Physics 280 Lecture 2
Summer 2016

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Objectives

- Review Lorentz Coordinate Transforms and principles of relativity
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- Review Lorentz Coordinate Transforms and principles of relativity
- Be able to understand and perform Lorentz Velocity Transforms
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- Be able to understand and perform Lorentz Velocity Transforms
- Understand relativistic momentum
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- Be able to understand and perform Lorentz Velocity Transforms
- Understand relativistic momentum
- Understand relativistic work and energy
“The laws of mechanics must be the same in all inertial frames of reference.”

\[ x' = x + ut \]

\[ t' = t \]
The principle of relativity: The laws of physics must be the same in all inertial reference frames.
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The constancy of the speed of light: The speed of light in vacuum has the same value, $(2.99 \times 10^8) \text{ m/s}$, in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.
We call the time measurement in the frame in which the event happens and is at rest the proper time (the clock here is at rest relative to the event) $t_p$. 

\[ \Delta t = \Delta t_p \sqrt{1 - \frac{u^2}{c^2}} \equiv \gamma \Delta t_p, \gamma \geq 1 \]

This is called **Time Dilation**. Moving clocks tick slower than clocks at rest.
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Length Contraction

The Lorentz Factor:

\[ \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}. \]

Length Contraction:
- That being measured is at rest: \( \Delta x \)
- That being measured is not at rest: \( L' = L \gamma \)

KEY: Objects are measured with longest length in their rest-frame.
Length Contraction

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Length Contraction:

That being measured is at rest

\[ \Delta x = \gamma \]

That being measured is not at rest

\[ \Delta x' \]
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KEY

Objects are measured with longest length in their rest-frame.
Lorentz Transforms

When motion is in x direction only:

\[ x' = \gamma (x - ut) \]
\[ y' = y \]
\[ z' = z \]

If we look at the reverse transform:

\[ x = \gamma (x' + ut') \]

and plug one into the other, we find:

\[ t' = \gamma (t - ux/c^2) \]

Implication: Not only do we have time dilation, but time and space are tied together.

Implication: The definition of simultaneous differs between observers. If something is simultaneous in one frame, it may not be so in another frame.
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The Lorentz Transforms between frame S and frame S’ where the relative speed between the frames is $v$ is:

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\begin{align*}
    x' &= \gamma (x - ut) \\
y' &= y \\
z' &= z \\
t' &= \gamma \left( t - \frac{ux}{c^2} \right)
\end{align*}
\]

These transforms are valid for infinitesimals as well:

\[
\begin{align*}
dx' &= \gamma (dx - udt) \\
dt' &= \gamma \left( dt - \frac{udx}{c^2} \right)
\end{align*}
\]
Lorentz Velocity Transformations

\[ dx' = \gamma (dx - udt) \]

\[ dt' = \gamma \left( dt - \frac{udx}{c^2} \right) \]

In order to not confuse relative velocity with the velocity of an object measured in frame \( S \), write the relative velocity as \( u \) and the velocity of a measured object as \( v_x \).
Lorentz Velocity Transformations

\[ dx' = \gamma (dx - udt) \]

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\[ \frac{dx'}{dt'} = \frac{(dx - udt)}{(dt - \frac{udx}{c^2})} \]
Lorentz Velocity Transformations

\[ dx' = \gamma (dx - udt) \]
\[ dt' = \gamma \left( dt - \frac{udx}{c^2} \right) \]

\[ \frac{dx'}{dt'} = \frac{dx - udt}{dt - \frac{udx}{c^2}} \]

\[ \frac{dx'}{dt'} = \frac{\frac{dx}{dt} - u}{1 - \frac{u}{c^2} \frac{dx}{dt}} \rightarrow \frac{dx'}{dt'} = \frac{v_x - u}{1 - \frac{uv_x}{c^2}} \]

