

Maxwell's equations:

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}\end{aligned}$$

Setting $\rho = 0$, $\mathbf{J} = 0$:

$$\begin{aligned}\nabla \cdot \mathbf{E} &= 0 \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}\end{aligned}$$

Take the curl of the third equation

$$\begin{aligned}\nabla \times (\nabla \times \mathbf{E}) &= \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t} \right) \\ \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} &= -\frac{\partial}{\partial t} \nabla \times \mathbf{B} \\ \nabla^2 \mathbf{E} &= \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \\ &= \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}\end{aligned}$$

Wave equation in 1D:

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = 0.$$

Advection equation in 1D:

$$\frac{\partial u}{\partial x} + \frac{1}{v} \frac{\partial u}{\partial t} = 0.$$