To argue (A), if we write the above equation in terms of potential energy, $\Delta U = q\Delta V$. Thus,

$$q(\mathsf{emf}) \quad = \quad q\Delta V_1 + q\Delta V_{bulb} + q\Delta V_2$$

Since $\Delta V_1 << \Delta V_{bulb}$, then the energy loss of electrons through a connecting wire is much smaller than through the bulb and $q\Delta V_1$ is negligible. As a result, $\mathsf{emf} \approx \Delta V_{bulb}$.

19.x.57

(a) Apply the Loop Rule (Conservation of Energy) to loop 2.

$$\begin{aligned}(V_F - V_C) + (V_C - V_D) + (V_D - V_E) + (V_E - V_F) &= 0 \\ -8\text{ V} + (V_C - V_D) + 4.5\text{ V} + 0 &= 0 \\ (V_C - V_D) &= 3.5\text{ V}\end{aligned}$$

(b) $V_C > V_D$ so V_C must be the positive terminal since it is at a higher potential.

19.X.58

The emf of the battery is equal to the potential difference across the wire, due to the Loop Rule.

$$\begin{aligned}\mathsf{emf} &= \Delta V = EL \\ E &= \frac{\mathsf{emf}}{L} \\ &= \frac{1.6\text{ V}}{0.48\text{ m}} = 3.33\ \frac{\text{V}}{\text{m}}\end{aligned}$$

If the wire is replaced with one of regular diameter, the electric field inside the wire remains the same. However, the current will increase since $i = neAuE$

19.x.59

Note that the battery remains the same in all experiments, and according to the Loop Rule, emf $= \Delta V$ where ΔV is the potential difference across the wire. Since $\Delta V = EL$, then emf $= EL$ where L is the length of the wire. This can be a useful relationship. Answers are shown in Table .

19.x.60

The resistor (thin wire) and thick connecting wires are in series. Applying conservation of charge (i.e. the Current Node Rule) to the resistor and wires leads to the conclusion that the electron current through the thin wire must equal the electron current through the thick wire. Use $i = neAuE$ and the fact that n and u are properties of the material in order to compare the quantities given in Table .

19.x.61

Experiment	Effect on current	Parameter that changed
Double L	i changes by a factor of 1/2	**E**; E changes by a factor of 1/2 because L increased by 2 and $i \propto A$.
Double A	i changes by a factor of 2	**A**; A changes by a factor of 1/2 and $i \propto A$.
Double L	i changes by a factor of 1/2	**E**; Two bulbs in series is like having a single filament that is twice as long. Doubling L changes E by 1/2 and $i \propto E$.
Double emf	i changes by factor of 2	**E**; 2emf results in $2E$ which results in $2i$ since $i \propto E$.

Table 2: Table with answers to Question 19.X.59.

resistor	**$<$, $=$, or $>$**	**thick wires**
i_R	$=$	i_w
n_R	$=$	n_w
A_R	$<$	A_w
u_R	$=$	u_w
E_R	$>$	E_w
v_R	$>$	v_w

Table 3: Table with answers to Question 19.X.59.

According to the Loop Rule, the emf of the battery is equal to the potential difference across the wire. Since $\Delta V = EL$, then emf $= EL$, and the electric field is $E = \frac{\text{emf}}{L}$. For two batteries, the emf increases by a factor of 2. A 36 cm length increases L by a factor of 3. As a result, E changes by a factor 2/3.

Since $i = neAuE$ and since the cross-sectional area is increased by a factor of 2 and since E is changed by a factor 2/3, then i increases by a factor $(2)(2/3) = 4/3$.

The magnetic field due to the wire is proportional to the current through the wire. As a result, the magnetic field component that causes the compass to deflect increases by a factor 4/3. Assuming that the wire is aligned with the N-S direction, then the magnetic field due to the wire is E-W and $\tan\theta = \frac{B_{wire}}{B_{earth}}$. Since $B_{earth} \approx 2 \times 10^{-5}$ T, then a 6° deflection means that the original magnetic field was 2.1×10^{-6} T. Increasing this by a factor of 4/3 gives a magnetic field of 2.8×10^{-6} T which results in a deflection of 8°.

