Ch. 9: Relativity

→ our intuition for motion of objects based on everyday experience with speeds $v \ll c$, $c = 3 \times 10^8$ m/s (speed of light)

⇓

Newton's laws & Newtonian mechanics

→ Newtonian mechanics fails at speeds $v \lesssim c$, e.g. $v = 0.99c$ for accelerated electrons

classically: $K = \frac{1}{2} m_e v^2$,

so if we transfer to such e^- energy of $4 \cdot K$ ⇒

$$\left.\begin{aligned} v &= 0.99c \\ K &= \tfrac{1}{2} m_e v^2 \end{aligned}\right\} \Rightarrow \quad \begin{aligned} v &= 1.98c \\ &\uparrow \\ 4K &= \tfrac{1}{2} m_e (2v)^2 \end{aligned}$$

increase energy
from K to 4K

$\Downarrow$

NOT consistent with experiments

In 1905, Einstein published his special theory of relativity & resolved the theoretical problems in explaining the numerous experimental data

The Principle of Newtonian Relativity

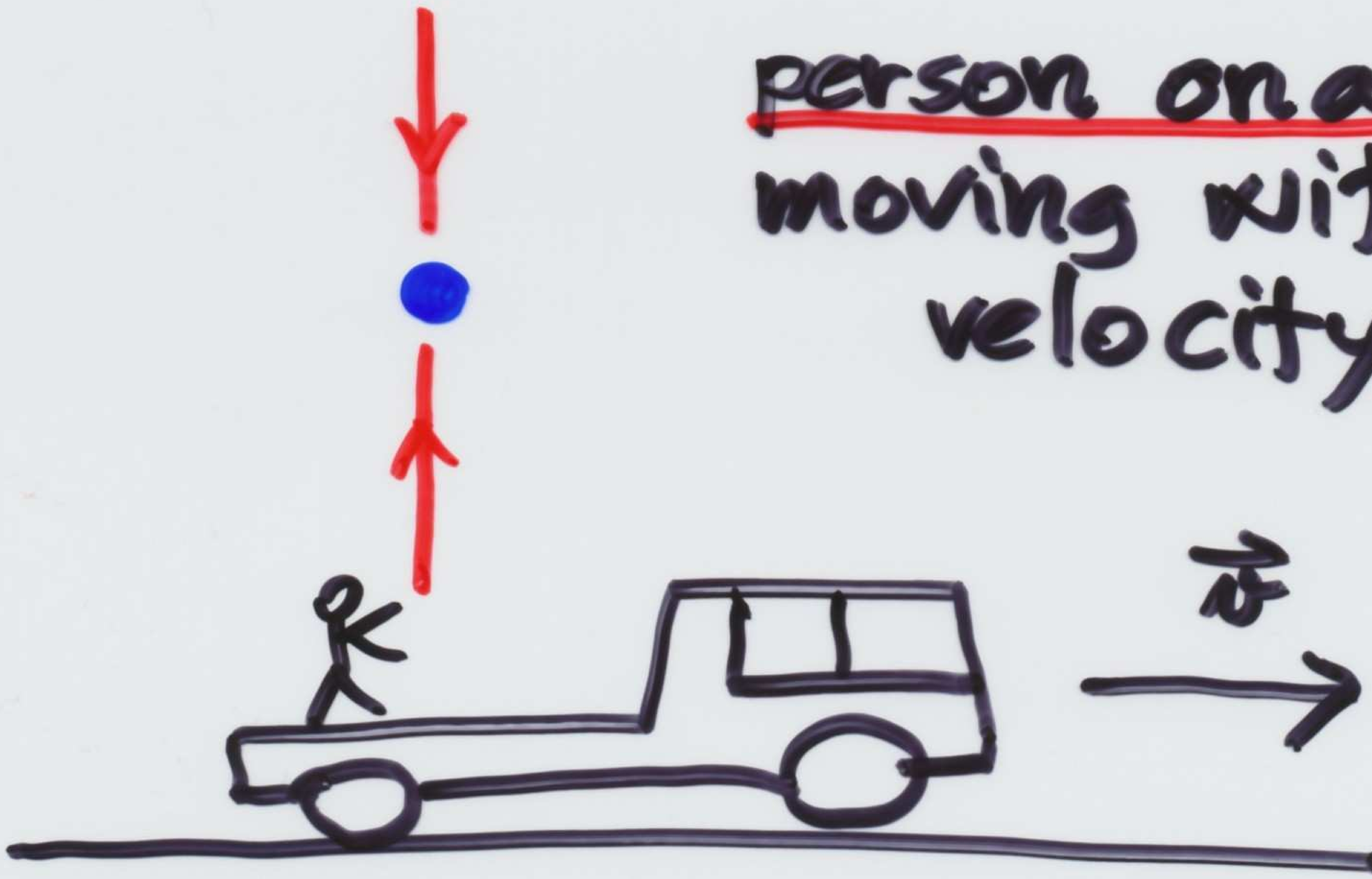
→ every physical event needs to be described in a specified frame of reference

→ an inertial frame ⇔ if no force is acting on the object, object is non-accelerated
(any frame moving with constant VELOCITY with respect to the inertial frame is also an inertial frame)

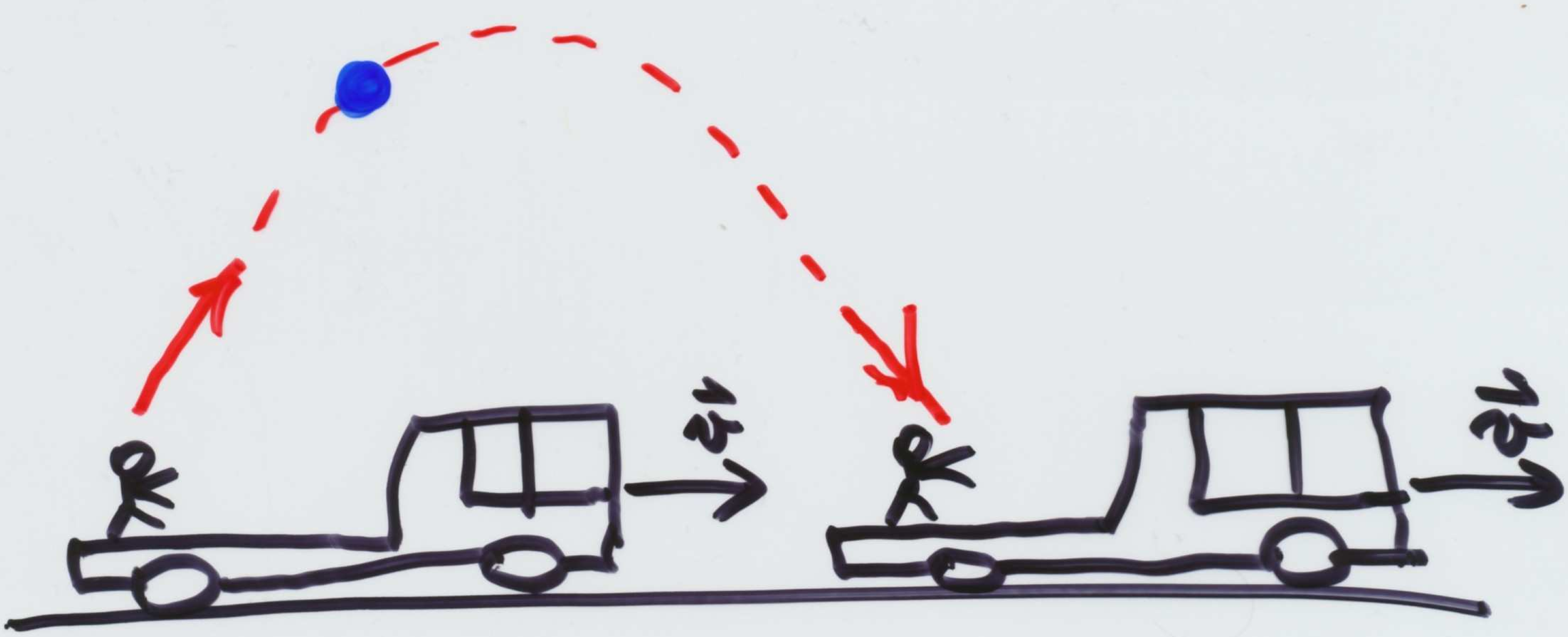
The laws of mechanics are the same in all inertial frames
(the principle of Newtonian relativity)

Example of a physical event observed from 2 DIFFERENT inertial frames:

(a) person on a truck moving with velocity $\vec{v}$



(b) observer on the road



beginning of event

end of event

→ (a) path of the ball is vertical

(b) path of the ball is a parabola

BUT

The same Newton's laws are valid in both frames (a) & (b)

→ same force

→ same energy conservation

→ same momentum conservation

How do we describe mathematically the events in different inertial frames?

