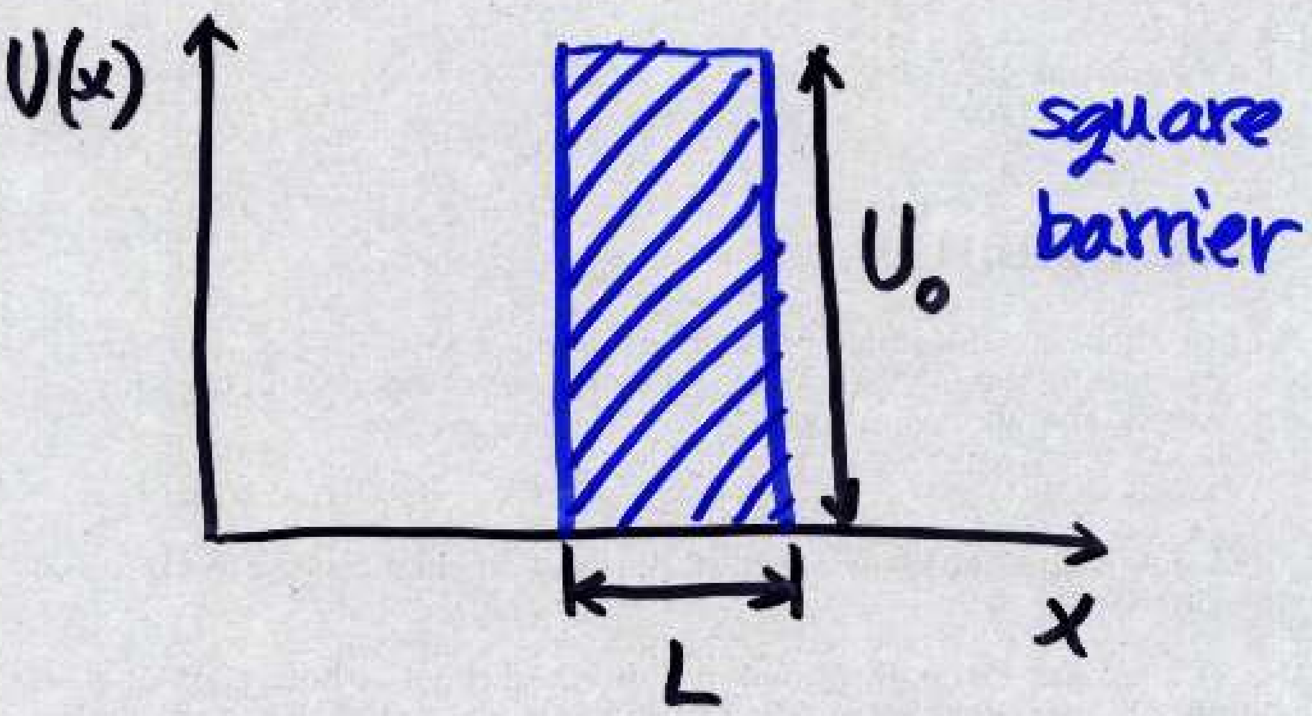
Tunneling through a potential energy barrier

→ what is an energy barrier?



$$U(x) = \begin{cases} 0, & x<0 \ \& \ x>L \\ U_0, & 0 \leq x \leq L; \ \underline{\underline{U_0 > 0}} \end{cases}$$

→ what happens to a classical object with kinetic energy E when it approaches the barrier?

Two possible scenarios:

