Due by 4:00 pm Friday, October 12.

Problem 1: Relativistic Correction to Harmonic Oscillator

Find the lowest-order relativistic correction to the energy levels of the 1-D harmonic oscillator. Hint: use ladder operators to compute expectation values.

Problem 2: Zeeman Effect

Here’s an easy one (see your class notes): Consider the $n = 2$ states of the Hydrogen atom, including the spin of the electron, under the influence of a strong magnetic field.

a. What is the energy degeneracy if $B = 0$?

b. Write down equations for the energies of these states, including the Coulomb Hamiltonian and the perturbation cause by the magnetic field.

c. Draw an energy level diagram that illustrates how increasing $B$ lifts the degeneracies (most of it, anyways). Clearly label each line in the diagram and indicate its slope.

Problem 3: Hydrogen without spin

Consider the Hydrogen atom in the absence of intrinsic spin for the electron.

a. What are the energy levels of the 1s and 2p states if the only contributions to fine structure come from the relativistic correction to the kinetic energy? Find the formulae for the energy shifts.

b. What happens to these energy levels if we turn on an external magnetic field? Describe if/how the energy levels get split and why (what’s the perturbing Hamiltonian?).

c. What is the resulting spectrum of possible transitions between the 2p and 1s states? Draw a diagram that labels the states and shows the transitions that could be observed (the transitions must obey the electric dipole selection rules $\Delta l = \pm 1, \Delta m = 0, \pm 1$).
Problem 4: Hyperfine splitting in almost Hydrogen

Determine the hyperfine energy splitting for the following “almost Hydrogen” atoms by appropriate modification of the Hydrogen formula.

a. muonic hydrogen which has a muon in place of the electron (same charge and g-factor but 207 times the mass).

b. positronium which has a positron in place of the proton (same mass and g-factor as an electron, but with positive charge).

c. muonium which has an anti-muon (same mass and g-factor as a muon, but opposite charge) in place of the proton.

Hint: Make sure you use the reduced mass to compute the effective “Bohr radius” in each case.