19.x.62

The graph shows that the potential difference across the connecting wires is not negligible, even though the question suggests neglecting this potential difference. Therefore, we will not neglect it. By estimation, $V_B - V_A \approx -0.5$ V and $V_D - V_C \approx -0.5$ V. According to the Loop Rule,

$$\begin{aligned}
(V_B - V_A) + (V_C - V_B) + (V_D - V_C) + (V_A - V_D) &= 0 \\
-0.5 \text{ V} + (V_C - V_B) + -0.5 \text{ V} + 3.5 \text{ V} &= 0 \\
(V_C - V_B) &\approx -2.5 \text{ V}
\end{aligned}$$

The potential difference across the resistor is about 5 times the potential difference across the connecting wires.

The conventional current through the resistor is $I = neAuE = neAu\frac{\Delta V}{L}$. We need to know the properties of the resistor in order to calculate the current through the resistor. No, there is not enough information to determine the current I in the circuit.

19.P.63

(a) (2)

(b)

$$\begin{aligned}
V_G - V_B + V_E - V_G + V_D - V_E + V_B - V_D &= 0\\
\mathsf{emf}_1 + V_E - V_G + \mathsf{emf}_2 + V_B - V_D &= 0\\
1.3\text{ V} + V_E - V_G + 1.3\text{ V} + V_B - V_D &= 0\\
2.6\text{ V} + V_E - V_G + V_B - V_D &= 0
\end{aligned}$$

(c) Since the wires are identical, $V_E - V_G = V_B - V_D$, so

$$\begin{aligned}
2.6\text{ V} + 2(V_B - V_D) &= 0\\
V_B - V_D &= \frac{2.6\text{ V}}{2}\\
&= 1.3\text{ V}
\end{aligned}$$

Since E is uniform in the wire, $\Delta V = EL$.

$$\begin{aligned}
E &= \frac{V_B - V_D}{L}\\
&= \frac{1.3\text{ V}}{0.26\text{ m}}\\
&= 5\ \frac{\text{V}}{\text{m}}
\end{aligned}$$

So, $E_B = 5\ \frac{\text{V}}{\text{m}}$. E at all points in the wire is $5\ \frac{\text{V}}{\text{m}}$.

(d) i is the same everywhere since charge is conserved. Thus,

$$\begin{aligned}
i &= nAuE\\
&= (7\times10^{28}\text{ m}^{-3})(\pi)\left(\frac{7\times10^{-4}\text{ m}}{2}\right)^2\left(5\times10^{-5}\ \frac{\text{m/s}}{\text{V/m}}\right)\left(5\ \frac{\text{V}}{\text{m}}\right)\\
&= 6.73\times10^{18}\text{ s}^{-1}
\end{aligned}$$

Since $\Delta V = EL$ and ΔV is the same, then E is the same. $E_B = 5\ \frac{\text{V}}{\text{m}}$.

(e) (C)

19.P.64

$$\begin{aligned}
i_{\mathrm{B}} &= i_{\mathrm{D}} \\
nA_{\mathrm{B}} uE_{\mathrm{B}} &= nA_{\mathrm{D}} uE_{\mathrm{D}} \\
A_{\mathrm{B}} E_{\mathrm{B}} &= A_{\mathrm{D}} E_{\mathrm{D}} \\
(1.4 \times 10^{-6}\ \mathrm{m}^2)E_{\mathrm{B}} &= (5.9 \times 10^{-8}\ \mathrm{m}^2)E_{\mathrm{D}} \\
E_{\mathrm{D}} &= \left(\frac{1.4 \times 10^{-6}\ \mathrm{m}^2}{5.9 \times 10^{-8}\ \mathrm{m}^2}\right) E_{\mathrm{B}} \\
&= (23.7)E_{\mathrm{B}}
\end{aligned}$$

Combine the loop equation and node equation and solve for E_{D} and E_{B}.

$$\begin{aligned}
1.8\ \mathrm{V} - 2E_{\mathrm{B}} L_{\mathrm{B}} - E_{\mathrm{D}} L_{\mathrm{D}} &= 0 \\
1.8\ \mathrm{V} - 2E_{\mathrm{B}} L_{\mathrm{B}} - (23.7)E_{\mathrm{B}} L_{\mathrm{D}} &= 0 \\
1.8\ \mathrm{V} - E_{\mathrm{B}} (2L_{\mathrm{B}} + 23.7L_{\mathrm{D}}) &= 0 \\
E_{\mathrm{B}} &= \frac{1.8\ \mathrm{V}}{2(0.25\ \mathrm{m}) + (23.7)(0.061\ \mathrm{m})} \\
&= 0.925\ \frac{\mathrm{V}}{\mathrm{m}} \\
E_{\mathrm{D}} &= 23.7E_{\mathrm{B}} \\
&= 21.9\ \frac{\mathrm{V}}{\mathrm{m}}
\end{aligned}$$