$E > U_0$

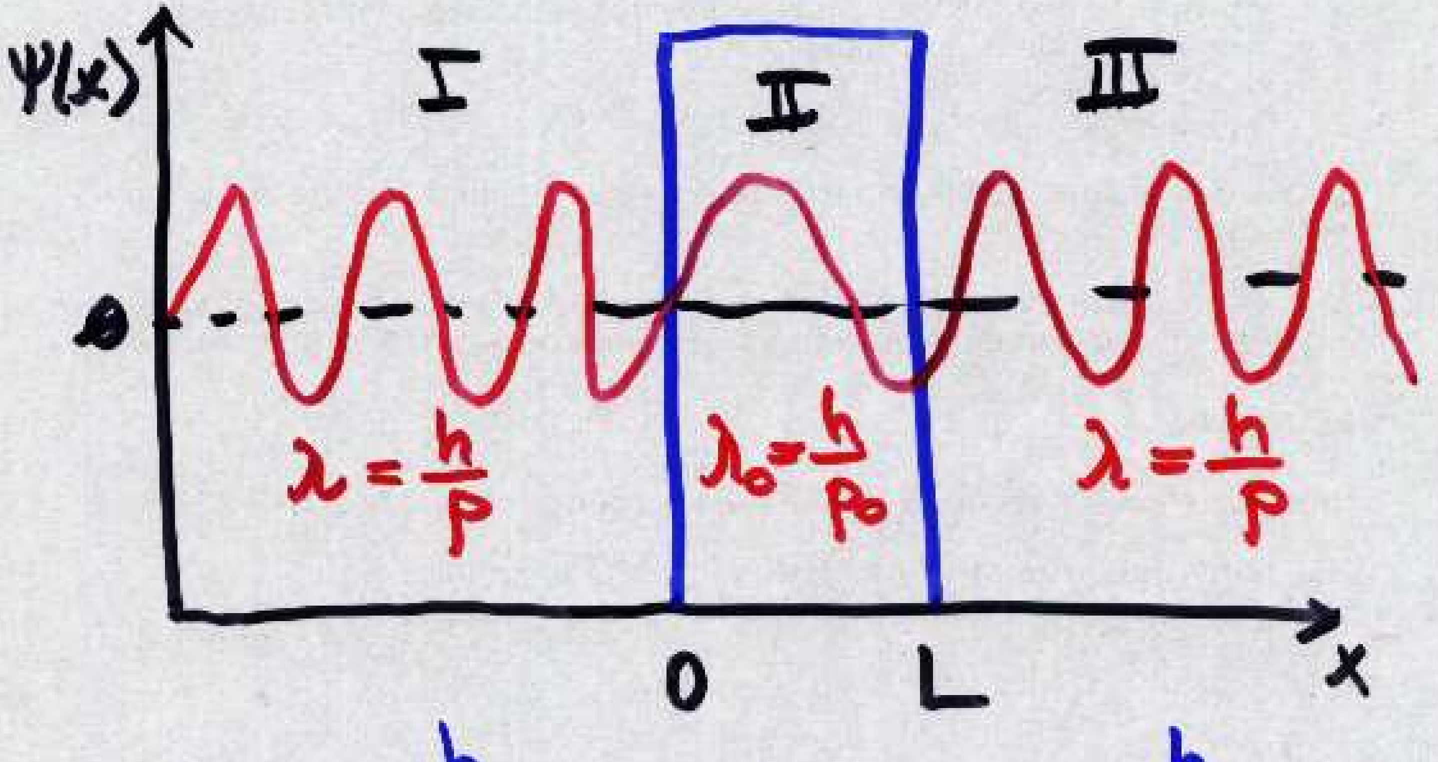
- object passes through the barrier
- inside the barrier, at $0 \leq x \leq L$, $E_T = E - U_0 > 0$ (smaller kinetic energy $\Rightarrow$ lower speed)
- at $x > L$, $E_T = E$, same kinetic energy

$E < U_0$

- the object CANNOT pass the barrier
- no object at $x > 0$ or at $x > L$!

→ what happens to a quantum particle approaching the barrier?

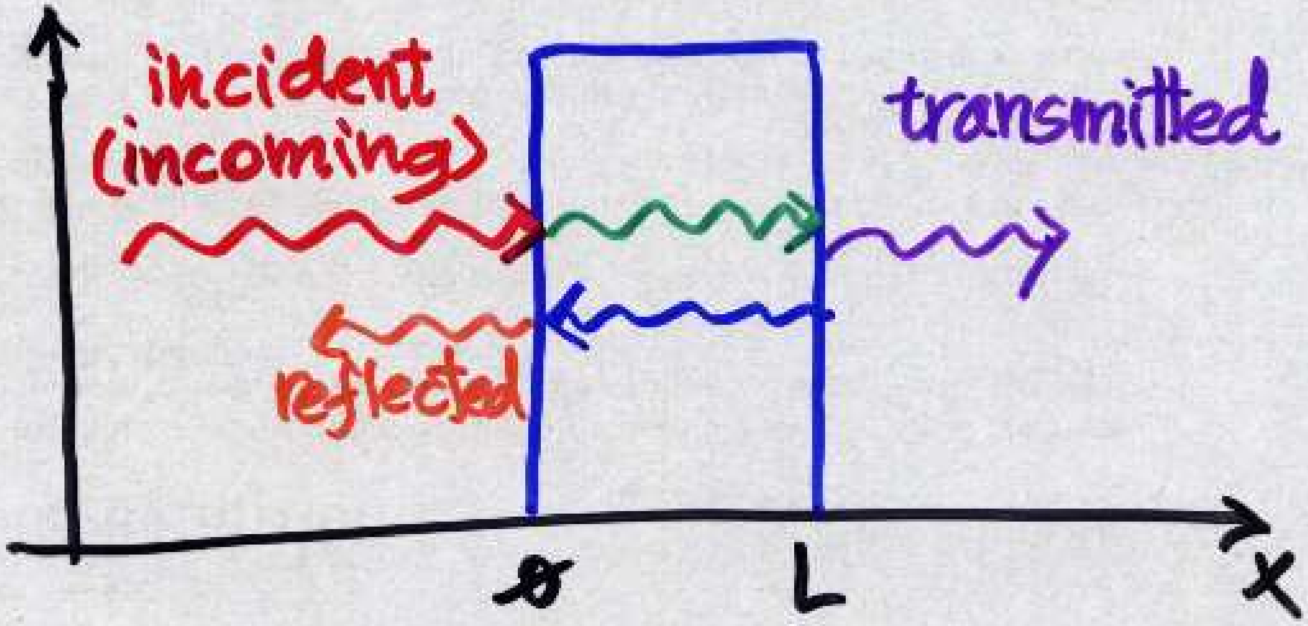
(a) $E > U_0$: situation similar to the case of a classical object, however, QP requires quantum mechanical description in terms of $\psi(x)$



