Some Newton-Cotes Formulae

Here are the basic, trapezoid, and simpson integration formulae discussed in class. Unlike Numerical Recipes, we adopt here the convention (consistent with the Python scripts on the course web page) that the range $[x_{min}, x_{max}]$ is divided into n equal intervals of width $\delta x = (x_{max} - x_{min})/n$ and that the function is evaluated at the $n+1$ points $x_0, x_1, \ldots, x_{n-1}, x_n$, where $x_i = i\,dx$, $i = 0, \ldots, n$. Note that n must be odd in the simpson formula.

$$
\begin{aligned}
I^{(basic)} &= \delta x \sum_{i=0}^{n} f(x_i) \\
I^{(trapezoid)} &= \delta x \left[\tfrac{1}{2} f(x_0) + \sum_{i=1}^{n-1} f(x_i) + \tfrac{1}{2} f(x_n) \right] \\
I^{(simpson)} &= \delta x \left[\tfrac{1}{3} f(x_0) + \tfrac{4}{3} \sum_{\substack{i=1 \\ i\,\text{odd}}}^{n-1} f(x_i) + \tfrac{2}{3} \sum_{\substack{i=2 \\ i\,\text{even}}}^{n-2} f(x_i) + \tfrac{1}{3} f(x_n) \right]
\end{aligned}
$$