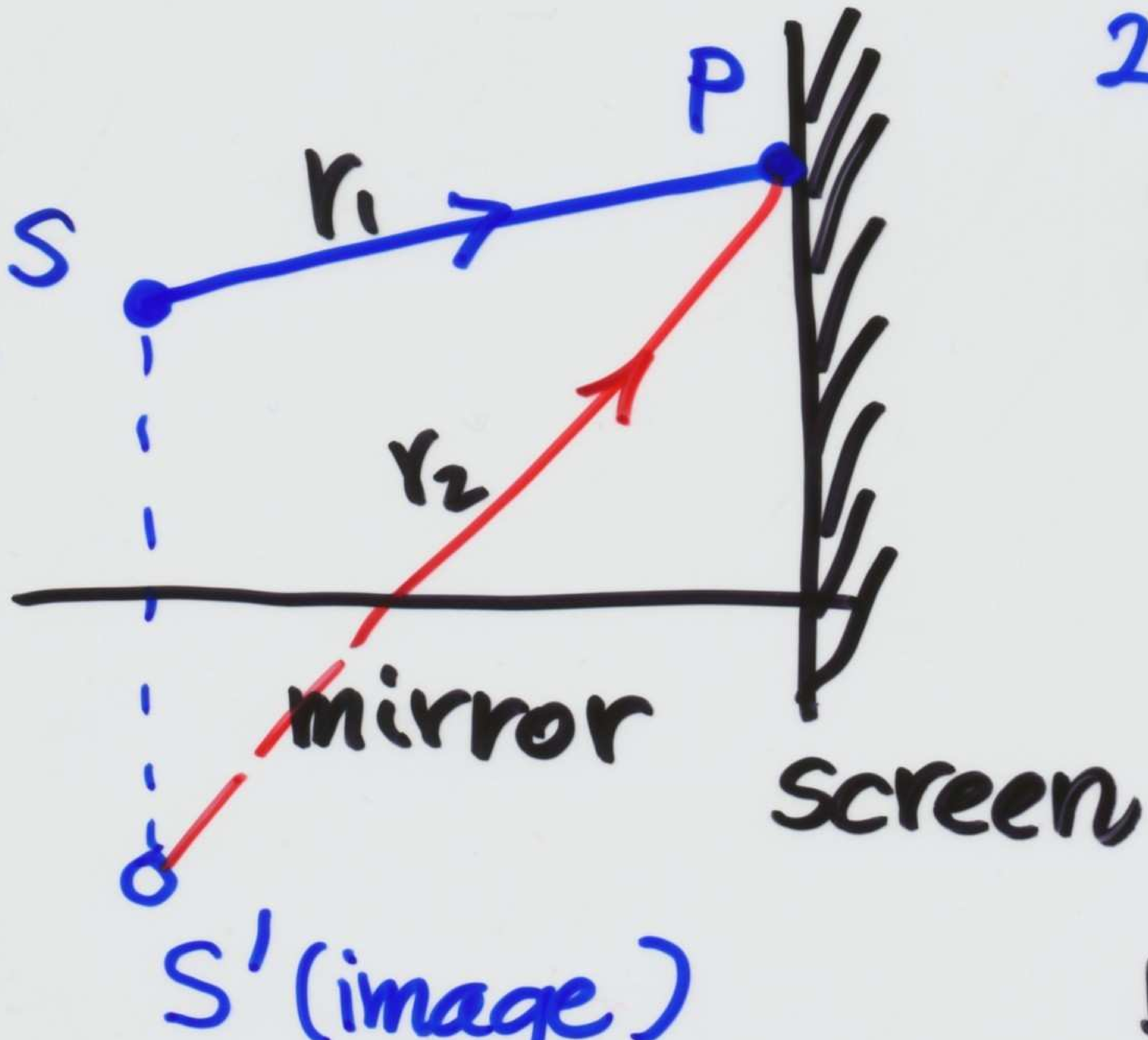
Phase change due to reflection

Lloyd's mirror



2 paths SP & S'P