$$\lambda = \frac{h}{\sqrt{2mE}} \quad \& \quad \lambda_0 = \frac{h}{\sqrt{2m(E-U_0)}}$$

$$\lambda_{I} = \lambda_{III} \quad \& \quad \lambda_{II} = \lambda_0$$

- in addition to the wave description, there is a finite probability for a QP to reflect back from I → II and from II → III borders:



probabilities for reflection, R, and transmission, T:

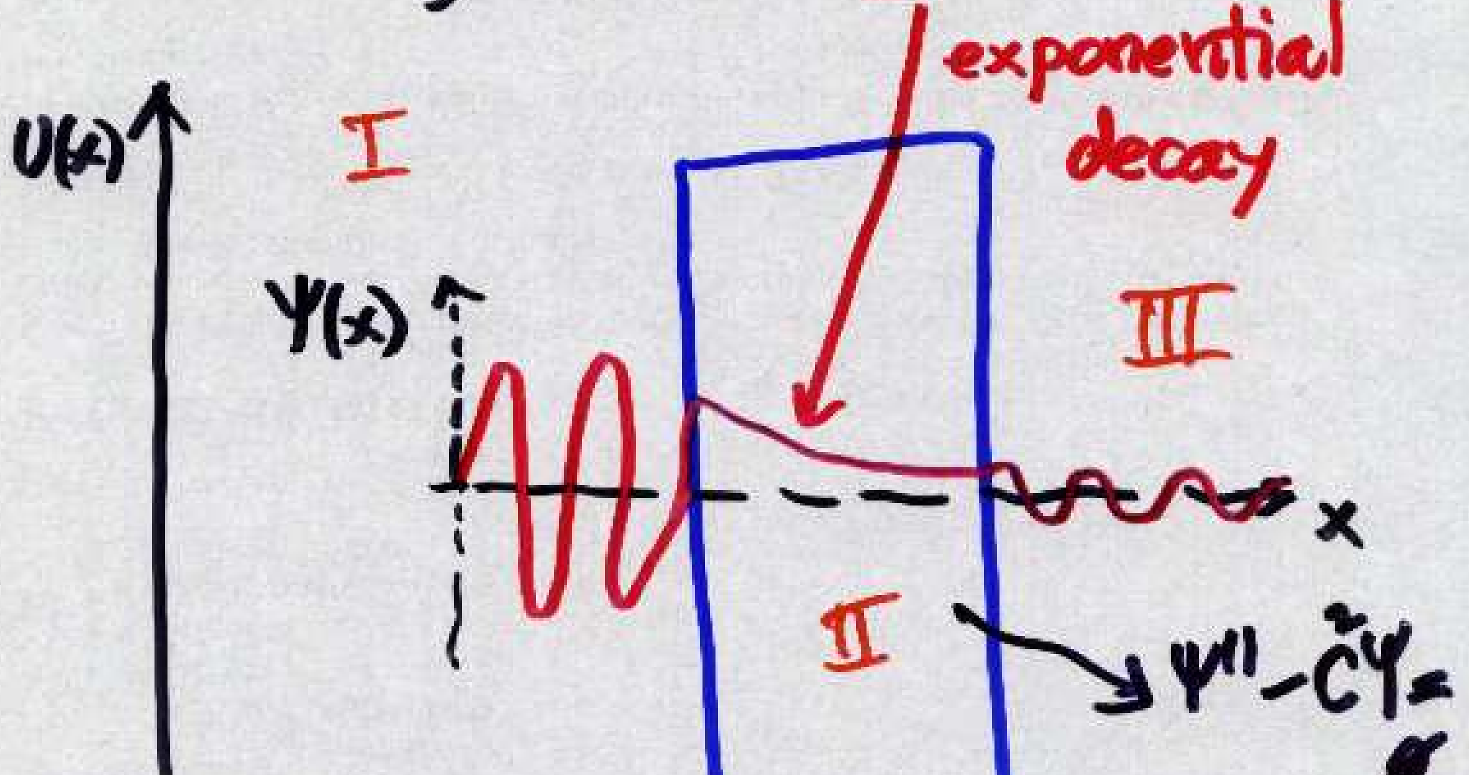
$$T + R = 1$$

→ QP incoming on the barrier is either reflected OR transmitted

(b) $E < U_0$: QUANTUM TUNNELING

UNLIKE the classical object, there is a FINITE probability, T, for the QP to tunnel to the other side of the barrier!

