Homework 4

Chapter 28

Problem 1. *In Homework 3, you showed that the sun emits $3.87 \cdot 10^{26}$ W of power. (a) Use Stefan's law to calculate the surface temperature at the photosphere ($r = 6.96 \cdot 10^8$ m). (b) Estimate the power needed to produce the same spectrum with an incandescent light bulb (i.e. to heat the bulb to the same temperature). Model the light bulb's tungsten filament as a cylinder 58.0 cm long and 45 μm in diameter with emissivity of 0.45. (c) Tungsten melts at 3695 K, so we cannot actually operate the filament at the same temperature as the sun. What temperature is the filament from (b) if we only radiate at 60 W?*
Because of their long length, tungsten filaments are usually coiled twice. See `http://en.wikipedia.org/wiki/Electrical_filament` *for some nice pictures.*
This problem is similar to P28.2 and P28.55.

(a)

$$P = \sigma A e T^4 \tag{1}$$

$$T = \left(\frac{P}{\sigma A e}\right)^{1/4} = \left(\frac{P}{\sigma 4\pi r^2 e}\right)^{1/4} = \left(\frac{3.87 \cdot 10^{26}\ \text{W}}{5.6696 \cdot 10^{-8}\ \text{W/m}^2\text{K}^4 \cdot 4\pi(6.96 \cdot 10^8\ \text{m})^2 \cdot 1}\right)^{1/4} = 5790\ \text{K}\ , \tag{2}$$

where we treat the sun as a black body ($e = 1$).
(b) Let the subscript w denote the tungsten filament and s denote the sun.

$$\frac{P_\text{w}}{P_\text{s}} = \frac{\sigma A_\text{w} e_\text{w} T^4}{\sigma A_\text{s} e_\text{s} T^4} = \frac{L_\text{w} 2\pi r_\text{w} e_\text{w}}{4\pi r_\text{s}^2} \tag{3}$$

$$P_\text{w} = \frac{L_\text{w} r_\text{w} e_\text{w}}{2r_\text{s}^2} P_\text{s} = \frac{0.58\ \text{m} \cdot \frac{45 \cdot 10^{-6}\ \text{m}}{2} \cdot 0.45}{2 \cdot (6.96 \cdot 10^8\ \text{m})^2} \cdot 3.87 \cdot 10^{26} = 2.35\ \text{kW} \tag{4}$$

(c)

$$T = \left(\frac{P}{\sigma A e}\right)^{1/4} = \left(\frac{P}{\sigma L 2\pi r e}\right)^{1/4} = \left(\frac{60\ \text{W}}{5.6696 \cdot 10^{-8}\ \text{W/m}^2\text{K}^4 \cdot 0.58\ \text{m} \cdot 2\pi \cdot \frac{45 \cdot 10^{-6}\ \text{m}}{2} \cdot 0.45}\right)^{1/4} = 2310\ \text{K} \tag{5}$$

Problem 2. *A simple pendulum as a length of 2.00 m and a mass of 3.00 kg. The amplitude of oscillation is 5.00 cm. Assuming that energy is quantized, calculate the quantum number of the pendulum.*

Using Planck's assumption that energy is quantized (Equation 28.2)

$$E_n = nhf = mgh_\text{max} = mgL(1 - \cos\theta_\text{max}) = mgL\left(1 - \sqrt{1 - (a/L)^2}\right) = 18.4\ \text{mJ} \tag{6}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = 352\ \text{mHz} \tag{7}$$

$$n = \frac{E_n}{hf} = \frac{18.4\ \text{mJ}}{6.63 \cdot 10^{-34}\ \text{J·s} \cdot 352\ \text{mHz}} = 7.87 \cdot 10^{31} \tag{8}$$

Problem 3. *Sodium has a work function of 2.46 eV. (a) Find the cutoff wavelength and cutoff frequency for the photoelectic effect. (b) What is the stopping potential if the incident light has a wavelength of 440 nm?*

(a) The cutoff wavelength is the wavelength where the incoming light has barely enough energy to free an electron, i.e. all of the photon's energy goes into overcoming the work function barrier.

$$hf = \phi \tag{9}$$

$$f = \frac{\phi}{h} = \frac{2.46\ \text{eV} \cdot 1.60 \cdot 10^{-19}\ \text{J/eV}}{6.63 \cdot 10^{-34}\ \text{J·s}} = 592\ \text{THz} \tag{10}$$

$$\lambda f = c \tag{11}$$

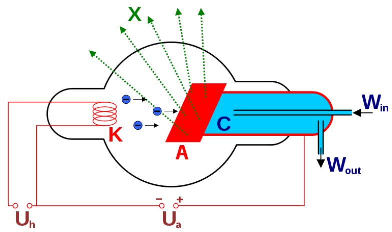
$$\lambda = \frac{c}{f} = 507\ \text{nm} \tag{12}$$

(b) The photon brings in hf, but much of that energy goes to overcoming the work function barrier. The left over energy $hf - \phi$ becomes the electron's kinetic energy. The stopping potential is the voltage change which matches that kinetic energy.

