potential and $V_D - V_K = 0$. Since $V_B = V_{+,bat}$ and $V_C = V_{-,bat}$, then $V_B - V_C = \Delta V_{bat} = 3.0$ V.

(c) Apply the loop rule to loop ABCDGHKLMNA (basically batteries, Bulb 3 and Bulb 1). Define all current directions to be from high potential to low potential (i.e. current points in the direction toward the negative terminal of the battery). Thus,

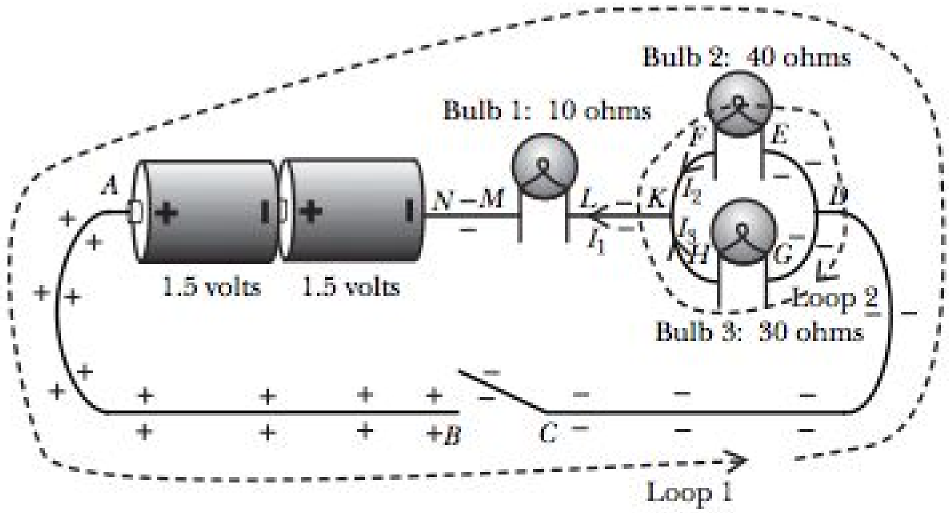

Figure 19: Surface charge distribution on the circuit for Question 20.P.84.

$$2emf - I_3R_3 - I_1R_1 = 0$$

Apply the loop rule to loop ABCDEFKLMNA (basically batteries, Bulb 2 and Bulb 1). Define all current directions to be from high potential to low potential (i.e. current points in the direction toward the negative terminal of the battery). Thus,

$$2emf - I_2R_2 - I_1R_1 = 0$$

Apply the node equation at K.

$$I_2 + I_3 = I_1$$

The above three equations can be used to solve for I_1, I_2, and I_3.

(d) $V_C - V_F$ is simply the potential difference across bulb 2 (since $V_C = V_D = V_E$), and $\Delta V_2 = I_2R_2$.

(e) $P_{bat} = \Delta V_{bat}I_{bat}$. Since the battery and bulb 1 are in series, then $P_{bat} = (3.0 \text{ V})I_1$.

(f) The equations are:

$$\begin{aligned} 3 - I_3(30) - I_1(10) &= 0 \\ 3 - I_2(40) - I_1(10) &= 0 \\ I_2 + I_3 &= I_1 \end{aligned}$$

You can solve this simultaneous equations using linear algebra or substitution or the solve function on a TI calculator. The solutions is $I_1 = 21/190 = 0.111$ A, $I_2 = 9/190 = 0.0474$ A, and $I_3 = 6/95 = 0.0632$ A.

(g) $i = i_1 = I_1/e = (0.111 \text{ A})/(1.602 \times 10^{-19} \text{ C} = 6.94e17)$ electrons per second.