→ wave function $\psi_{II}(x)$



→ the thicker the barrier (large L) the less is T (tunneling prob.)

→ tunneling probability or transmission coefficient

$$\boxed{\begin{gathered} T \approx e^{-2CL} \\ C = \frac{\sqrt{2m(U_0 - E)}}{\hbar} \end{gathered}}$$

$U_0 - E > 0$

→ important:

- dependence on L is the strongest : $\propto e^{-\alpha L}$
- weaker but still strong dependence on E :

$$e^{-\alpha\sqrt{U_0 - E}}$$

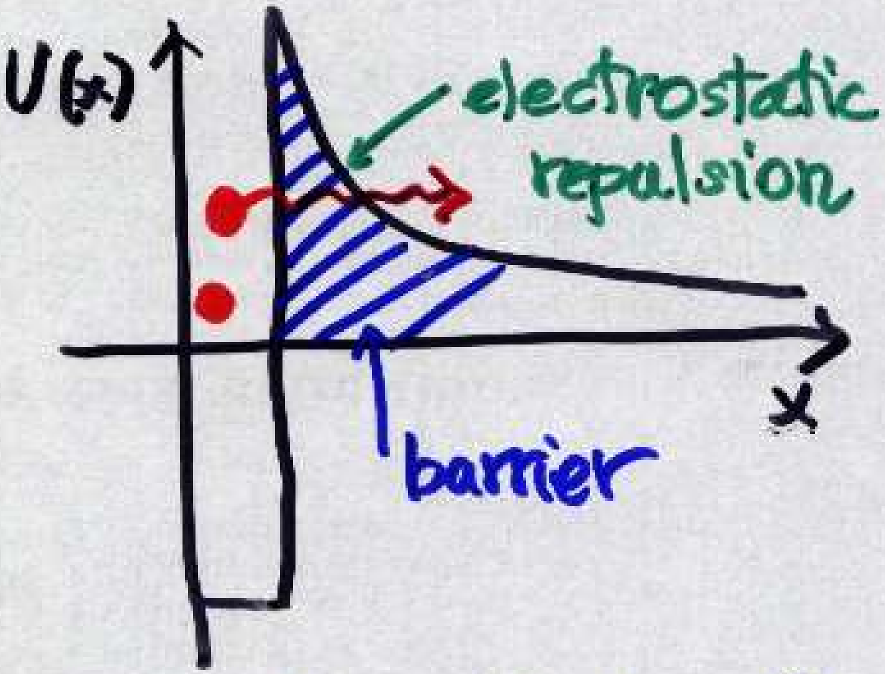
→ reflection coefficient (reflection probability) $R = 1 - T$

Applications of Tunneling

→ α- decay:

- α particle (nucleus of He-atom)
- heavy nuclei (+ charged) naturally decay
- α particle 2+ charge

} coulomb repulsion



● α-particle tunnels through the barrier

→ nuclear fusion (inverse to decay):

- two protons fuse to form a deuterium nucleus

→ scanning tunneling microscope (STM)

Chapter 28 Quiz Questions

(1) What is the minimal energy that binds an electron in a metal?

a. The stopping potential.

[✓ b.] The work function.

c. A quantum oscillation.

d. A thermal release.

e. The cutoff frequency.

(2) The maximum kinetic energy of photoelectrons depends on:

[✓ a.] the frequency of light

b. the intensity of light

c. the # of photons that hit the metal per second

d. the speed of light

e. the # of quanta

Q1

$hf = \phi + K_{max}$

[redacted]

(3) In the photoelectric effect, K_{max} and f are:

a. inversely proportional

b. K_{max} proportional to f^2

c. $K_{max} \propto f^{-2}$

d. not related

[✓ e.] linearly related

(4) Photoelectric current magnitude depends on:

a. light frequency

[✓ b.] light intensity

c. light speed

d. index of refraction

e. not related

Q2

[redacted]

(5) The momentum of photon frequency f is given by:

a. mv
b. mvr
[✓ c. hf/c]
d. hf/m
e. hc/f

$$P = \frac{h}{\lambda}$$

$$\lambda = \frac{c}{f}$$

(6) The Compton wavelength of the electron is:

[✓ a. h/mc]
b. m/f
c. fc/m
d. mhc/f
e. f/mr

(7) The de Broglie wavelength of a particle is given by:

[✓ a. h/mv]

b. cv/m

c. h/mc

d. mc/v

e. hf/mv

(8) A photon collides with an electron. After the collision, the wavelength of the scattered photon is:

[✓ a. greater than or equal to the initial wavelength]

b. equal to the initial wavelength

c. less than the initial wavelength

(9) Which type of hot body radiation requires the highest temperature?

