

Homework 3

Chapters 24 and 27

Problem 1. Assume that the intensity of solar radiation incident on the cloudtops of the Earth is 1370 W/m^2 . (a) Calculate the total power radiated by the Sun. (b) Determine the maximum values of the electric and magnetic fields in the sunlight at their source in the photosphere (<http://en.wikipedia.org/wiki/Sun#Photosphere>) a distance of $6.96 \cdot 10^8 \text{ m}$ from the center of the sun.

(a) The intensity at Earth is the total power spread over a sphere of radius $r_E = 150 \cdot 10^9 \text{ m}$ (the Earth-Sun distance).

$$\frac{P}{4\pi r_E^2} = I_E \quad (1)$$

$$P = 4\pi r_E^2 I_E = 4\pi (150 \cdot 10^9 \text{ m})^2 \cdot 1370 \text{ W/m}^2 = 3.87 \cdot 10^{26} \text{ W} . \quad (2)$$

(b) The intensity in the photosphere is

$$I_P = \frac{P}{4\pi r_P^2} = \frac{4\pi r_E^2 I_E}{4\pi r_P^2} = I_E \left(\frac{r_E}{r_P} \right)^2 = 63.6 \text{ MW/m}^2 . \quad (3)$$

The intensity of light is related to the peak electric field by Equation 24.27

$$I = \langle S \rangle = \frac{1}{2\mu_0 c} E_{\max}^2 . \quad (4)$$

So

$$E_{\max} = \sqrt{2\mu_0 c I} = \sqrt{2 \cdot 4\pi \cdot 10^{-7} \cdot 3 \cdot 10^8 \cdot 63.6 \cdot 10^6} = 219 \text{ kV/m} . \quad (5)$$

In a light wave, $E_{\max} = cB_{\max}$ so

$$B_{\max} = \frac{E_{\max}}{c} = 730 \text{ } \mu\text{T} \quad (6)$$

Problem 2. 11500 Hz sound waves moving at 350 m/s hit a wall with two slits 50.0 cm apart. (a) At what angle is the first minimum located? (b) At what angle is the first minimum located if the sound is replaced by 10.0 GHz microwaves?

The first minimum occurs when the waves are 180° out of phase (due to a path length difference of $\lambda/2$).

$$d \sin(\theta_{\min}) = \left(n + \frac{1}{2} \right) \lambda \quad n = 0, 1, 2, \dots \quad (7)$$

$$\theta_{\min} \approx \frac{\left(n + \frac{1}{2} \right) \lambda}{d} \quad n = 0, 1, 2, \dots \quad (8)$$

So the first minimum ($n = 0$) is located at

$$\theta_{\min} = \arcsin \left(\frac{\lambda}{2d} \right) . \quad (9)$$

(a) For $f = 1150 \text{ Hz}$ sound, $\lambda = vT = v/f = 3.04 \text{ cm}$ and

$$\theta_{\min} = \arcsin \left(\frac{3.04 \text{ cm}}{2 \cdot 50.0 \text{ cm}} \right) = 30.4 \text{ mrad} = 1.74^\circ . \quad (10)$$

(b) For $f = 10.0 \text{ GHz}$ light, $\lambda = c/f = 3.00 \text{ cm}$ (pretty much the same wavelength as the sound in (a)) and

$$\theta_{\min} = \arcsin \left(\frac{3.00 \text{ cm}}{2 \cdot 50.0 \text{ cm}} \right) = 30.0 \text{ mrad} = 1.72^\circ . \quad (11)$$

Problem 3. Two slits are separated by 1 mm. A beam of 440 nm light incident on the slits produces an interference pattern. (a) Determine the number of intensity maxima observed in the angular range $-20.0^\circ < \theta < 20.0^\circ$. (b) If the light is shifted to a longer wavelength, will the number of maxima in that range increase or decrease?

(a) Following the path-length argument outlined in the previous problem for interference minima, we have maxima located at

$$d \sin(\theta_{\max}) = n\lambda \quad n = 0, 1, 2, \dots \quad (12)$$

$$n_{20^\circ} = \frac{d \sin(\theta)}{\lambda} = \frac{1 \cdot 10^{-3} \text{ m} \cdot \sin(20^\circ)}{440 \cdot 10^{-9} \text{ m}} = 777.32 . \quad (13)$$

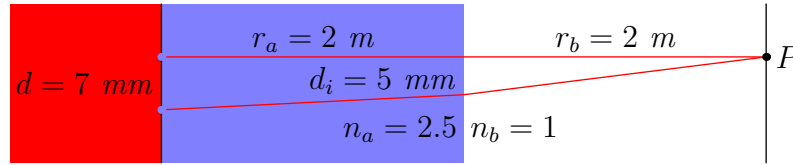
So the number of maxima in the specified range is

$$2 \cdot \text{floor}(n_{20^\circ}) + 1 = 1555 \quad (14)$$

(b) A longer wavelength will increase the spacing between the interference maxima, decreasing the total number in the specified angle range.

Problem 4. A plane wave of monochromatic light moving to the right through a strange material with index of refraction n_a hits a barrier with two slits separated by d . Some light passes through each slit, and after moving a distance r_a to right, leaves the strange material and enters air with a refractive index n_b . The interface bends the light rays according to Snell's law (bending shown on the diagram, so you don't need Snell's law to solve the problem). A distance r_b to the right of the interface is a screen, on which an interference pattern appears. The first minimum of the interference pattern appears at point P , directly to the right of the top slit. Find the frequency of light.