⇓

Galilean transformation of coordinates

→ consider 2 inertial frames,

S & S'

S: observer on the road

S': observer on the truck $\longrightarrow \vec{v}$

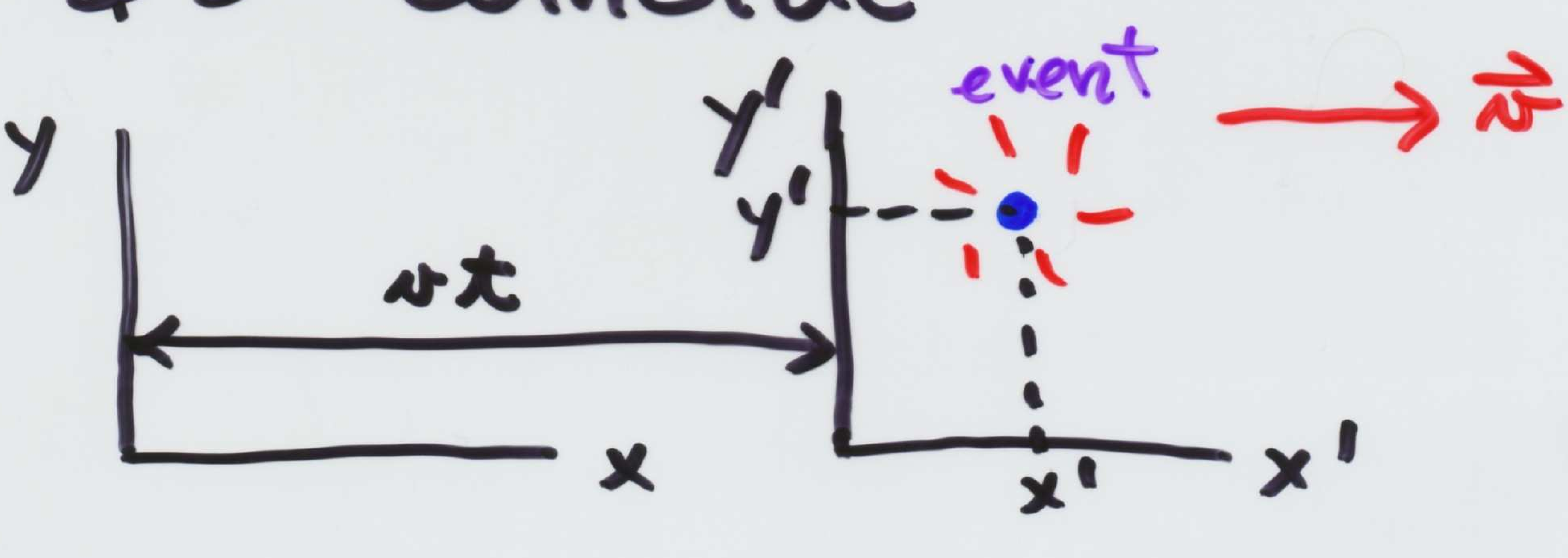
→ in S, an event is described by

(x, y, z, t)

→ in S', the same event is described by

(x', y', z', t')

→ it is typically assumed that at $t = t' = 0$, the frame S & S' coincide:



Galilean transformation of coordinates:

$$x' = x - vt \qquad x = x' + vt$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z' \quad \text{(inverse)}$$

$$t' = t \qquad t = t'$$

Galilean transformation of velocities:

u_x, u_y, u_z & u'_x, u'_y, u'_z

$$u_x \stackrel{\text{def.}}{=} \frac{dx}{dt} \quad \& \quad u'_x = \frac{dx'}{dt'}$$

but $dt' = dt$

& $dx' = dx - v\,dt$

$$\Rightarrow u'_x = u_x - v \qquad u_x = u'_x + v$$

$$u'_y = u_y \qquad u_y = u'_y \quad \text{(inverse)}$$

$$u'_z = u_z \qquad u_z = u'_z$$

The Michelson-Morley Experiment

Motivation: propagation of light

⇛ Is there a medium through which light propagates?

In 19th century, scientists believed that EM waves propagate through a medium = ether.

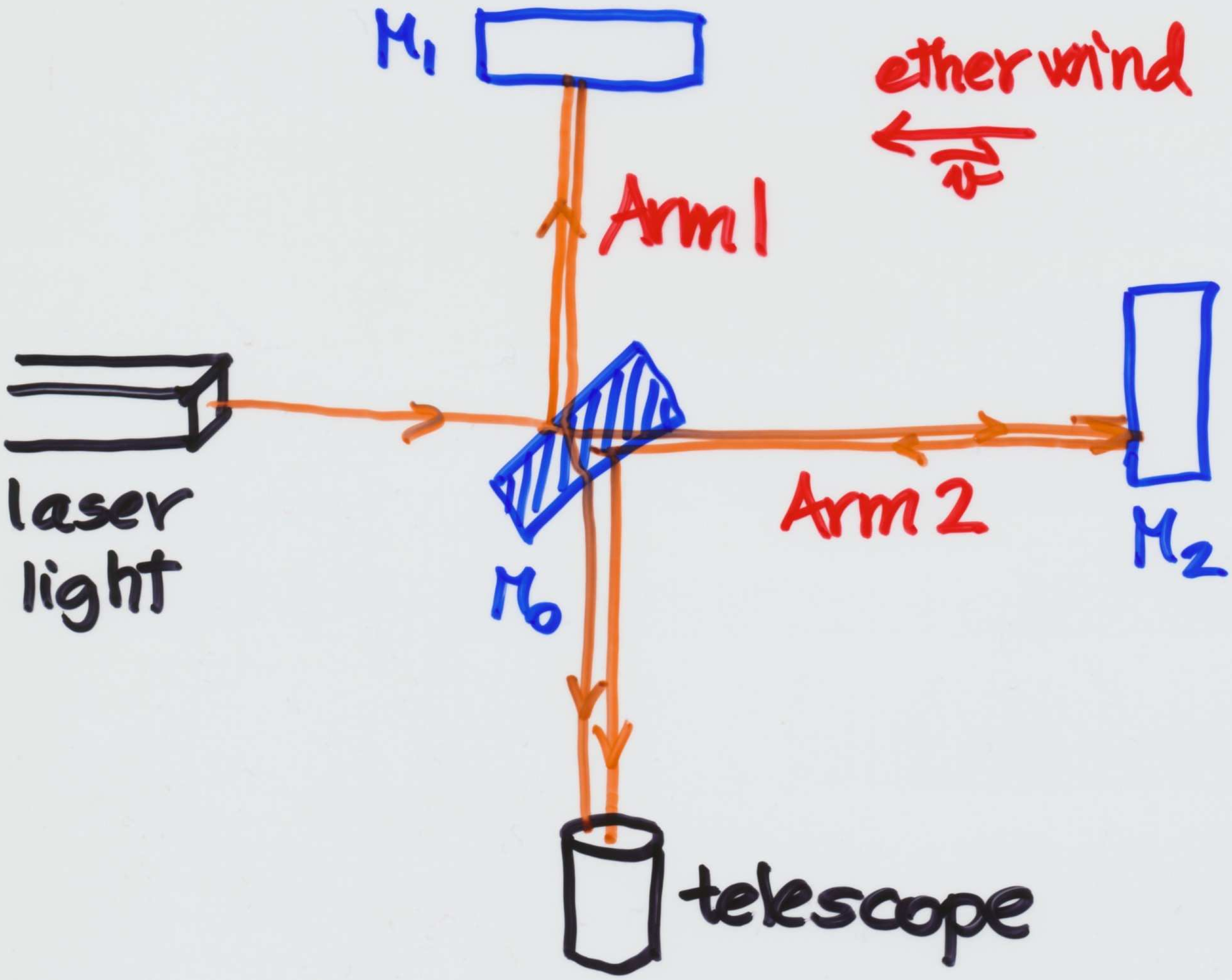
Ether would define an absolute frame of reference in which the speed of light is c (3×10^8 m/s).