a. infrared

[✓ b. blue]

c. red

d. yellow

e. green

$\lambda_{max} T = const$

(10) Light can be considered as both wave and :

a. fluid

b. attractor

c. α-particle

d. electron

[✓ e. photon]

Q5

(11) Heisenberg's uncertainty principle states that it is fundamentally impossible to make simultaneous measurements, with infinite accuracy, of a particle's position and :

a. wavelength

b. work function

c. angular momentum

d. energy

[✓ e. momentum]

$\Delta x \Delta p_x \gtrless \frac{\hbar}{2}$

~~$\Delta T \Delta P$~~

Q6

(12) A particle in a 1D infinite potential well is known as a particle in a box. When the walls of the well are located at x=0 and at x=L, the wave function of the QP must have the value $\psi(0)=\_$ and $\psi(L)=\_$.

[✓ a.] 0 & 0

b. 0 & L

c. L & 0

d. 0 & $\sqrt{L}$

e. $\sqrt{L}$ & $\sqrt{L}$

(13) A particle in a 1D infinite potential well has the walls located at $x=0$ and $x=L$. If the boundary conditions are to be satisfied, the particle must have wavelength $\lambda=$

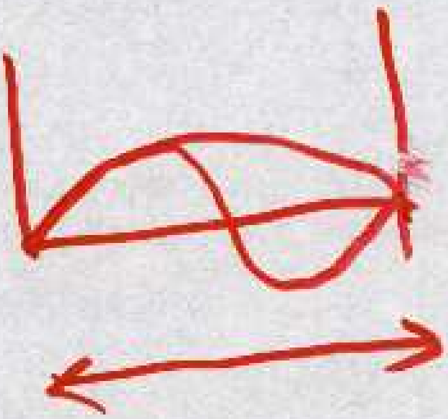
~~a.~~ $2L/n$

b. 0

c. L

d. $1/L$

e. $L/2n$



$2L=\lambda$

$L=\lambda$

Q8

(14) In the quantum phenomenon of tunneling through a potential energy barrier, the barrier consists of:

a. an attractive potential across a FINITE region of space

b. an attractive potential across an INFINITE region of space

[✓ c.] a repulsive potential across a FINITE region

d. a repulsive potential across an INFINITE region

Q9

(15) When a particle is in a box of length L, the separation between energy levels with quantum numbers $\underline{n}$ and $\underline{n+1}$ is:

a. $\frac{h^2}{8mL^2}$

b. $\frac{h^2 n}{8mL^2}$

c. $\frac{h^2 n}{4mL^2}$

[✓ d.] $\frac{h^2(2n+1)}{8mL^2}$

e. $\frac{h^2(n+1)^2}{8mL^2}$

$$E_n = \frac{n^2h^2}{8mL^2}$$

$$E_{n+1} - E_n$$

Q16

(16) When a surface with a work function of 5eV is illuminated by 400 nm wavelength photons, photoelectrons are:

a. not emitted

b. emitted with $K_{max} = 1.89eV$

c. emitted with $K_{max} = 3.11eV$

d. emitted with $K_{max} = 5 eV$

e. not emitted but photons of 5eV energy are emitted

$$hf_c = \phi$$

$$\frac{hc}{\lambda_c} = \phi \rightarrow \lambda_c = \frac{hc}{\phi} = \frac{1240\,eV\,nm}{5\,eV}$$

Q11

(17) The maximum change in wavelength in photons scattering from electrons occurs when the photon's scattering angle is:

a. 0°

b. 45°

c. 90°

[✓ d. 180°]

e. 270°

$$\lambda - \lambda_0 = \lambda_c (1 - \cos\theta)$$

Q12

(18) A quantum particle restricted to $-L \leq x \leq L$ has a wave function $\Psi(x)$ defined within that region. The integral

$$\int_{-L}^{+L} |\Psi(x)|^2 \, dx$$

will be equal to:

a. 2L

b. 0

[✓ c. 1]

d. 1/L

e. $\frac{1}{2}L^2$

Q13

(19) Two observers are each traveling at 0.49c toward each other. Observer A sends a pulse of light toward observer B and notices that the pulse leaves her vehicle at a speed of 1.00c. At what speed does observer B record the pulse moving?

a. 0.49c

b. 0.98c

[✓ c. 1.00c]

d. 1.49c

e. 1.98c

Q14

(20) A meter stick is moving along its long axis near the speed of light. A stationary observer will observe that the meter stick is:

a. longer than 1m

[✓ b. shorter than 1m]

c. exactly 1m in length

d. moving faster than the speed of light

e. at rest

Q15

(21) A clock is moving near the speed of light. A stationary observer will observe that the clock is:

[✓ a. running slow]
b. running fast
c. at rest
d. stopped
e. is lengthening in the direction of motion