In order to not confuse relative velocity with the velocity of an object measured in frame S, write the relative velocity as \( u \) and the velocity of a measured object as \( v_x \).
Two spacecraft A and B are moving in opposite direction. An observer on the Earth measures the speed of spacecraft A to be 0.750c and the speed of spacecraft B to be 0.850c. Find the velocity of spacecraft B as observed by the crew on spacecraft A.
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Identify \( u = 0.750c \) and \( v_x = -0.850c \) such that

\[
v'_x = \frac{v_x - u}{1 - \frac{v_x u}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.977c
\]
The requirements of conservation of momentum require that our equation for momentum, $p = mv$ is modified:

$$
\vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m\vec{v}
$$

What happens when $v \to c$?
Relativistic momentum and Newton’s Second Law

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This in turns modifies Newton’s Second Law (\( F \) in direction of \( v \)):

\[
F = \frac{d}{dt} \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{ma}{\left(\sqrt{1 - \frac{v^2}{c^2}}\right)^3}
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a = \frac{F}{m} \left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}
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Relativistic momentum and Newton’s Second Law

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What happens when \( v \to c \)?
An electron, which has a mass of $9.11 \times 10^{-31}$ kg, moves with a speed of 0.750$c$. Find the magnitude of its relativistic momentum and compare this value with the momentum calculated from the classical expression.
Relativistic momentum: example

An electron, which has a mass of $9.11 \times 10^{-31}$ kg, moves with a speed of $0.750c$. Find the magnitude of its relativistic momentum and compare this value with the momentum calculated from the classical expression.

**Classical:**

$$p = mu = 9.11 \times 10^{-31} \text{kg} \left(0.750 \times 3.0 \times 10^8 \text{m/s}\right) = 2.05 \times 10^{-22} \text{kg} \cdot \text{m/s}$$
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**Relativity:**

\[
p = \frac{mu}{\sqrt{1 - \frac{u^2}{c^2}}} = 3.10 \times 10^{-22} \text{kg} \cdot \text{m/s}
\]
Relativity plus conservation of momentum \[ \implies p = \gamma mv \]
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Finally, the work energy theorem implies that

$$K = W = \int_{x_1}^{x_2} Fdx \to [\text{Algebra and calc}] \to (\gamma - 1) mc^2$$
There is a component, $mc^2$ which is independent of velocity and is thus called **Rest Energy**. Thus the total energy is:

$$E = K + mc^2 = \gamma mc^2$$
Relativity and Energy

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What is the energy if the particle is at rest?
Relativity and Energy

There is a component, $mc^2$ which is independent of velocity and is thus called **Rest Energy**. Thus the total energy is:

$$E = K + mc^2 = \gamma mc^2$$

What is the energy if the particle is at rest? With some algebra (see textbook) we get an equation for energy:

$$E^2 = (mc^2)^2 + (pc)^2$$

when $p = 0$:

$$E = mc^2$$

and when $m = 0$ (e.g. photons):

$$E = pc$$
Physicists tend to find that it is easier to express energy in terms of eV, the energy it takes to bring an electron across a potential of 1 Volt. Find the rest energy of a proton in units of electron volts.

*Given:* \(1 \text{eV} = 1.602 \times 10^{-19} \text{J}, \ m_p = 1.673 \times 10^{-27} \text{kg}.

\[E_p = m_p c^2 = 1.673 \times 10^{-27} \text{kg} \left(3.00 \times 10^8 \text{m/s}\right) = 1.504 \times 10^{-10} \text{J} = 938 \text{MeV} \]
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If the total energy of a proton is three times its rest energy, what is the speed of the proton?
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\[ E = 3mc^2 = \gamma mc^2 \]

Solve for \( \gamma \):

\[ \frac{9}{\sqrt{1 - \frac{u^2}{c^2}}} = 1 \]

\[ u = 0.943c \]
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Solve for \( u \):

\[ 9 = \frac{1}{1 - \frac{u^2}{c^2}} \rightarrow u = 0.943c \]
For this particular proton, what is the momentum?
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Recall that $E^2 = p^2 c^2 + (m_p c^2)^2$ and from the previous problem we know that for this particular proton (not all protons) that $E = 3m_p c^2$. An answer in units MeV/c is fine.
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p^2 c^2 = 8 \left( m_p c^2 \right)^2
\]

\[
p = \sqrt{8 \frac{m_p c^2}{c}} = \sqrt{8 \left( 938 \text{ MeV}/c \right)} = 2.65 \times 10^3 \text{ MeV}/c
\]