NAME: Solutions

ID#: ____________________________

SIGNATURE: ________________________

PHYS-201: FUNDAMENTALS OF PHYSICS III
Exam # 2
6 February 2012
CAT 61, 8:00-8:50 AM

RULES:

1. Do not turn over this sheet until instructed to do so.

2. No talking is allowed during the exam. Cell phones and other communication devices need to be turned off.

3. Indicate clearly where the answer is located. Numerical results without units will not be fully credited.

4. This is a closed-book exam: No books or lecture notes allowed!

5. You may use a simple calculator.

6. You need to sign the exam attendance sheet when handing in the exam.

Problem 1: [/30 points] ________

Problem 2: [/30 points] ________

Problem 3: [/30 points] ________

Problem 4: [/10 points] ________

Bonus Problem: [/20 points] ________

Total: [/120 points] ________
**Electromagnetic Waves**

**Problem 1.** A 15 mW helium-neon laser (\(\lambda = 632.8 \text{ nm}\)) emits a beam of circular cross section with a diameter of 2 mm.

(A) Find the maximum electric field in the beam.

(B) Calculate the total energy contained in a 1 m length of the beam.

(C) Find the momentum carried by a 1 m length of the beam.

\[
A)
\quad P = 15 \text{ mW}
\]

\[
P = \frac{\lambda A}{c} = \frac{15 \text{ mW}}{c} = \frac{A E \cdot B}{2 \pi \mu_0} = \frac{A E_{\text{max}}^2}{2 \pi \mu_0}
\]

\[
E_{\text{max}} = \sqrt{\frac{2 \pi \mu_0 P}{A}} = \sqrt{\frac{2 \pi \mu_0 (15 \text{ mW})}{A}} = \sqrt{\frac{2 \pi \mu_0 (15 \times 10^{-3} \text{ W})}{A}}
\]

\[
E_{\text{max}} = 1897 \text{ V/m}
\]

\[
B)
\quad U_{\text{total}} = \left(\frac{P_{\text{avg}}}{cA}\right) L = \frac{P_{\text{avg}}}{c} L
\]

\[
= \frac{15 \times 10^{-3} \text{ W}}{3 \times 10^8 \text{ m/s}} \times 1 \text{ m}
\]

\[
= 5.0 \times 10^{-11} \text{ J}
\]

\[
C)
\quad P = \frac{U}{c} = \frac{5.0 \times 10^{-11} \text{ J}}{3 \times 10^8 \text{ m/s}} = 1.67 \times 10^{-19} \text{ kg m/s}
\]
Electromagnetic Waves
Problem 2. A possible means of space flight is to place a totally reflecting aluminized sheet into orbit around the Earth and then use the pressure of the light (EM radiation) from the Sun to push this “solar sail”. Suppose that the sail has an area of $6 \times 10^5$ m$^2$ and mass of 6,000 kg. Assume a solar intensity of 1,370 W/m$^2$.

(A) What force is exerted on the sail?

(B) What is the sail’s acceleration?

(C) How long does it take the sail to reach the Moon, which is $3.84 \times 10^8$ m away from the Earth. Ignore all gravitational effects and assume that the acceleration calculated in (B) remains constant throughout the trip.

\[ A) \quad P_{\text{rad}} = \frac{2 \pi \frac{W}{m^2}}{C} = \frac{2 \cdot 1370 \frac{W}{m^2}}{5 \times 10^8 \frac{W}{m^2}} = 9.13 \times 10^{-6} \text{ Pa} \]

\[ F = P_{\text{rad}} A = 9.13 \times 10^{-6} \frac{N}{m^2} \cdot 6 \times 10^5 \text{ m}^2 = 54.8 N \]

\[ B) \quad F = ma \]

\[ \alpha = \frac{F}{m} = \frac{54.8 N}{6000 \text{ kg}} = 9.13 \times 10^{-4} \text{ m/s}^2 \]

\[ C) \quad d = \frac{1}{2} a t^2 \quad \Rightarrow \quad t = \sqrt{\frac{2d}{a}} = \left( \frac{2 \times 3.84 \times 10^8 \text{ m}}{9.13 \times 10^{-4} \text{ m/s}^2} \right)^{1/2} = 916992.9 \text{ s} \]

\[ \approx 254 \text{ hrs} \]
Radiation Pressure
**Photoelectric Effect**

**Problem 3.** Photons of energy 5 eV are incident on an electron emitter with the total absorbed power of 2 mW and a work function 3 eV.

(A) Calculate the wavelength of incoming photons.

(B) How many photons per second hit the emitter?

(C) Calculate the fraction of photons that are absorbed by the emitter and which contribute to the photocurrent, if the measured photocurrent at positive voltage is 200 pA (1 pA = $10^{-12}$ A).

\[ A) \] \[
E = hf = \frac{hc}{\lambda}
\]
\[
\lambda = \frac{hc}{E} = \frac{1240 \text{ eV \cdot nm}}{5 \text{ eV}} = 248 \text{ nm}
\]

\[ B) \] \[
P_{\text{power}} = P = \frac{E}{t}
\]
\[
\frac{\# \text{photons}}{s} = P \cdot \frac{1 \text{ photon}}{5 \text{ eV}} = 2 \times 10^{-3} \frac{J}{s} \cdot \frac{1 \text{ photon}}{5 \text{ eV}} \cdot \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}}
\]
\[
= 2.5 \times 10^{15} \frac{\text{photon}}{s}
\]

\[ C) \] \[
\text{One electron per photon}
\]
\[
I = 200 \text{ pA} = 200 \times 10^{-12} \text{ A} \cdot \frac{1 \text{ electron}}{1.6 \times 10^{-19} \text{ A}}
\]
\[
= 1.25 \times 10^9 \frac{\text{electron}}{s}
\]
\[
\text{fraction absorbed} = \frac{1.25 \times 10^9}{2.5 \times 10^{15}} = 5 \times 10^{-7}
\]
Interference in Thin Films

Problem 4. A material with an index of refraction $n_A = 1.3$ is used as an antireflective coating film on a piece of glass with an index of refraction $n_B = 1.5$.

(A) Light with a wavelength of 500 nm is incident on this coating film on the glass. Calculate the minimal thickness of the film that would most reduce the reflection of this light.

\[ 2nt = (m + \frac{1}{2}) \lambda \]

Minimum thickness $m=0$

\[ t_{\text{min}} = \frac{\lambda}{4n} = \frac{500 \text{ nm}}{4 \cdot 1.3} \]

\[ = 96.15 \text{ nm} \]
Interference in Thin Films
The Diffraction Grating

**Bonus Problem.** Show that whenever white light is passed through a diffraction grating of any spacing size \(d\), the violet end (with the wavelength of \(\lambda_V = 400\) nm) of the continuous visible spectrum in the third order always overlaps with red end (with the wavelength of \(\lambda_R = 750\) nm) at the other end of the second-order spectrum.

\[ d \sin \theta = m \lambda \]

\[ \sin \theta = \frac{m \lambda}{d} \]

\[
\begin{array}{ccc}
\text{m} & \text{Violet} & \text{Red} \\
1 & \frac{400\text{ nm}}{d} & \frac{750\text{ nm}}{d} \\
2 & \frac{800\text{ nm}}{d} & \frac{1500\text{ nm}}{d} \\
3 & \frac{1200\text{ nm}}{d} & \frac{2250\text{ nm}}{d}
\end{array}
\]

\( m = 3 \)

\( m = 2 \)

\( m = 1 \)
The Diffraction Grating