Q16

(22) Two identical clocks are synchronized. One remains on Earth while the other goes in orbit around the Earth for a year as measured by the clock on Earth. After both clocks are again on Earth, which is true:

a. both clocks are still synchronized

b. the clock that made the trip is stopped

c. The clock that made the trip runs slower after the return

[✓ d.] the clock that made the trip does not have the same time as the clock that stayed on Earth

Q17

(23) At what fraction of c must a meter stick be moving to appear to be 75cm in length to a stationary observer?

a. 0.57c

b. 0.59c

c. 0.63

d. 0.65

~~e.~~ 0.66

$$L = \frac{L_0}{\gamma}$$

$$\gamma = \frac{L_0}{L} = \frac{1\cancel{m} \cdot 4}{3\cancel{m}}$$

$$\gamma = \frac{4}{3}$$

$$1 - \left(\frac{v}{c}\right)^2 = \frac{9}{16}$$

$$1 - \frac{9}{16} = \left(\frac{v}{c}\right)^2$$

$$\frac{v}{c} = \sqrt{\frac{7}{16}}$$

Q18

(24) The total energy of a body is three times its rest energy. How fast in m/s is the body traveling?

[✓ a. 2.83×10^8 m/s]
b. 3.5×10^8 m/s
c. 2.99×10^8 m/s
d. 1.51×10^8 m/s
e. 0.11×10^8 m/s

$E = \gamma mc^2$

$E = \gamma E_R$

$\gamma = 3$

$$\frac{v}{c} = \sqrt{1 - \frac{1}{9}}$$

$$v = 3 \times 10^8 \text{ m/s } \frac{\sqrt{8}}{3}$$

$$E - E_R = K = (\gamma - 1)mc^2$$

Q19

(25) An astronaut who is 40 years old goes on a journey at speeds close to the speed of light and returns to Earth after 20 years as measured by Earth time. How old in years would she be as measured by her time?

a. 20

b. 40

c. 60

[✓ d.] older than 40 but less than 60

e. older than 60

Q20

(26) Two fireworks explode, one after the other, at the same place on the 4th of July. An observer notices that the time interval between the two events was 5s. A second observer, flying high, passes the fireworks at a speed of 0.6c. He measures the time interval to be:

a. 8.33s

[✓ b.] 6.25s

c. 4.00s

d. 2.00s

e. 5.00s

$$T = \gamma \overset{\swarrow\ 5s}{T_0}$$

$$\gamma = \frac{1}{\sqrt{1-0.36}} = \frac{1}{0.8} = \frac{5}{4}$$

Q21

(27) The half-life of a muon is 2.2 μs when measured in a reference frame in which the muon is at rest. What is a half-life of a muon in a reference frame relative to which it moves at a speed $v = 0.8c$?

a. 8.13 μs

b. 2.75 μs

~~c.~~ 3.67 μs

d. 15.00 μs

e. 1.32 μs

$$t = \gamma t_0$$

$$t_0 = 2.2 \mu s$$

$$\gamma = \frac{1}{\sqrt{1-0.64}} = \frac{5}{3}$$

Q22

(28) A spaceship moving past the earth with a speed of 0.8c blasts it with pulsed laser photons every 10 seconds. When observers on Earth measure the time interval between flashes they find its value is:

$\gamma = \frac{5}{3}$

a. 13.4 s

[✓ b.] 16.7 s

c. 9.2 s

d. 8.0 s

e. 22.0 s

Q23

(29) An astronaut traveling with a speed of 0.9c holds a stick of 1m length in her hand. If she measures its length, she will obtain a value of:

~~a.~~ 1m

b. 0.9 m

c. 2.21 m

d. 0.42m

e. 0.81m

Q24

(30) Which of the following is a postulate of relativity theory?

a. space and time are invariant

b. mass and energy are equivalent

[✓ c.] the speed of light in a vacuum is the same for all observers regardless of the source velocity

d. energy is conserved in elastic collisions

e. gravity can bend light

Q25

(31) Which of the objects listed below has no rest mass?

a. an electron

b. a proton

[✓ c.] a photon

d. a cork floating on water

e. an astronaut in space

(32) A particle has a kinetic energy $K=5m_oc^2$. The total energy of the particle is:

a. m_oc^2

[✓ b.] $6m_oc^2$

c. $5.5m_oc^2$

d. $4m_oc^2$

e. unknown

$$E_T = K + E_R$$

$$5m_oc^2 + m_oc^2$$

Q26

(33) What is the ground state energy of a Bohr hydrogen atom?

a. + 11.2 eV

b. − 4.5 eV

c. + 4.5 eV

[✓ d.] − 13.6 eV

e. + 13.6 eV

(34) If Planck's constant is 6.63×10^{-34} J·s, what is the angular momentum of an electron in its 2nd Bohr orbit?