Note that $E_{\mathrm{F}} = E_{\mathrm{B}} = 0.925\ \frac{\mathrm{V}}{\mathrm{m}}$

(e)

$$\begin{aligned}
i_{\mathrm{D}} &= nAuE_{\mathrm{D}} \\
&= (4 \times 10^{28}\ \mathrm{m}^{-3})(5.9 \times 10^{-8}\ \mathrm{m}^2)(5 \times 10^{-4}\ \frac{\mathrm{m/s}}{\mathrm{V/m}})(21.9\ \frac{\mathrm{V}}{\mathrm{m}}) \\
&= 2.58 \times 10^{19}\ \mathrm{s}^{-1}
\end{aligned}$$

19.P.66

(a) Apply Conservation of Energy (Loop Rule) to the circuit. Then,

$$\begin{aligned}
\mathsf{emf} - 2\Delta V_{\mathrm{Cu}} - \Delta V_{\mathrm{nichrome}} &= 0 \\
\mathsf{emf} - 2E_{\mathrm{Cu}} L_{\mathrm{Cu}} - E_{\mathrm{nichrome}} L_{\mathrm{nichrome}} &= 0
\end{aligned}$$

Apply conservation of charge (Node Rule) to the wires. Thus, Use this constant to find i_1 in this new circuit.

$$\begin{aligned} i_1 &= nA_1 u \frac{2}{3}\frac{\text{emf}}{L} \\ &= \frac{2}{3}(9 \times 10^{17}\ \text{s}^{-1}) \\ &= 6 \times 10^{17}\ \text{s}^{-1} \end{aligned}$$

It makes sense that this current is greater than with two identical bulbs because it is easier to push current through a thick filament than a thin filament.

19.P.70

(a) Apply the loop rule. Call the thin wire 1 and the thick wire 2.

$$2\text{emf} - E_1 L_1 - E_2 L_2 = 0$$

Apply the node rule.

$$\begin{aligned} i_1 &= i_2 \\ nA_1 uE_1 &= nA_2 uE_2 \\ A_1 E_1 &= A_2 E_2 \\ E_1 &= \frac{A_2}{A_1} E_2 \\ &= \frac{(0.35)^2}{(0.25)^2} E_2 \\ &= 1.96\ \text{E}_2 \\ &\approx 2E_2 \end{aligned}$$

Substitute into the loop equation.

$$\begin{aligned} 2\text{emf} - 2E_2 L_1 - E_2 L_2 &= 0 \\ 2\text{emf} - E_2(2L_1 + L_2) &= 0 \\ E_2 &= \frac{2\text{emf}}{2L_1 + L_2} \\ &= \frac{2(1.5\ \text{V})}{2(0.5\ \text{m}) + 0.15\ \text{m}} \\ &= 2.6\ \frac{\text{V}}{\text{m}} \end{aligned}$$

So, $E_1 = 2E_2 = 5.2\ \frac{\text{V}}{\text{m}}$.

(b) $\bar{v} = uE$, so calculate drift speed and Δt for each wire seperately since $\bar{v}$ is different for each wire.

$$\begin{aligned}\bar{v}_1 &= (7 \times 10^{-5}\ \frac{\text{m/s}}{\text{V/m}})(5.2\ \frac{\text{V}}{\text{m}}) \\ &= 3.64 \times 10^{-4}\ \text{m/s}\end{aligned}$$

$$\begin{aligned}\Delta t_{_1} &= \frac{L_{_1}}{\bar{v}_1} \\ &= \frac{0.5\ \text{m}}{3.64 \times 10^{-4}\ \text{m/s}} \\ &= 1374\ \text{s}\end{aligned}$$

For wire 2,

$$\begin{aligned}\bar{v}_2 &= uE_{_2} \\ &= 1.82 \times 10^{-4}\ \text{m/s} \\ \Delta t_{_2} &= \frac{0.15\ \text{m}}{1.82 \times 10^{-4}\ \text{m/s}} \\ &= 824\ \text{s}\end{aligned}$$

$$\begin{aligned}\Delta t &= \Delta t_{_1} + \Delta t_{_2} \\ &= 1374\ \text{s} + 824\ \text{s} \\ &\approx 2200\ \text{s} \\ &\approx (2200\ \text{s})\left(\frac{1\ \text{mm}}{60\ \text{s}}\right) \\ &= 36.6\ \text{min}\end{aligned}$$

