Final Exam

Name

TA/ Section #

Recitation Time

June 12, 2007, 10:30am

You have 2 hours to complete the exam. Please answer all questions clearly and completely, and that you clearly indicate your final answer to each problem (put a box around the final answer, for example). Make sure that you show all of your work. If your exam sheets do not have enough room for you to write on, please ask for more paper.

You may use a calculator, and, of course, reference the formula sheet, attached. Beyond that, the exam is entirely closed book.
Physical Constants

\[ G = 6.67 \times 10^{-11} \frac{N m^2}{kg^2} \]
\[ g = 9.8 m/s^2 \simeq 10 m/s^2 \]
\[ c = 3 \times 10^8 m/s \]

Some useful math relations

\[ \sin \theta \simeq \theta \quad \text{small angle} \]
\[ \cos \theta \simeq 1 - \theta^2/2 \quad \text{small angle} \]
\[ \frac{dC}{dt} = 0 \]
\[ \frac{d(t^n)}{dt} = nt^{n-1} \]
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula} \]

Projectile Relations

\[ \Delta \vec{r} = \vec{r}_f - \vec{r}_i \]
\[ \vec{r} = xi + yj \]
\[ \vec{v} = \frac{d\vec{r}}{dt} \]
\[ \vec{a} = \frac{d\vec{v}}{dt} \]
\[ \vec{r}(t) = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \]
\[ \vec{v}(t) = \vec{v}_i + \vec{a}t \]
\[ v_{x,f}^2 = v_{x,i}^2 + 2a_xx \]

Circular Motion

\[ a_c = \frac{v^2}{r} \]
\[ a_t = \frac{dv}{dt} \quad \text{tangential acceleration} \]

Newton’s Laws

\[ \vec{F} = ma \]
\[ \sum \vec{F} = 0 \quad \text{equilibrium} \]
Some Specific Forces

\[ \vec{F}_g = -mg\hat{j} \]
\[ F_s = -kx \]
\[ F_G = -\frac{GMm}{r^2} \]

Friction

\[ F_r = \mu F_N \]
\[ R = \frac{1}{2} D p Av^2 \]
\[ F_{vis} = -bv \]

Work/Kinetic Energy

\[ W = \vec{F} \cdot \Delta \vec{r} \]
\[ = \int_{x_1}^{x_2} F_x dx \quad (1 - d) \]
\[ K = \frac{1}{2}mv^2 \]
\[ W = \Delta K \]
\[ P = \frac{dW}{dt} \]
\[ E_{mech} = K + U \]

Potential Energy

\[ \Delta U \equiv -W_{int} \]
\[ \frac{dU}{dx} = -F_x \]
\[ U_g = mgy \]
\[ U_s = \frac{1}{2}kx^2 \]
\[ U_G = -\frac{GMm}{r} \]

Multiple Particle Systems

\[ \vec{r}_{COM} = \frac{1}{M} \sum_i m_i \vec{r}_i \]
\[ \vec{P} = M\vec{v}_{com} \frac{d\vec{P}}{dt} = \vec{F}_{ext} \]

1-d elastic collision

\[ p_{1f} = p_{1i} \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \]
\[ p_{2f} = p_{1i} \left( \frac{2m_2}{m_1 + m_2} \right) \]
Angular Relations

\[ \omega = \frac{d\theta}{dt} \]
\[ \alpha = \frac{d\omega}{dt} \]
\[ \tau = Fr \sin \phi \]
\[ L = I \omega \]
\[ \frac{dL}{dt} = \tau \]
\[ \tau = I \alpha \]
\[ K_R = \frac{1}{2} I \omega^2 \]
\[ v_t = \omega R \]
\[ W = \tau \Delta \theta \]
\[ \sum \tau = 0 \] rotational equilibrium

Moment of Inertia

\[ I = \sum m_i R^2 \]
\[ I_{\text{ring}} = MR^2 \]
\[ I_{\text{disk}} = \frac{1}{2} MR^2 \]
\[ I_{\text{sphere}} = \frac{2}{5} MR^2 \]
\[ I_{\text{rod@end}} = \frac{1}{3} \ell^2 \]
\[ I = I_{\text{com}} + M h^2 \]
1. [20 points] A cannon fires a 5kg cannonball from a cliff 2m above a flat field (as shown).

The ball is fired at an elevation of 30 degrees, and with a velocity of 20 m/s. Ignore air resistance.

(a) What is the kinetic energy of the cannonball when it is first launched?
(b) What is the initial velocity of the cannonball (expressed as a vector)?
(c) What is the maximum height (above the ground) attained by the cannonball?
(d) How long (in time) does the cannonball fly before hitting the ground (y=0)?
(e) How far from the cliff is the cannonball at the end of the flight?
(f) What is the kinetic energy of the cannonball at that time?
2. [20 points] There are two blocks on a table, one with a mass of 3 kg, and one with a mass of 2 kg, as shown. A force of 20 N is applied to the first block, as shown. The coefficient of static friction between each block and the table is 0.2, and the coefficient of kinetic friction between the blocks and table is 0.3.

(a) Draw a free-body diagram showing all of the forces on the left-hand block, and another showing all of the forces on the right-hand block. If you can’t calculate any force directly, simply label them with a variable.
(b) Under the circumstances, do the blocks overcome static friction (will they move)? A yes or no will not suffice. You must demonstrate your answer numerically.
(c) What is the contact force between the two blocks?
(d) What is the acceleration of the entire system?
3. [20 points] A 5kg box is sitting on an inclined plane with an angle of inclination of 30° as shown. The plane has a coefficient of kinetic friction of 0.3.

(a) What is the gravitational potential energy of the block relative to the floor?
(b) What is the force of friction on the block as it slides?
(c) How much work does gravity do on the block during the time it slides to the ground?
(d) How much work does friction do on the block during the time it slides to the ground?
(e) Assuming it starts from rest, what is the kinetic energy of the block when it hits the ground?
(f) How fast is the block going by the time it hits the ground?
4. [20 points] A 100 kg student, angry about his final exam score charges at a speed of 3m/s at his professor (m=70kg), who is initially at rest. The student starts, at t=0, at a position $x = -2m$, and the professor at $x = 4m$.

(a) Where is the center of mass of the system initially?
(b) What is the momentum of the system initially?
(c) What is the center of mass velocity of the system initially?
(d) The student then collides with the professor. Assuming he collides elastically, what is the final speed of the professor?
(e) If they, instead, were to collide perfectly inelastically, what would the final speed be of the two?
(f) **E.C.** If they do collide inelastically, how much kinetic energy is lost?
5. [20 points] A merry-go-round has a radius of 2m, and a mass of 100 kg (as shown). 

(a) How much torque do I apply to the merry-go-round?
(b) What is the angular acceleration of the merry-go-round?
(c) After 10 seconds, what is the angular velocity of the merry-go-round?
(d) After 10 seconds, what is the rotational kinetic energy of the merry-go-round?
(e) After 10 seconds, what is the angular momentum of the merry-go-round?
(f) A 20 kg dog jumps onto the edge of the merry-go-round while in motion. What is the total moment of inertia of the merry-go-round/dog system?
(g) What is the angular velocity of the system after the dog jumps on the merry-go-round?