(h) $V_C - V_F = \Delta V_2 = I_2 R_2 = (0.0474 \text{ A})(40\ \Omega) = 1.90$ V

(i) $P_{bat} = (3.0 \text{ V})I_1 = (3.0 \text{ V})(0.111 \text{ A}) = 0.333$ W

(j) $\Delta V = EL$ for a uniform electric field. Thus, $E_2 = (1.9 \text{ V})/(0.008 \text{ m}) = 238$ V/m

20.P.85

$$\begin{aligned} W &= \int_0^Q \frac{q}{C} dq \\ &= \frac{1}{C}\int_0^Q q dq \\ &= \frac{1}{C}\frac{q^2}{2}\bigg|_0^Q \\ &= \frac{1}{2}\frac{Q^2}{C} \end{aligned}$$

20.X.86

$$\Delta V_{\text{bat}} = \mathsf{emf} - I r_{\text{int}}$$

When short-circuited, $\Delta V_{\text{bat}} = 0$ and the internal resistance of the battery can be calculated to be $r_{\text{int}} = \frac{6 \text{ V}}{12 \text{ A}} = 0.5\ \Omega$. If a 1 Ω resistor is connected, then $\Delta V_{\text{bat}} = \Delta V_{\text{R}} = IR$, and

$$\begin{aligned} IR &= \mathsf{emf} - I r_{\text{int}} \\ I(R + r_{\text{int}}) &= \mathsf{emf} \\ I &= \frac{6 \text{ V}}{(1\ \Omega + 0.5\ \Omega)} \\ &= 4 \text{ A} \end{aligned}$$

20.P.87

(a)

$$\begin{aligned} \Delta V &= \mathsf{emf} - I r_{\text{int}} \\ 0 &= \mathsf{emf} - I r_{\text{int}} \\ r_{\text{int}} &= \frac{\mathsf{emf}}{I} = \frac{9 \text{ V}}{18 \text{ A}} = \frac{1}{2}\Omega \end{aligned}$$

(b)

$$\begin{aligned} P &= (\mathsf{emf})I \\ &= (9\ \text{V})(18\ \text{A}) = 162\ \text{W} \end{aligned}$$

(c)

162 J

(d)

$$\begin{aligned} \Delta V_{_\text{R}} &= \mathsf{emf} - Ir_{_\text{int}} \\ IR &= \mathsf{emf} - Ir_{_\text{int}} \\ I(R + r_{_\text{int}}) &= \mathsf{emf} \\ I &= \frac{\mathsf{emf}}{R + r_{_\text{int}}} = \frac{9\ \text{V}}{(10\ \Omega + 0.5\ \Omega)} = 0.857\ \text{A} \end{aligned}$$

(e)

$$\begin{aligned} P &= \Delta V I \\ &= \left(\frac{I}{R}\right) I \\ &= \frac{I^2}{R} = \frac{(0.857\ \text{A})^2}{10\ \Omega} = 0.0735\ \text{W} \end{aligned}$$

(f) It reads $\Delta V = \mathsf{emf} - Ir_{_\text{int}} = 9\ \text{V} - (0.857\ \text{A})(\frac{1}{2}\Omega) = 8.57\ \text{V}$

20.P.88

(a) A sketch of the electric field and the surface charge is shown in Figure 20.

(b) See Figure 20

(c) Apply the loop rule to the wire. The electric field is the same everywhere in the wire. Thus emf $= \Delta V_{wire} = E_{wire} L_{wire}$. Thus $E_{wire} = \text{emf}/L_{wire} = (12\ \text{V})/(0.4\ \text{m}) = 30\ \text{V/m}$.

(d) $\Delta V_{wire} = E_{wire} L_{wire} = (30\ \text{V/m})(0.05\ \text{m}) = 1.5\ \text{V}$.

(e) Conventional current flows into the negative terminal of the ammeter. As a result, it will read a negative current. To calculate the magnitude of the current, apply the loop rule, emf $= \Delta V_{wire} = IR$. Solving for I gives, $I = (12\ \text{V})/(50\ \Omega) = 0.24\ \text{A}$.

(f) $i = I/e = (0.24\ \text{A})/(1.602 \times 10^{-19}\ \text{C}) = 1.5 \times 10^{18}$ electrons per second. In 60 seconds, $(1.5 \times 10^{18})(60) = 9 \times 10^{19}$ electrons flow from the negative terminal of the battery.