(c) The minimum time required to reach steady state is given by the speed of light. Using $L = L_{_1} + L_{_2} = 0.65$ m,

$$\begin{aligned}c &= \frac{L}{\Delta t} \\ \Delta t &= \frac{L}{c} \\ &= \frac{0.65\ \text{m}}{3 \times 10^{8}\ \text{m/s}} \\ &\approx 2 \times 10^{-9}\ \text{s}\end{aligned}$$

This is 2 ns, which is much smaller than the time interval for a mobile electron to travel around the circuit.

(d) The electron current is

$$
\begin{aligned}
i_1 &= nA_1 uE_1 \\
&= (9\times10^{28}\ \mathrm{m}^{-3})\pi\left(\frac{0.25\times10^{-3}\ \mathrm{m}}{2}\right)^2(7\times10^{-5}\ \frac{\mathrm{m/s}}{\mathrm{V/m}})(5.2\ \frac{\mathrm{V}}{\mathrm{m}}) \\
&= 1.6\times10^{18}\ \mathrm{s}^{-1}
\end{aligned}
$$

Thus, 1.6×10^{18} electrons pass from the thin wire to the thick wire every second.

19.P.71

(a) According to conservation of charge (the node rule), $i_1 = i_2$. Thus,

$$
\begin{aligned}
i_1 = i_2 & \\
nAu_1E_1 &= nAu_2E_2 \\
u_1E_1 &= u_2E_2 \\
E_2 &= \left(\frac{u_1}{u_2}\right)E_1 \\
&= \frac{1}{3}E_1
\end{aligned}
$$

(b) Neglect the potential difference (energy loss) across the connecting wires. Apply the loop rule to the circuit with the single bulb (bulb 1, which we will call 0 to distinguish this case from the previous case in part (a)).

$$
\mathsf{emf} - E_0L = 0
$$

The current is

$$
\begin{aligned}
i_0 &= nAu_1E_0 \\
&= nAu_1\left(\frac{\mathsf{emf}}{L}\right)
\end{aligned}
$$

Now apply the loop rule to the circuit with two bulbs.

$$
\begin{aligned}
\mathsf{emf} - E_1L - E_2L &= 0 \\
\mathsf{emf} - E_1L - \left(\frac{1}{3}\right)E_1L &= 0 \\
\mathsf{emf} - \frac{4}{3}E_1L &= 0 \\
E_1 &= \frac{3}{4}\frac{\mathsf{emf}}{L}
\end{aligned}
$$

The current through bulb 1 is

$$\begin{aligned} i_1 &= nAu_1 E_1 \\ &= nAu_1 \left(\frac{3}{4}\frac{\mathsf{emf}}{L}\right) \end{aligned}$$

But we know that

$$i_0 = nAu_1 \left(\frac{\mathsf{emf}}{L}\right)$$

So, substitution gives

$$i_1 = \frac{3}{4} i_0$$

Evidently replacing Bulb 2 with a wire increased the current through the battery by a factor of 4/3. Or, to say it in another way, if you originally have a battery and bulb 1 and you connect bulb 2 in series with bulb 1, the current through the battery and bulbs will be 3/4 of the original current.

(c)

$$\begin{aligned} i_{\text{Cu}} &= i_{\text{filament}} \\ n_{\text{Cu}} A_{\text{Cu}} u_{\text{Cu}} E_{\text{Cu}} &= n_{\text{fil}} A_{\text{fil}} u_{\text{fil}} E_{\text{fil}} \end{aligned}$$

The cross-sectional area of a wire is at least 100 times larger than the area of a filament. The mobility of copper is about 10 times larger than tungsten. Thus, the electric field in the copper wire is at least 1000 times smaller than in the tungsten.

The electric fields in all the copper wires are the same because $i = nAuE$ and i (and n, A, and u) is the same in all the copper wires. Thus, E must also be the same.

(d) The graph is shown in Figure 10. Gray lines are for the circuit with two bulbs (part a).

19.X.72

(a) The loop rule gives

$$\begin{aligned} \mathsf{emf} - EL &= 0 \\ E &= \frac{\mathsf{emf}}{L} \\ &= \frac{1.7\text{ V}}{0.75\text{ m}} \\ &= 2.27\ \frac{\text{V}}{\text{m}} \end{aligned}$$