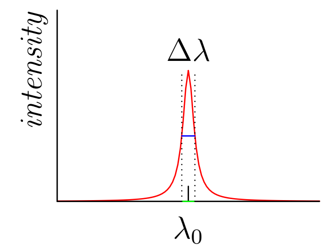
Homework 1

Chapters 12 and 13

Problem 1. *It would take a huge potential energy barrier to confine an electron to the nucleus of an atom (diameter $d \approx 10$ fm). (a) Use the Heisenberg uncertainty principle to find the momentum uncertainty of such a bound electron. (b) Use the monemtum uncertainty from (a) to find the minimum binding energy U. Note that the total energy $E = K + U < 0$ for a bound particle. You may use the non-relativistic form of kinetic energy even though it's not particularly valid for this situation.*

Problem 2. *The time/energy Heisenberg uncertainty principle is the source of an natural linewidth $\Delta\lambda$ in photons emitted from atoms when electrons change orbitals. (a) Calculate the frequency of light emitted in the $n_2 \to n_1$ transition for Hydrogen. (b) Assuming that transition has an average lifetime of $\tau = 1.6$ ns, estimate the relative uncertainty in the energy of the emitted photon.*



Problem 3. *In the particle-wave duality, localized particles are modeled as wave packets, with both a group speed and a phase speed. Between Equations 28.13 and 28.16, the text shows that the group speed v_g of a wave function ψ is the same as the particle speed u. Treat the particle as a non-relativistic de Broglie wave, and use $v_p = \lambda f$ to show that the phase speed $v_p = u/2 \neq u = v_g$.*

Problem 4. *An electron that has an energy of approximately 6 eV moves between rigid walls 1.00 nm apart. Find (a) the quantum number n for the energy state that the electron occupies and (b) the precise energy of the electron.*

Problem 5. *The wave function for a particle confined to moving in a one-dimensional box is*

$$\psi(x) = A \sin\left(\frac{n\pi x}{L}\right) \quad (1)$$

Use the normalization condition to show that

$$A = \sqrt{\frac{2}{L}} \quad (2)$$

HINT*: Because the box length is L, the wave function is zero for $x < 0$ and for $x > L$.*

Problem 6. BONUS PROBLEM. *Particles incident from the left are confronted with a step in potential energy. Located at $x = 0$, the step has a height U. The particles have energy $E > U$. Classically, we would expect all the particles to continue on, although with reduced speed. According to quantum mechanics, a fraction of the particles are reflected at the barrier. Prove that the reflection coefficient R for this case is*

$$R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2} \,, \quad (3)$$

where $k_1 = 2\pi/\lambda_1$ and $k_2 = 2\pi/\lambda_2$ are the wave numbers for the incident and transmitted particles. Proceed as follows. Impose the boundary conditions $\psi_1 = \psi_2$ and $d\psi_1/dx = d\psi_2/dx$ at $x = 0$ to find the relationships between B and A. Then evaluate $R = B^2/A^2$.
Assume the wave function $\psi_1 = Ae^{ik_1x} + Be^{-ik_1x}$ satisfies the Schrödinger equation in region 1, for $x < 0$. Also assume that $\psi_2 = Ce^{ik_2x}$ satisfies the Schrödinger equation in region 2, for $x > 0$. These assumptions will be derived in the posted solutions in case you are interested, but they are pretty straightforward.