HINT. I've drawn the paths taken to P from each slit. The vertical distance between the two rays at the strange-material/air interface is d_i . You should use a geometric argument to determine the phase difference between the two paths.



The path length in mediums a and b for the top t and bottom b slits are given by

$$L_{ta} = 2 \text{ m} \qquad L_{tb} = 2 \text{ m} \qquad (15)$$

$$L_{ba} = \sqrt{2^2 + (0.007 - 0.005)^2} \text{ m} = (2 + 1.00 \cdot 10^{-6}) \text{ m} \qquad L_{bb} = \sqrt{2^2 + 0.005^2} \text{ m} = (2 + 6.25 \cdot 10^{-6}) \text{ m} \qquad (16)$$

The wavelength depends on the index of refraction according to $\lambda = \frac{c}{n_f}$, and the phase along a path is given by $\phi = \frac{2\pi}{\lambda}L$, so

$$\frac{\Delta\phi}{2\pi} = \frac{\phi_b - \phi_t}{2\pi} = \frac{L_{ba}}{\lambda_a} + \frac{L_{bb}}{\lambda_b} - \frac{L_{ta}}{\lambda_a} + \frac{L_{tb}}{\lambda_b} = \frac{f}{c} [n_a(L_{ba} - L_{ta}) + n_b(L_{bb} - L_{tb})] . \qquad (17)$$

At frequency for which this is the first minimum,

$$\Delta\phi = \pi , \qquad (18)$$

so

$$f = \frac{c}{2[n_a(L_{ba} - L_{ta}) + n_b(L_{bb} - L_{tb})]} = \frac{3.00 \cdot 10^8 \text{ m/s}}{2(2.5 \cdot 1.00 \cdot 10^{-6} \text{ m} + 6.25 \cdot 10^{-6} \text{ m})} = \mathbf{17.1 \text{ THz}} . \qquad (19)$$

Problem 5. An oil film ($n = 1.5$) floats on the surface of a bowl of water. The film is illuminated by a white light placed directly above the bowl. Red light at $\lambda = 650 \text{ nm}$ is the most strongly reflected color. (a) How thick is the oil? (b) What color is most strongly transmitted?

(a) Constructive interference on reflection follows Equation 27.11

$$2nt = \left(m + \frac{1}{2}\right) \lambda \qquad m = 0, 1, 2, \dots \qquad (20)$$

Where the $+\frac{1}{2}$ is due to the 180° phase change from a reflection off the interface in the direction of increasing index of refraction (for the path that bounces off the air-oil interface).

The strength of the reflected color decreases as m increases, due to some light being absorbed by the oil and decoherence from scattering. Since red light is the most strongly reflected, $m = 0$, and the thickness of the film is

$$t = \frac{\lambda}{4n} = \mathbf{108 \text{ nm}} \qquad (21)$$

(b) Constructive interference on transmission occurs when

$$2nt = m\lambda \qquad m = 1, 2, 3, \dots \qquad (22)$$

Because of absorption, the strongest transmission will be for $m = 1$, so the most strongly transmitted color is

$$\lambda_t = 2nt = \mathbf{325 \text{ nm}} \qquad (23)$$

which is in the ultraviolet.

Problem 6. BONUS PROBLEM. Derive the Equation 27.8, which gives the average intensity on a screen far from a single slit relative to the maximum intensity I_{max} at $\theta = 0$.

$$I_{avg} = I_{max} \cos^2 \left(\frac{\pi d \sin(\theta)}{\lambda} \right) \qquad (24)$$

HINT. Remember from Chapter 24 that for plane waves

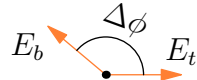
$$I = \frac{1}{2\mu_0 c} E_{max}^2 \qquad (25)$$

where the electric field is perpendicular to the direction of propagation. Assume the screen is far enough away that the waves emanating from the slits can be treated as plane waves.

From the path-length argument we've used in Problems 2 and 3, we know the phase difference between the light from each slit will be

$$\Delta\phi = \frac{2\pi}{\lambda} d \sin(\theta) \quad (26)$$

Drawing a phasor diagram for the Electric field, we have



$$\Delta\phi = \frac{2\pi}{\lambda} d \sin(\theta) \quad (27)$$

The maximum electric field is thus given by

$$E_{\max} = 2E_0 \cos\left(\frac{\Delta\phi}{2}\right) \quad (28)$$

where $E_0 = E_t = E_b$. The intensity is then given by

$$I = \frac{1}{2\mu_0 c} E_{\max}^2 \quad (29)$$

$$= \frac{1}{2\mu_0 c} 4E_0^2 \cos^2\left(\frac{\Delta\phi}{2}\right) \quad (30)$$

$$= I_{\max} \cos^2\left(\frac{\Delta\phi}{2}\right), \quad (31)$$

where we made the substitution $I_{\max} = I(\Delta\phi = 0)$. Plugging in for $\Delta\phi$,

$$I = I_{\max} \cos^2\left(\frac{\pi d \sin(\theta)}{\lambda}\right), \quad (32)$$

which is what we set out to show.