A.A. Michelson & E.W. Morley

Objective:

determine the speed of the Earth through space with respect to ether

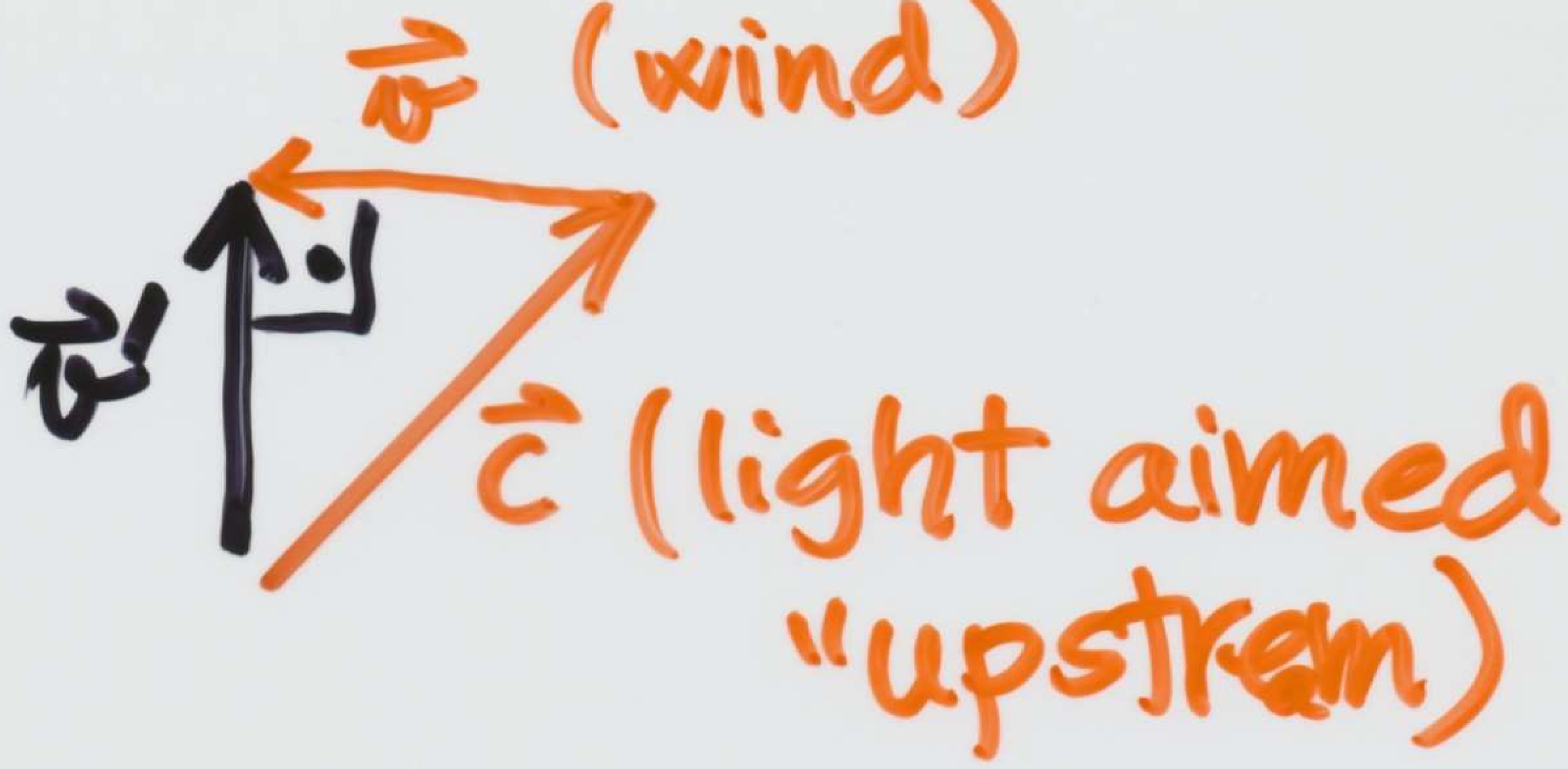
Michelson Interferometer



→ $-\vec{v}$ velocity of Earth with respect to "ether"

→ Arm 2 : light → $c-v$
light ← $c+v$

→ Arm 1:



→ velocity of light perpendicular to the ether wind:

$$v'^2 = c^2 - v^2 \Rightarrow v' = \sqrt{c^2 - v^2}$$

→ light in Arm 1 & Arm 2 travels the same path lenght but presumably at different speeds ⇒ should yield interference pattern @ a fixed λ of monochromatic light

⇓

NO! ⇒ no ether $\exists$

Einstein's principle of relativity

special theory of relativity is based upon two postulates:

1. the principle of relativity
   (all laws of physics the same in all inertial reference frames)

2. the speed of light in vacuum has the same value in all inertial frames (there is NO ether for EM propagation)

1. includes all laws of mechanics, electricity & magnetism (Maxwell's equations), optics, thermodynamics etc.

   ⇒ there is no preferred or absolute inertial reference frame

Consequences of Special Relativity

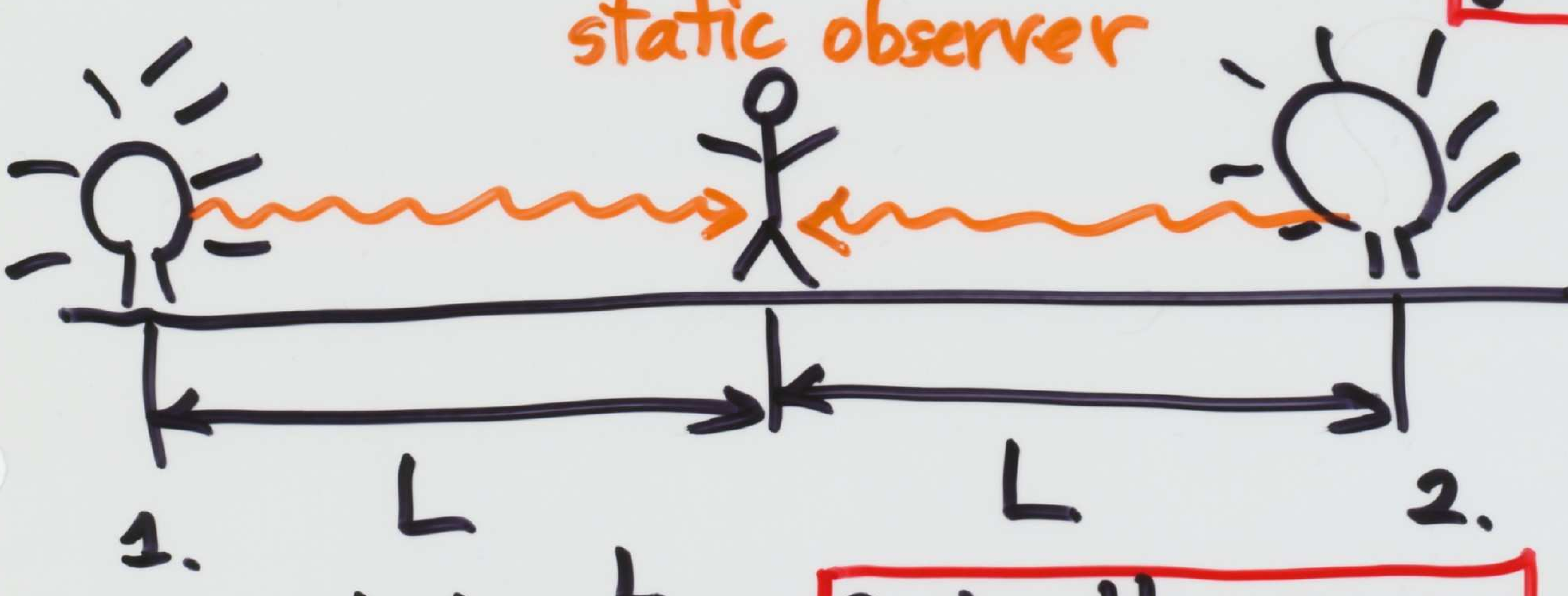
(A) Time measurement depends on the reference frame in which the measurement is made.

→ two events which are simultaneous in one frame may NOT be simultaneous in another inertial frame

Example:

(a) at $t=0$, 1. & 2. send a light signal

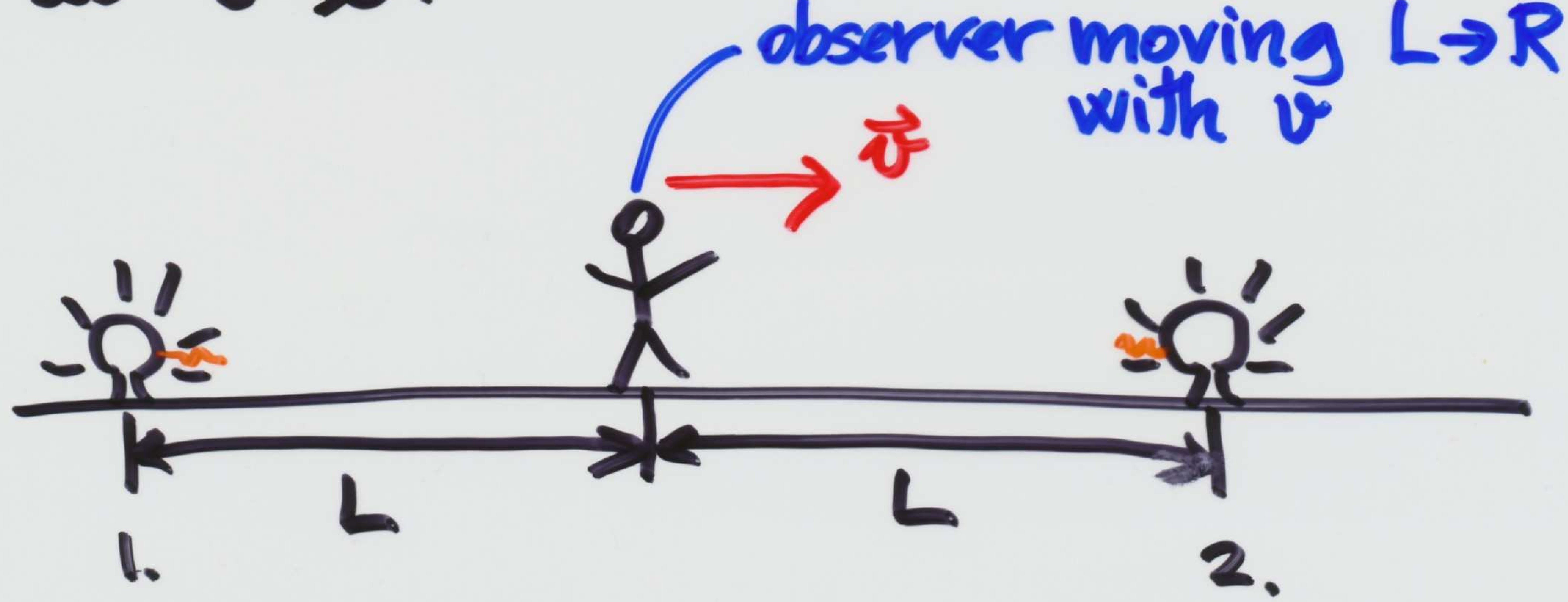
$\boxed{\text{frame S}}$



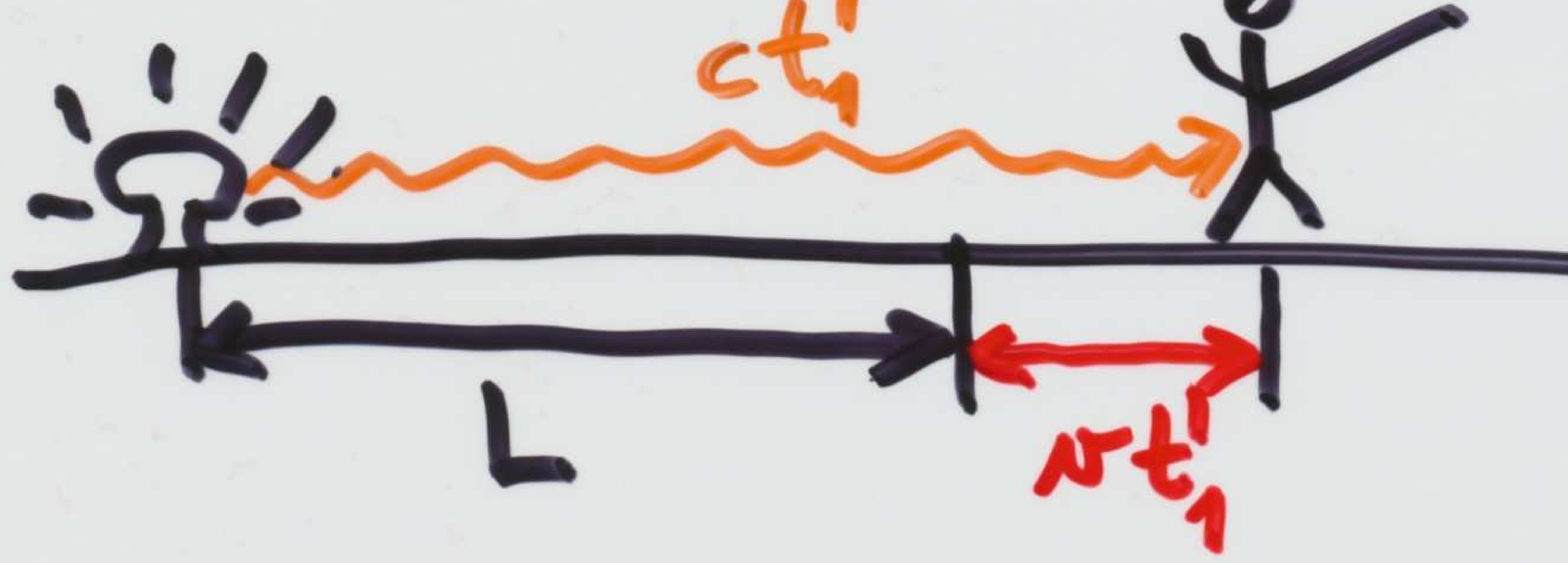
at $t_{1,2} = \frac{L}{c} \Rightarrow \boxed{\text{2 simultaneous s.}}$

(b) frame S' is moving with the speed v from left to right:

at $t' = 0$:



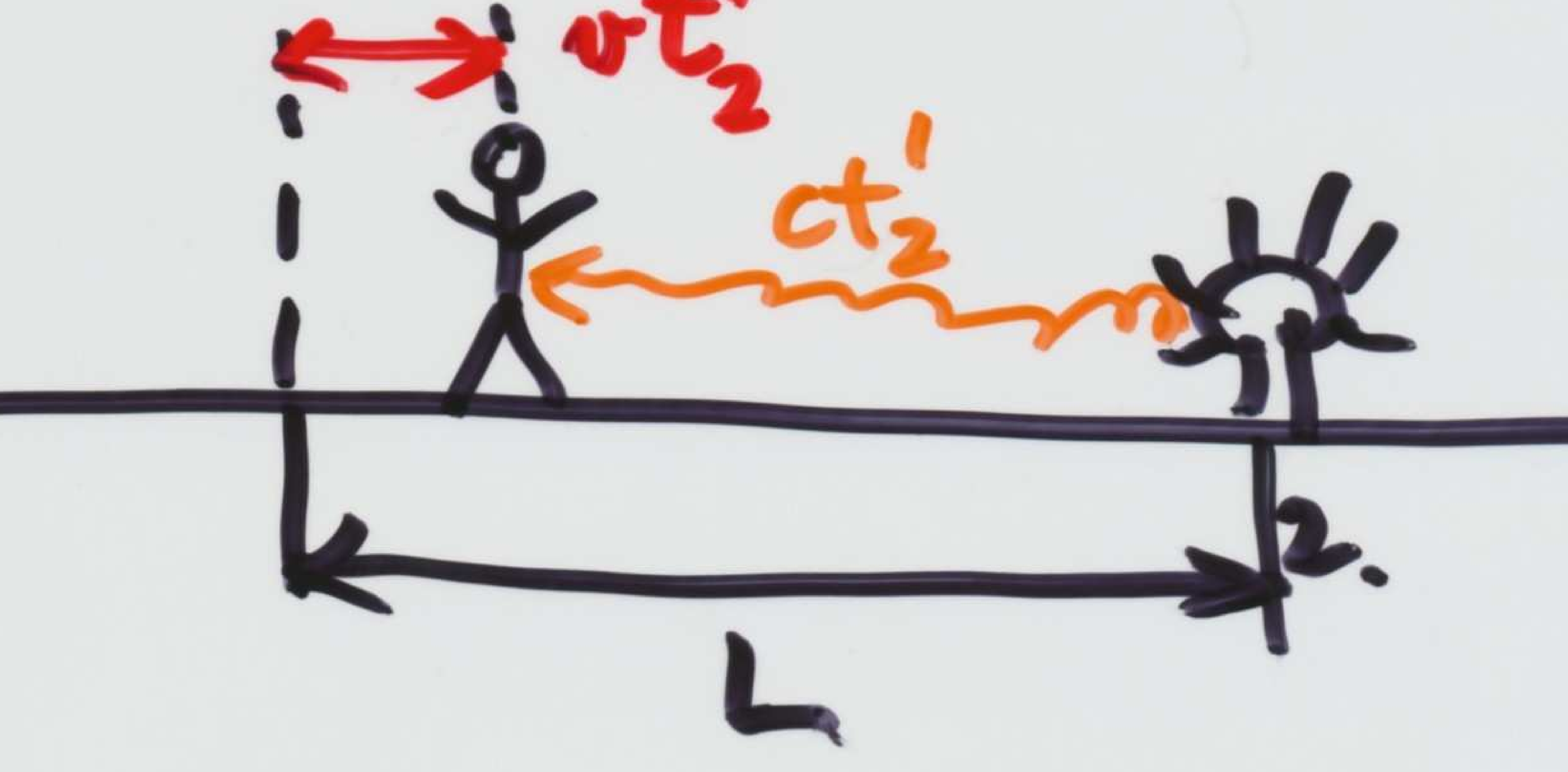
at t_1' signal from 1. hits the observer:



$$ct_1' = L + vt_1'$$
$$\Rightarrow t_1' = \frac{L}{c-v}$$

at t_2' signal from 2. hits the observer:

$$ct_2' = L - vt_2'$$
$$\Rightarrow t_2' = \frac{L}{c+v}$$



$$\boxed{t_2' < t_1'}$$

Signal from the light source 2. hits the moving observer FIRST, then the light from the source 1. hits the observer.