a. 1.05×10^{-34} kg m^2/s

[✓ b.] 2.11×10^{-34} kg m^2/s

c. 3.05×10^{-34} kg m^2/s

d. 2.05×10^{-34} kg m^2/s

e. 1.92×10^{-34} kg m^2/s

$\overset{L}{\boxed{m_e v_n r_n}} = n\hbar$

$r_n = n^2 a_0$

$n = 2$

$L = 2\hbar = $

$= \frac{h}{\pi}$

Q27

(35) An electron in a hydrogen atom makes a transition from the n=4 to n=3 energy state. The energy of the emitted photon is:

- a. 0.54 eV
- ~~b.~~ 0.66 eV
- c. 0.85 eV
- d. 1.51 eV
- e. 10.2 eV

$$E_n = \frac{1}{n^2}(-13.6eV)$$

$$E_4 - E_3 = 13.6eV\left(\frac{1}{9} - \frac{1}{16}\right)$$

(36) An electron in a H-atom makes a transition from the n=3 to the n=1 energy state. The wavelength of the emitted photon is:

- a. 1006 nm
- b. 209 nm
- c. 306 nm
- ~~d.~~ 103 nm
- e. 821 nm

$$\Delta E = 13.6eV\left(1 - \frac{1}{9}\right)$$

$$\Delta E = hf = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,eV \cdot nm}{12.1}$$

(37) Which statement is NOT a basic postulate of the Bohr model?

a. The angular momentum of an electron in an orbit is quantized.

b. An electron in an allowed orbit does not radiate.

c. Radiation is emitted when an electron jumps from a more energetic initial state to a less energetic final state.

d. The electron travels in a circular orbit about the proton under the influence of the electrostatic force of attraction.

[✓ e.] The electric force on the electron balances the gravitational force. Q29

(38) The ionization energy of a hydrogen atom is the energy required to reach:

[✓ a.] a state in which the electron and the proton are separated from each other

b. the first excited state of the hydrogen atom

c. the state in which the radius of the electron is 0.529×10^{-10} m

d. the state with energy of 6.63×10^{-34} J

e. either (a) or (c) above

Q30

(39) Destructive interference between waves from two sources occurs when the path difference is:

a. λ
b. 2λ
c. 3λ
d. 4λ
[✓ e. $\frac{1}{2}\lambda$]

(40) When a light ray in a less dense medium is reflected at the surface of a denser medium, the reflected ray has a phase difference of:

a. zero radians
b. $\pi/2$ radians
c. $\pi/3$ radians
d. $\pi/4$ radians
[✓ e. π radians] 180°

Q31

(41) A thin film of thickness t and index of refraction n is surrounded by air on both sides. Destructive interference will occur in light of wavelength λ in air upon reflection from the film when:

a. $\frac{nt}{2} = m\lambda$

b. $\frac{2t}{n} = m\lambda$

c. $\frac{2n}{t} = m\lambda$

d. $2mt = n\lambda$

[✓ e.] $2\underline{n}t = m\lambda$

$m = 1, 2, 3, \ldots$

Q32

(42) Light spreads out after it posses through a small aperture. This phenomenon is called:

a. interference

b. Arago bright spot

[✓ c. diffraction]

d. polarization

e. thin film interference

(43) The limiting angle of a circular aperture is:

a. $\sin\theta = \frac{\lambda a}{2}$

[✓ b. $\theta_m = \frac{1.22\lambda}{D}$]

c. $\theta = \frac{n_1}{n_2}$

d. $\cos\theta = n\lambda D$

e. $\lambda = D\sin\theta$

(44) A particle restricted to $-L \le x \le L$ has a wave function $\Psi(x)$ defined within that region. The average positon OR EXPECTATION value $\langle x \rangle$ can be calculated from

a. $\int_{-L}^{L} |\Psi(x)|^2 \, dx$

b. $\int_{-L}^{L} \Psi(x) \, dx$

c. $\int_{-L}^{L} x \, \Psi(x) \, dx$

[✓ d.] $\int_{-L}^{L} x \, |\Psi(x)|^2 \, dx$

$\Psi(x) = f(x) + i\,g(x)$

$\Psi^*(x) = f(x) - i\,g(x)$

$\Psi(x) \in \mathbb{R}$

$\Psi^*(x) = \Psi(x)$ $\omega = \frac{E}{}$

$\Psi(x,t) = A\sin(\;)e^{i\omega t}$

Q34

(45) Which of the following phenomena involves tunneling?

a. photoelectric effect

[✓ b. α-decay & scan tunneling microscope]

c. Compton effect & x-ray scattering

d. blackbody radiation

e. inverse photoelectric effect and interference

(46) If $F = -kx$, then $\frac{k}{m}$ is

a. A

b. ω

[✓ c. ω^2]

d. $A\omega$

e. $A\omega^2$

Q35

(47) The speed of a wave on a stretched string can be calculated from

a. $\sqrt{\frac{m}{k}}$

[✓ b.] $\sqrt{\frac{T}{\mu}}$

c. $\sqrt{\frac{T}{m}}$

d. $\sqrt{\frac{g}{L}}$

(48) A uniform cord has a mass of 1.2 kg and a length of 24 m. What is its mass per unit length?

a. 0.50 kg/m

[✓ b.] 0.05 kg/m

c. 5 kg/m

d. 5×10^{-3} kg/m

e. 0.01 kg/m

Q36

(49) When the driving force has the same frequency as a natural frequency of a system, the response is referred to as

a. the Doppler effect
b. damping
c. ultraviolet catastrophe
[✓ d.] resonance
e. traveling wave

(50) The angular frequency of a wave is doubled. By what factor has the period changed?