$$K_\text{max} = hf - \phi = h\frac{c}{\lambda} - \phi = 5.98 \cdot 10^{-20}\ \text{J} = 0.358\ \text{eV} \tag{13}$$

$$\Delta V_S = K_\text{max}/e = 0.374\ \text{V} \tag{14}$$

Problem 4. *X-rays generation (e.g. for medical imaging) can be modeled as an inverse photoelectric effect (basically the regular photoelectric effect played backwards in time). An electron beam is fired into an anode, which absorbs the electrons and emits radiation (the X-rays). If the electrons are accelerated by* 50 *kV towards a Tungsten surface* ($\phi = 4.5$ *eV*)*, find the wavelength of the emitted photons predicted by this model.*



HINT. *Follow the energy flow through the system. Ignore anode heating.*

The kinetic energy of the incoming electrons (due to the accelerating voltage) is $e\Delta V = 50$ keV. They gain an additional $\phi = 4.5$ eV of energy while "dropping into" the lower energy bound states of the metal, but this energy is much less than the energy from the accelerating voltage, so we'll ignore it.
Assuming all the energy from the electons is converted into photons (since we're ignoring anode heating), we get

$$E = hf = \frac{hc}{\lambda} \tag{15}$$

$$\lambda = \frac{hc}{E} = \frac{hc}{e\Delta V} = \frac{1240 \text{ eV}\cdot\text{nm}}{5 \text{ keV}} = 0.0248 \text{ nm} \tag{16}$$

The production of X-rays in tubes is more accurately modeled as a combination of X-ray flourescence and bremsstrahlung. Our "inverse photoelectric effect" prediction is the gives the shortest wavelength (highest frequency) of the emitted photons. For more details, see `http://en.wikipedia.org/wiki/X-ray#Medical_physics`.

Problem 5. *(a) What value of* n_i *is associated with the* 94.96 *nm spectral line in the Lyman series of hydrogen? (b) Could this wavelength be associated with the Paschen series or the Balmer series?*

(a) The Lyman series transitions all end in the ground state $n_f = 1$ Using the generalized Rydberg equation (Equation 11.28)

$$\frac{1}{\lambda} = R_{\text{H}} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \tag{17}$$

$$-\frac{1}{n_i^2} = \frac{1}{\lambda R_{\text{H}}} - \frac{1}{n_f^2} \tag{18}$$

$$n_i = \left(\frac{1}{n_f^2} - \frac{1}{\lambda R_{\text{H}}} \right)^{-1/2} = \left(1 - \frac{1}{94.96 \cdot 10^{-9} \text{ m} \cdot 1.10 \cdot 10^7 \text{ m}^{-1}} \right)^{-1/2} = 5 \,. \tag{19}$$

(b) This wavelength cannot have come from the Balmer ($n_f = 2$) or Paschen ($n_f = 3$) series because the shortest wavelength for any series is given in the limit that $n_i \to \infty$

$$\frac{1}{\lambda_{\text{min}}} = \frac{R_{\text{H}}}{n_f^2} \tag{20}$$

$$\lambda_{\text{min}} = \frac{n_f^2}{R_{\text{H}}} \,. \tag{21}$$

For the Balmer series $\lambda_{\text{min}} = 365$ nm, and for the Paschen series $\lambda_{\text{min}} = 820$ nm. Both of these series-minimum wavelengths are larger than the wavelength of our spectral line.

Problem 6. BONUS PROBLEM. *Use Bohr's assumptions in Section 11.5 to derive a formula for the allowed energy levels in singly ionized helium* (He^+)*.*

Following the derivation of hydrogen energy levels in the book, we have our analog for Equation 11.19 (total energy of the electron)

$$E = K + U_e = \frac{1}{2} m_e v^2 - k_e \frac{Ze^2}{r} \tag{22}$$

where Z is the atomic number of our element (1 for H, 2 for He).

Applying Newton's second law and Coulomb's law to the electron's circular motion,

$$\frac{m_e v^2}{r} = F = \frac{k_e Z e^2}{r^2} \ , \tag{23}$$

so the kinetic energy of the electron is (our Equation 11.20 analog)

$$K = \frac{1}{2} m_e v^2 = \frac{k_e Z e^2}{2r} = \frac{U_e}{2} \ . \tag{24}$$

Plugging this expression for K into eq. 22

$$E = -\frac{k_e Z e^2}{2r} \tag{25}$$

We can find the allowed value for r by substituting angular momentum conservation $m_e v r = n\hbar$ into eq. 23

$$m_e v^2 = \frac{k_e Z e^2}{r} \tag{26}$$

$$m_e \left(\frac{n\hbar}{m_e r}\right)^2 = \frac{k_e Z e^2}{r} \tag{27}$$

$$n^2\hbar^2 = k_e Z e^2 r m_e \tag{28}$$

$$r = \frac{n^2\hbar^2}{m_e k_e Z e^2} \ , \tag{29}$$

which is our analog to Equation 11.22.
Plugging this expression for r into our electron energy forumula

$$E_n = -\frac{m_e k_e^2 Z^2 e^4}{2n^2\hbar^2} = Z^2 \cdot E_{n\mathrm{H}} \tag{30}$$

$$E_{n,\mathrm{He}} = 2^2 E_{n,\mathrm{H}} = -\frac{54.42 \text{ eV}}{n^2} \tag{31}$$

which is our analog to Equation 11.25.
Note that the only difference between this derivation and the book's hydrogen derivation is the replacement $k_e e^2 \to k_e Z e^2$ in Coulomb's law. This is also the only place the constant e comes into the derivation. Simply matching and replacing e^2 with Ze^2 in Equations 11.23 and 11.24 would produce the correct answer without following every step of the derivation.