$\Downarrow$

The two events that are simultaneous in the frame S are NOT simultaneous in the frame S'.

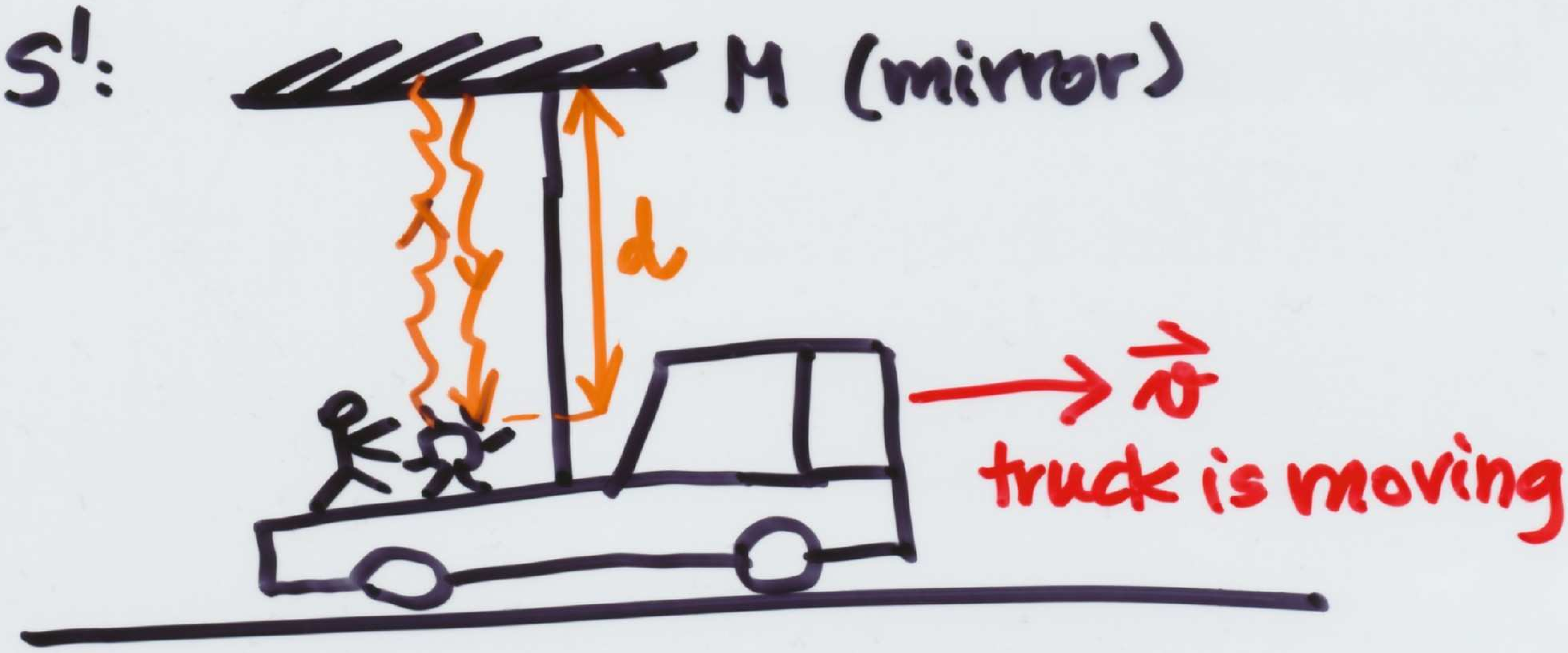
However: when $v \ll c$, the effect is very small. At $v \lesssim c$, the effect of the time relativity becomes practically important.

Simultaneity of events not absolute but depends on the state of motion of the observer.

Time Dilation

Consider an observer on a truck which moves L → R (frame S') with v, send a light signal up onto a mirror & then receives it back:

(a) S':

