QUANTUM MECHANICS I

PHYS 516

Problem Set # 1
Distributed: Jan. 8, 2010
Due: Jan. 20, 2010

The Frobenius Method is a classical workhorse for finding solutions of relatively simple ordinary differential equations. (The link is to th course website: `http://www.physics.drexel.edu/~bob/PHYS516_10/Frobenius.pdf`). Here are two examples:

$$\begin{aligned}\left(\frac{d^2}{dr^2}+\frac{A}{r^2}+\frac{B}{r}+C\right)R(r) &= 0\\ \left(\frac{d^2}{dr^2}+\frac{D}{r^2}+E+Fr^2\right)R(r) &= 0\end{aligned}$$

The first is a general form for the radial part of the quantum wavefunction for a Coulomb potential. The second is a general form for the radial part of the quantum wavefunction for a harmonic oscillator potential.

1. Use the Frobenius method to determine the quantization condition for square-integrable radial functions $\int_0^\infty R^2(r)dr < \infty$:

Coulomb: on the coefficients A, B, C;

Harmonic Oscillator: on the coefficients D, E, F.

2. For the Coulomb problem in three dimensions determine the coefficients A, B, C in the

a. relativistic case (Klein-Gordon Equation)

b. nonrelativistic case (NR Schrödinger Equation).

3. Determine the coefficients D, E, F for the nonrelativistic harmonic oscillator in

i. one dimension

ii. two dimensions

iii. three dimensions

4. Compute the energy of an electron (in eV) in the most tightly bound state about each of these nuclei:

	$Z = 1$ Proton	$Z = 26$ Iron Nucleus	$Z = 82$ Lead Nucleus
N.R. Schrödinger Equation	-13.58		
Relativistic Klein-Gordon Eq.			

In computing the relativistic energy, subtract off the electron rest energy mc^2 and enter $E - mc^2$ in the table above.

In this problem, if there are any surprises, explain:

a. What?

b. Where?

c. Why?

d. What does it mean?

e. What to do about it?