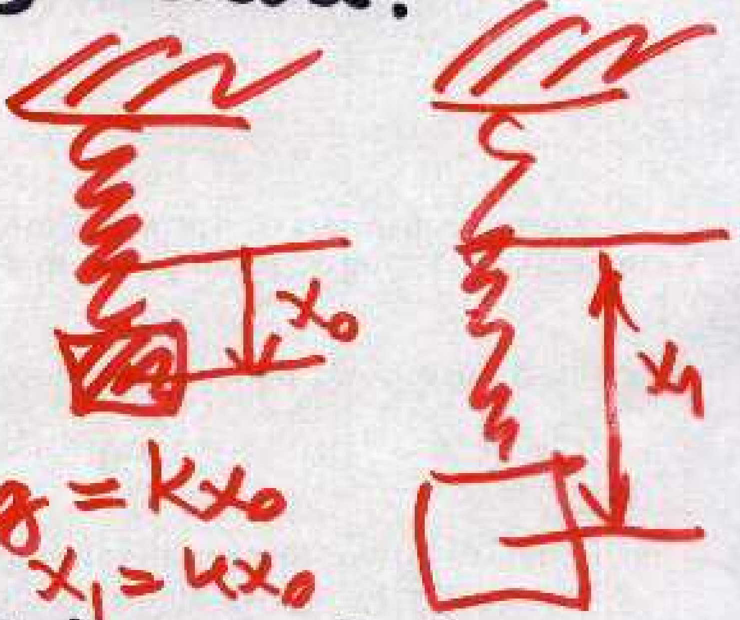
a. does not change
b. 1/4
[✓ c.] 1/2
d. 2
e. 4

$\omega \propto \frac{1}{T}$

Q37

(51) When the weight suspended from a spring is replaced by a weigth with 4-times as much mass, by what factor does the extension of the spring increase?

a. 1/3
b. 1/2
c. 1
d. 2
[✓ e. 4]



$mg = kx_0$
$x_1 = 4x_0$

(52) A simple pendulum is at its maximum displacement from equilibrium. Which of the following is also at its max?

a. speed
[✓ b. acceleration]
c. period
d. frequency
e. kinetic energy

Q38

(53) At every instant, the ratio E/B associated with an EM wave is

a. ε_0

b. μ_0

c. H

d. c

e. π

(54) The resonance frequency of an LC circuit is

a. $\frac{1}{2\pi\sqrt{LC}}$

b. $\sqrt{\frac{L}{C}}$

c. $\frac{1}{\sqrt{\varepsilon_0\mu_0}}$

d. $\frac{1}{2\pi\sqrt{L^2+C^2}}$

e. $\frac{2\pi}{\sqrt{LC}}$

(55) When a charged particle is accelerated, it radiates

a. waves that have particle's speed

b. longitudinal waves

c. electrons

d. mechanical waves

[✓ e.] EM waves

(56) $\mu_0^{-1}\vec{E}\times\vec{B}$ is called

a. Maxwell's third equation

b. the Lorentz force

c. Faraday's relation

[✓ d.] the Poynting vector

e. the coil energy density

Q40

(57) The light from the Sun takes 8.333 minutes to get to the Earth. How far is the Earth from the Sun?

$(c = 3 \times 10^8 \text{ m/s})$

a. 1.2×10^{10} m

b. 2.4×10^{12} m

[✓ c.] 1.5×10^{11} m

d. 1.1×10^{11} m

e. 2.2×10^{11} m

$$c = \frac{d}{t}$$

$$d = c \cdot t$$

Q41

(58) The half-life of a muon is 2.2μm when measured in a reference frame in which the muon is at rest. How fast is it moving [redacted] relative to an observer in an inertial reference frame who measures the value of the muon's half-life as 4.4μs?

~~a.~~ 0.866c

b. 0.75c

c. 0.97c

d. 0.72c

e. 0.50c

$\gamma = 2$

$$\frac{1}{\sqrt{1-(\frac{v}{c})^2}} = 2$$

$$1-(\frac{v}{c})^2 = \frac{1}{4}$$

$$(\frac{v}{c})^2 = \frac{3}{4}$$

$$\frac{v}{c} = \frac{1}{2}\cdot\sqrt{3}$$

Q42