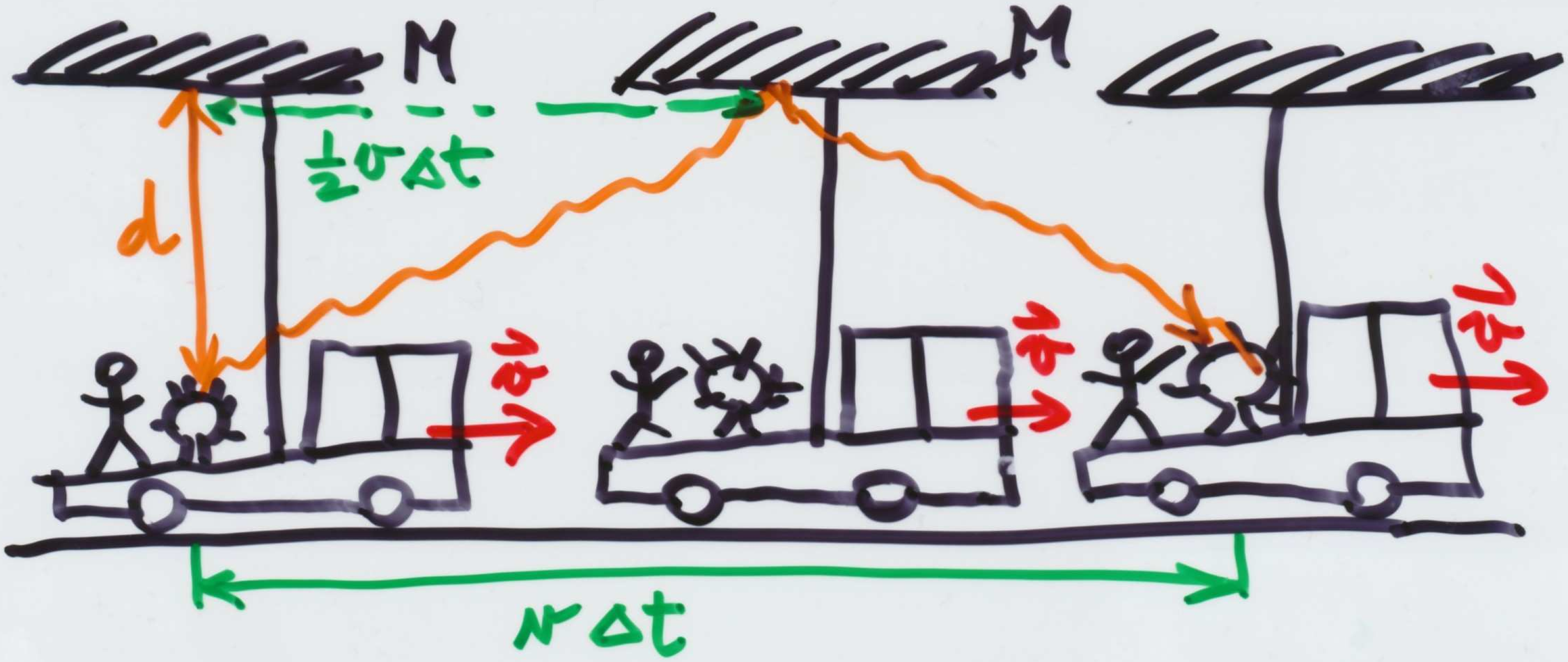


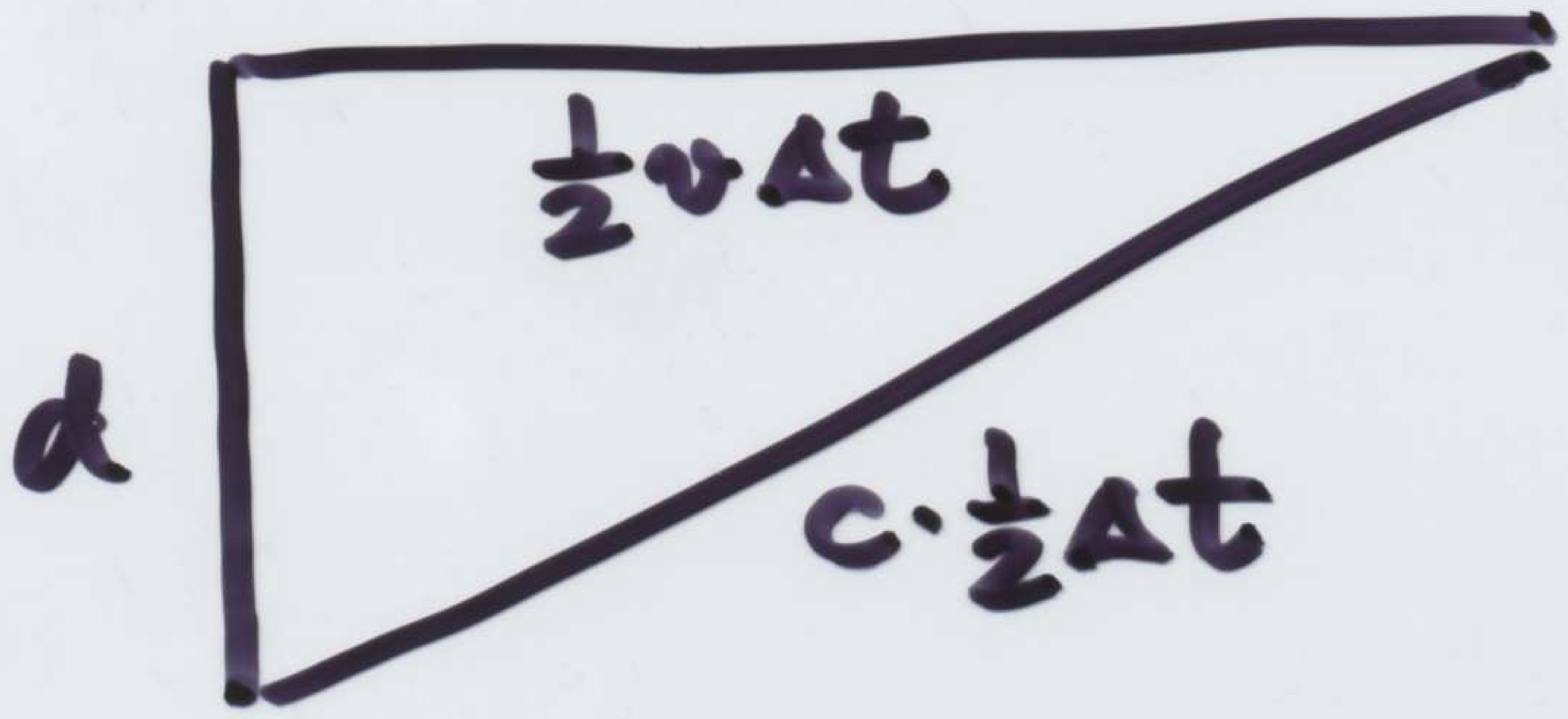
time interval $= \frac{2d}{c}$ c... speed of light

$$\boxed{\Delta t_p = \frac{2d}{c}}$$

Δt_p is measured in the frame S' where the two events occur at the same spatial position!

(b) S: observer standing on the road watching the observer S' on the truck:





$$\left(\tfrac{1}{2}c\Delta t\right)^2 = d^2 + \left(\tfrac{1}{2}v\Delta t\right)^2 \quad \Big| \cdot 4$$

$$(c^2 - v^2)\,\Delta t^2 = 4\,d^2$$

$$\Rightarrow \Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \underbrace{\frac{2d}{c}}_{=\Delta t_p} \cdot \underbrace{\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}}_{\gamma}$$

$$\boxed{\Delta t = \Delta t_p \cdot \gamma} \qquad \gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

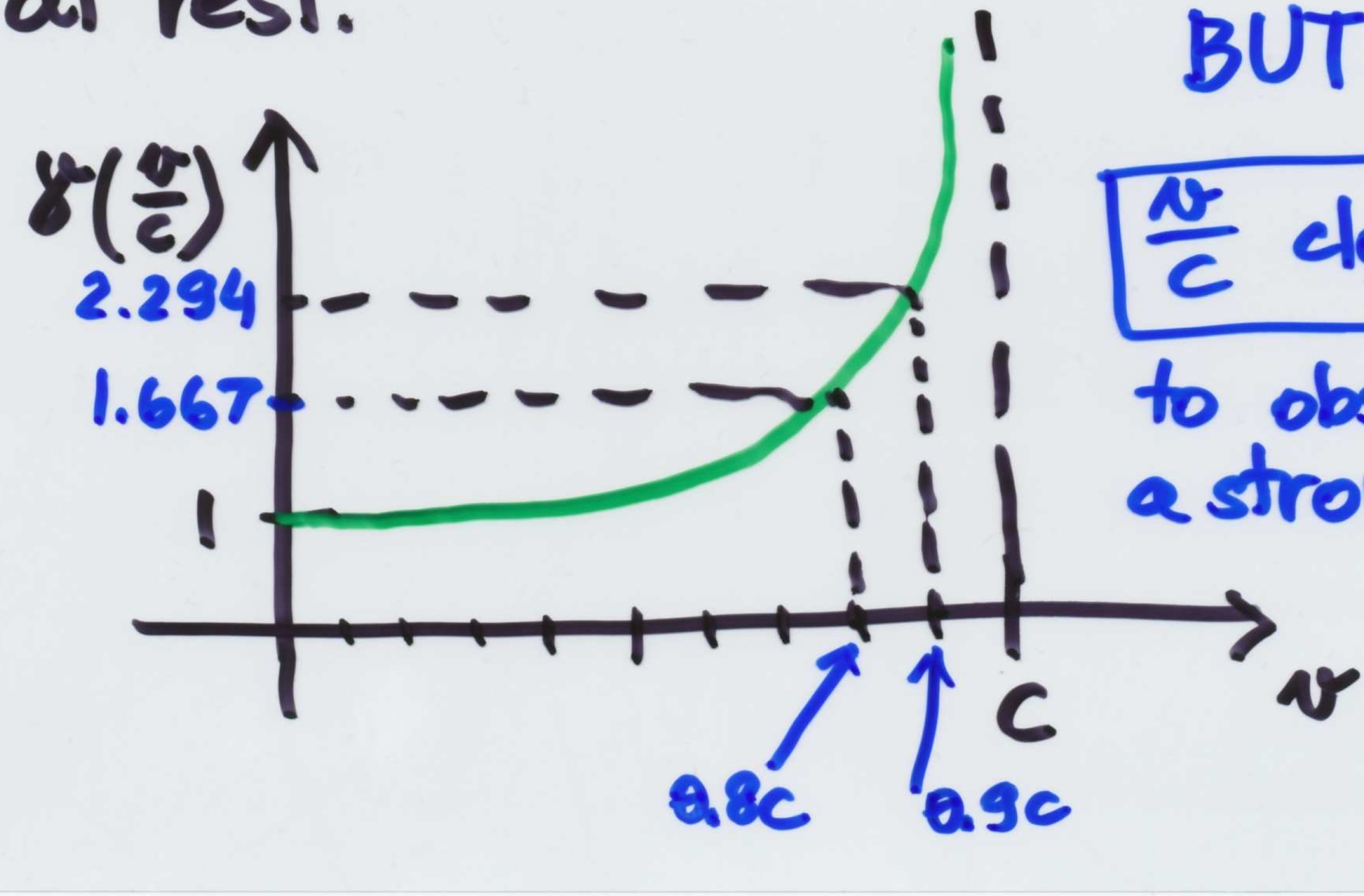
The time interval Δt measured in S is <u>longer</u> than the proper time Δt_p measured in the proper frame.

→ TIME DILATION

Proper frame $\stackrel{\text{def.}}{=}$ the frame in which the two events are measured at the same spatial coordinate.

OR

A moving clock is measured to run more slowly than the clock at rest.



BUT

$\boxed{\frac{v}{c} \text{ close to } 1}$

to observe a strong effect

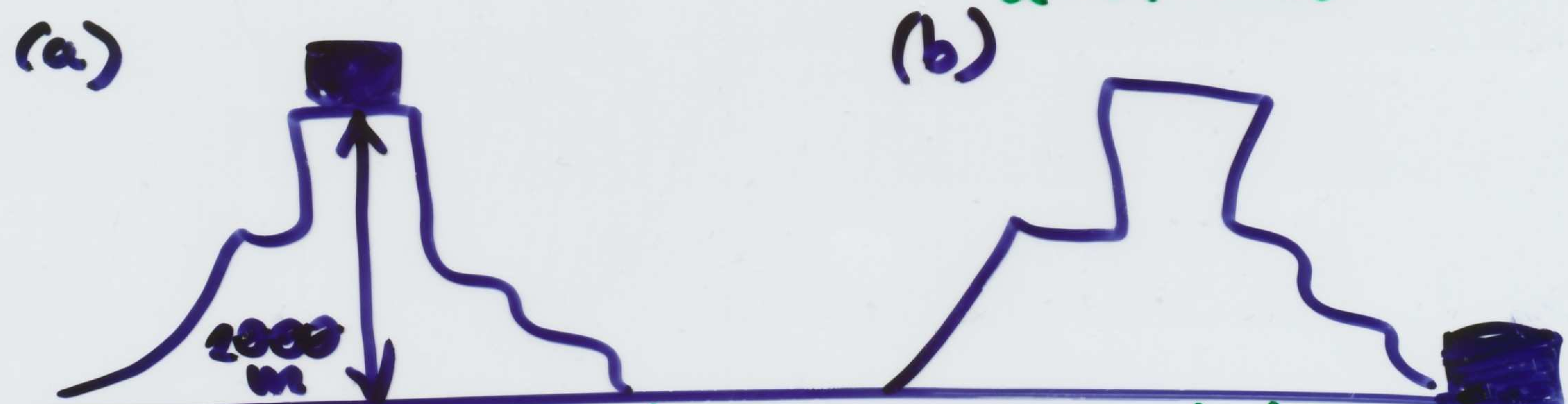
Experimental Verification of Time Dilation = Muon Decay

→ muons... unstable elementary particles with the charge $-e_0$ and a mass: $m_\mu = 207\, m_e$

(produced by INT between cosmic radiation & atoms in the high atmosphere)

→ lifetime of muons: $\Delta t_p = 2.2\,\mu s$
(Q: Identify the proper frame for muon decay!)

Experiment: count muons at different altitudes



muon detector at 2000m

muon det. at sea level

Results:

→ muon detector at 2000m detected ~1000 muons during a time interval t_0

→ muon detector at the sea level detected ~542 muons during t_0

Classical explanation:

$v = 0.98\,c$ (speed of muons)

$h = 2000\,m$

$\Delta t = \frac{h}{v} = 6.8 \times 10^{-6}\,s = 6.8\,\mu s$

$\tau = 2.2\,\mu s$ (lifetime of muons)

$$N = N_0 e^{-\Delta t/\tau} \Rightarrow N = 1000 \cdot e^{-\frac{6.8}{2.2}}$$

$$N \doteq 45$$

→ resalt ~10-times too low

Relativistic explanation:

→ time dilation

→ $\tau = 2.2\,\mu s$ is true only in the proper inertial reference frame which moves with muon with $v = 0.98c$

→ in the reference frame of the observer on Earth

$\tau' = 2.2\,\mu s \cdot \gamma$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \doteq 5 \qquad v = 0.98c$$

$\Rightarrow \tau' = 11\,\mu s$

$$N = N_0 e^{-\Delta t/\tau'} = 1000 \cdot e^{-\frac{6.8}{11}}$$

$\approx \underline{\underline{539}}$ correct order of magnitude!

Quiz: A crew on a spaceship watches a movie, 2hs long. The spaceship is moving with a high speed $v = 0.8c$ away from the Earth. An observer on the Earth watches the movie on the spaceship through a strong telescope. How long will the movie be for the Earth-bound observer?

(a) shorter than 2hs

$2hs/\gamma = 1.20\ hs$?

~~(b)~~ longer than 2hs

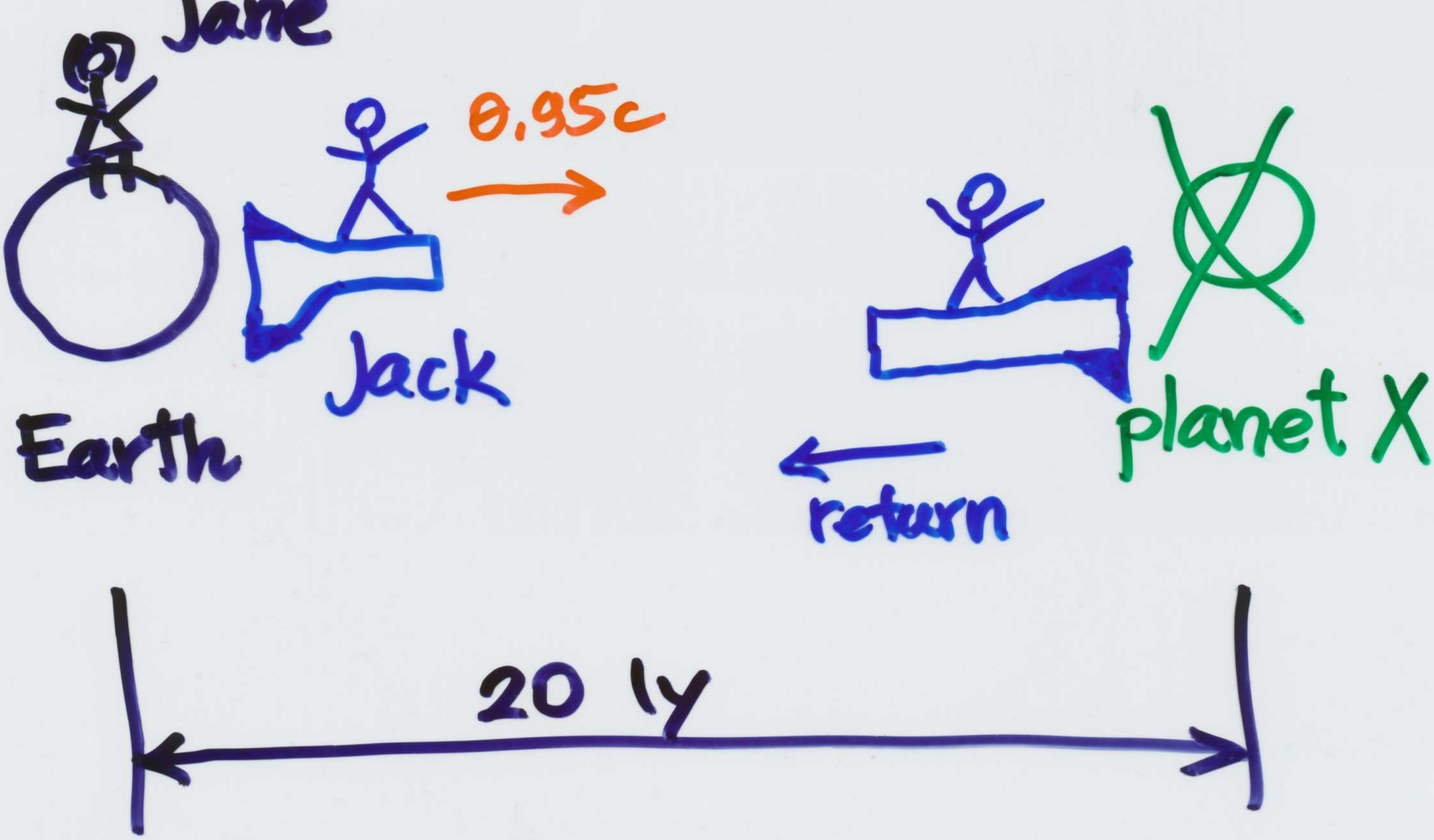
$2hs \times \gamma = 3.33\ hs$?

(c) equal to 2hs

$\frac{v}{c} = 0.8 \Rightarrow \gamma = 1.667$

The Twin Paradox

(a consequence of time dilation)



$$\underline{1\,ly} = 3 \times 10^{8}\,m/s \cdot 1\,y =$$

$$= 9.46 \times 10^{15}\,m \quad \text{(distance)}$$

→ Jack is away from Earth:

$$T_0 = \frac{2 \cdot 20\,ly}{0.95c} = \frac{40\,\cancel{c} \cdot y}{0.95\,\cancel{c}} =$$

$$\doteq \underline{\underline{42\,ys}} \quad \text{(Jane's frame)}$$

→ Jane has aged 42 ys at the time of Jack's return:

Jane's age: $(20+42)\,ys = \underline{\underline{62\,ys}}$

→ Jack's frame: proper frame
⇒ proper time T_p

$$T_p = \frac{T_0}{\gamma} \underset{\uparrow}{=} \frac{42}{3.2}\,ys = \underline{\underline{13\,ys}}$$

$\gamma\left(\frac{v}{c}=0.95\right) \doteq 3.2$

Jack's age: $(20+13)\,ys = \underline{\underline{33\,ys}}$

→ PARADOX?

From Jack's point of view, Jane was travelling with $v = 0.95c$ away from him, so Jane's age should also be 33 ys

→ RESOLUTION

Jane & Jack's reference frames are not equivalent: Jack's spaceship needs to first accelerate to $v = 0.95c$ & on return decelerate & accelerate in the opposite direction

⇓

no paradox

Jane's age : 62 ys

Jack's age : 33 ys

Quiz: Suppose you are an astronaut on a spaceship travelling in space with $v = 0.95c$. How would you prefer to be paid:

~~(a)~~ Earth-based clock OR (b) ship's c.

Length Contraction

proper length = spatial distance between two points measured by someone at rest relative to these two points

Any observer moving with respect to these two points will measure A SHORTER LENGTH ⇒ contraction of the length with respect to the proper length

Example:

1. event : Jack leaves the Earth in the spaceship $v = 0.95c$

2. event : Jack reaches planet X
in 13/2 ys = 6.5 ys
in Jack's frame

(A) Observer on Earth : Jane

Earth — planet X ⇒ proper distance L_p

$$L_p = T_0 \cdot v = 20 \text{ Lys}$$

T_0: 42ys

(B) Observer on the spaceship : Jack

T_p ... proper time : 6.5ys

v .... the same : 0.95c

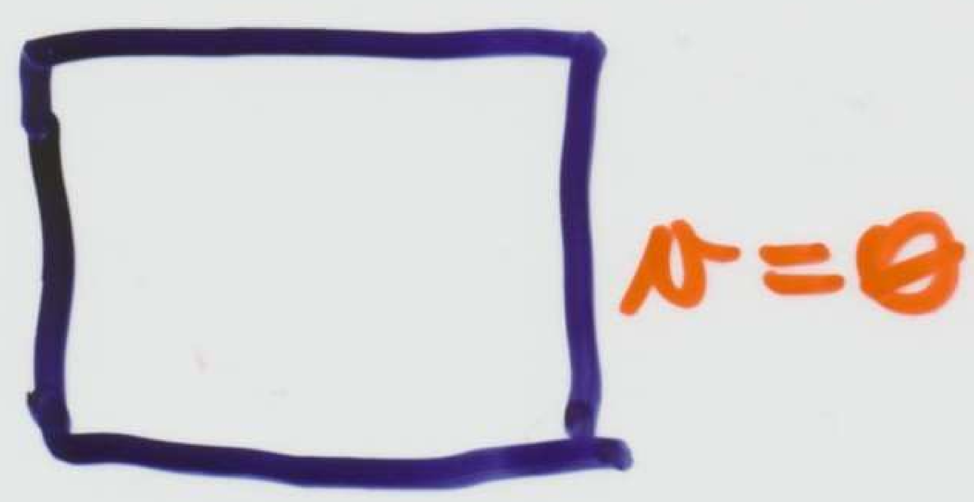
⇓

$$L = v T_p = v \frac{T_0}{\gamma} = \frac{L_p}{\gamma} = \underline{\underline{6.25 \text{ ly}}}$$

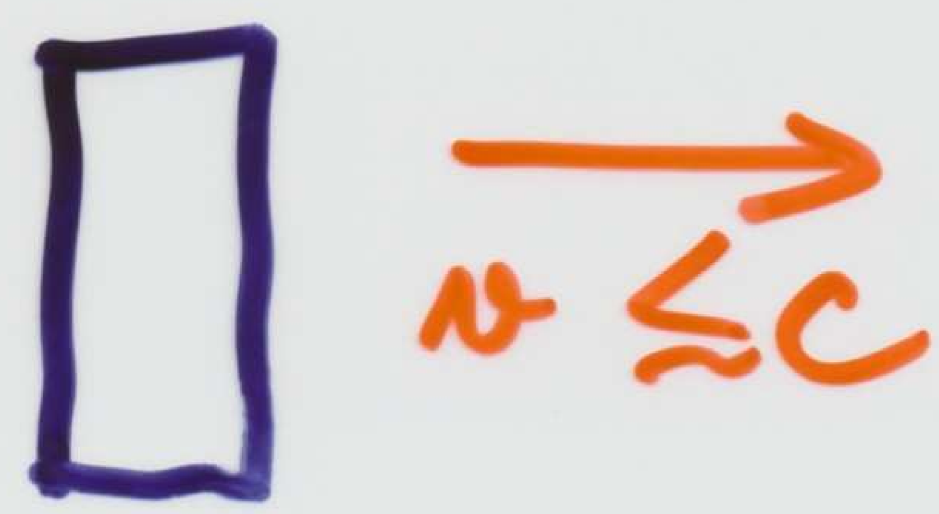
$$\Rightarrow L = \frac{L_p}{\gamma}$$ Length contraction

Length contraction only occurs in the direction of motion

$\Rightarrow$ object deformation



square at rest



Quiz: You are taking a trip to another star with a spaceship ($v = 0.99c$). Do you need to pack up clothes in a smaller size due to length contraction?

(a) Yes

(b) No

at rest

$v \lesssim c$

The Lorentz Transformations

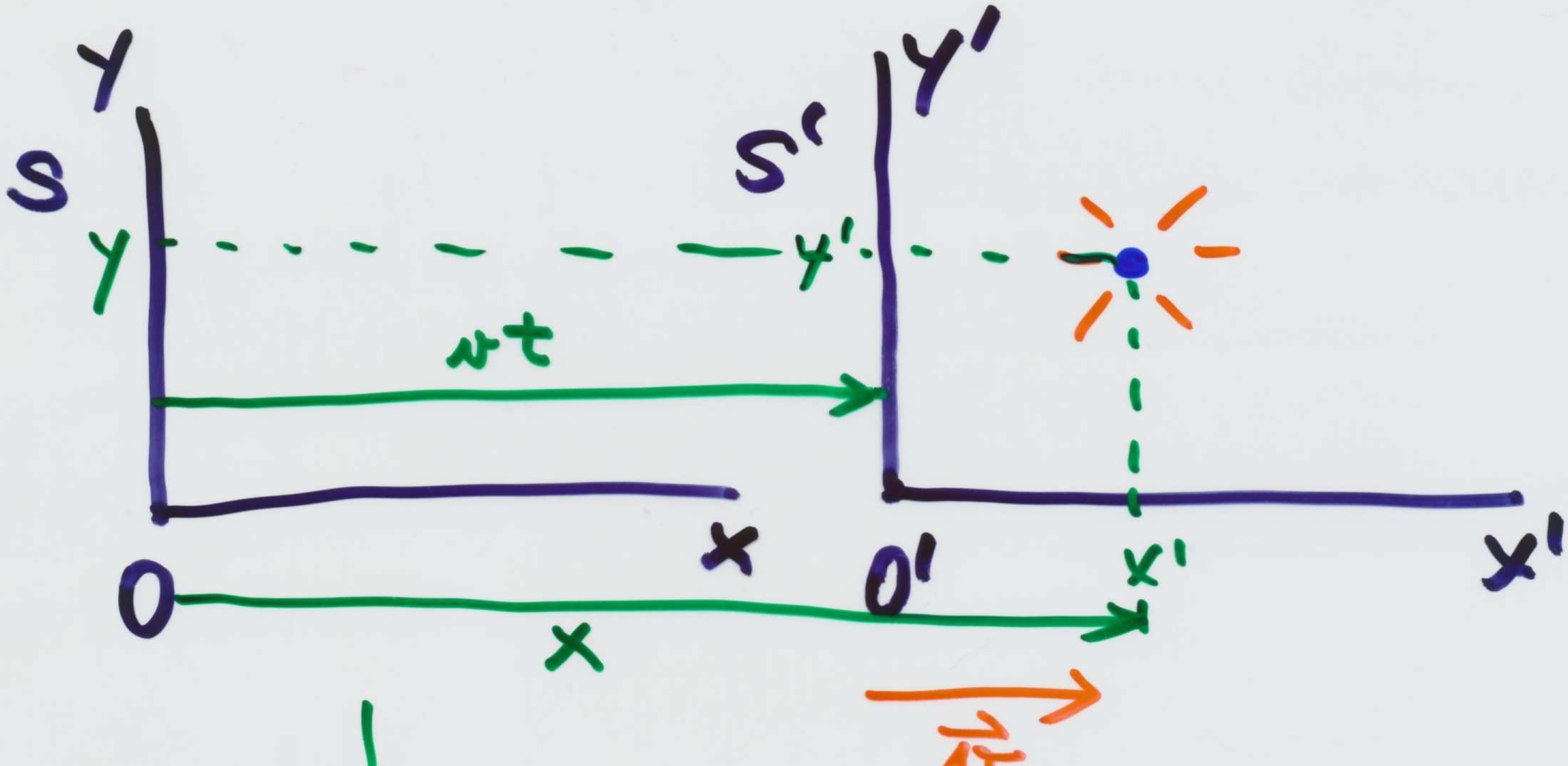
→ to replace Galilean transformations when v close to c

$S \rightarrow S'$

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$S' \rightarrow S$ (inverse)

$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' + \frac{vx}{c^2}\right)$$



$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

→ one of essential differences

space-time coordinates

$(t \neq t')$

(x, y, z, t) in S

(x', y', z', t') in S'

→ when $\frac{v}{c} \ll 1 \Rightarrow$

$\left.\gamma \to 1 \quad \frac{v^2}{c^2} \to 0 \right\} \Rightarrow$

Galilean transformations

$x' = x - vt$ $\qquad$ $x = x' + vt$

$y' = y$ $\qquad$ $y = y'$

$z' = z$ $\qquad$ $z = z'$

$t' = t$ $\qquad$ $t = t'$