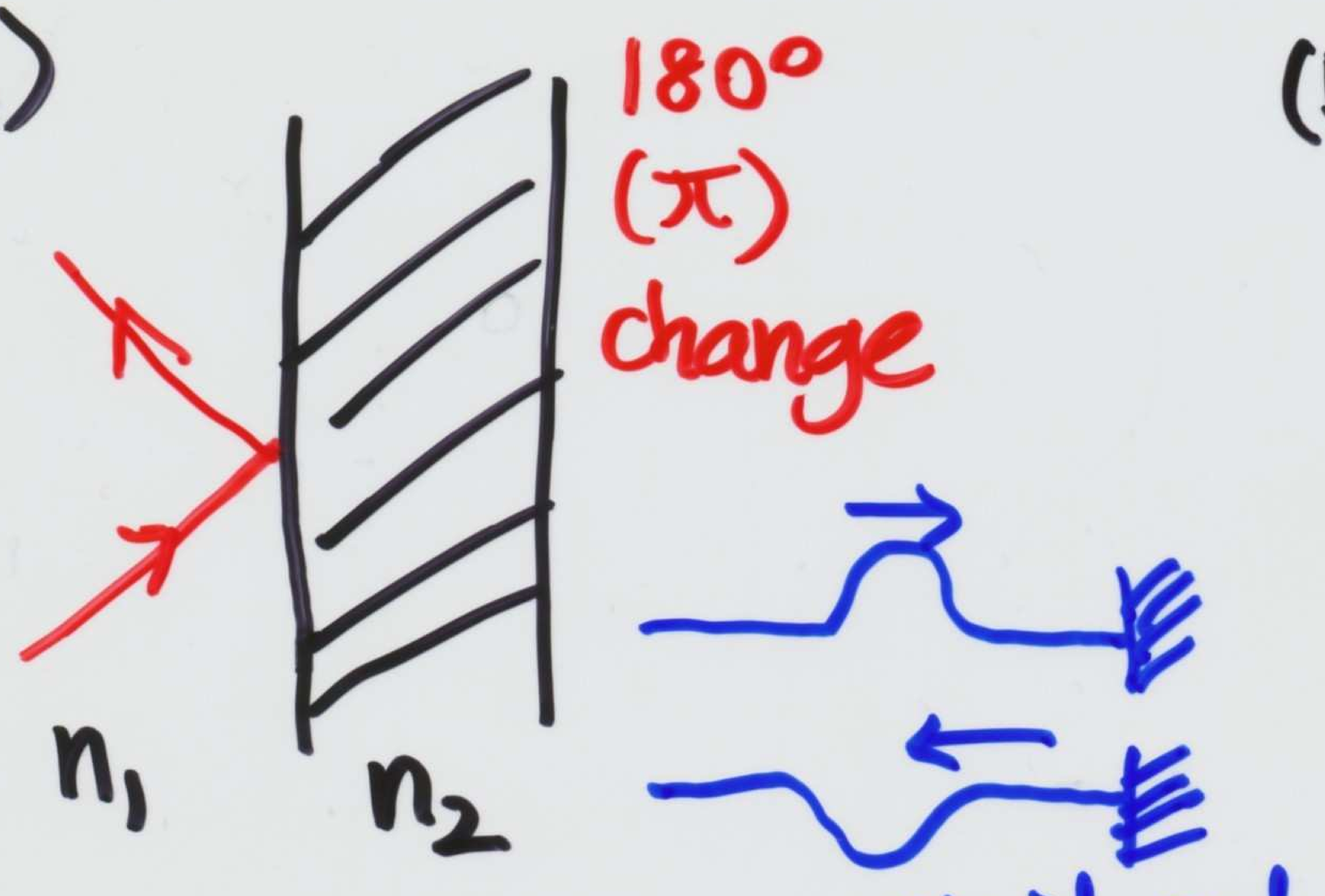
$\delta = r_2 - r_1 \ni$ interference

BUT

the positions of bright & dark fringes are REVERSED

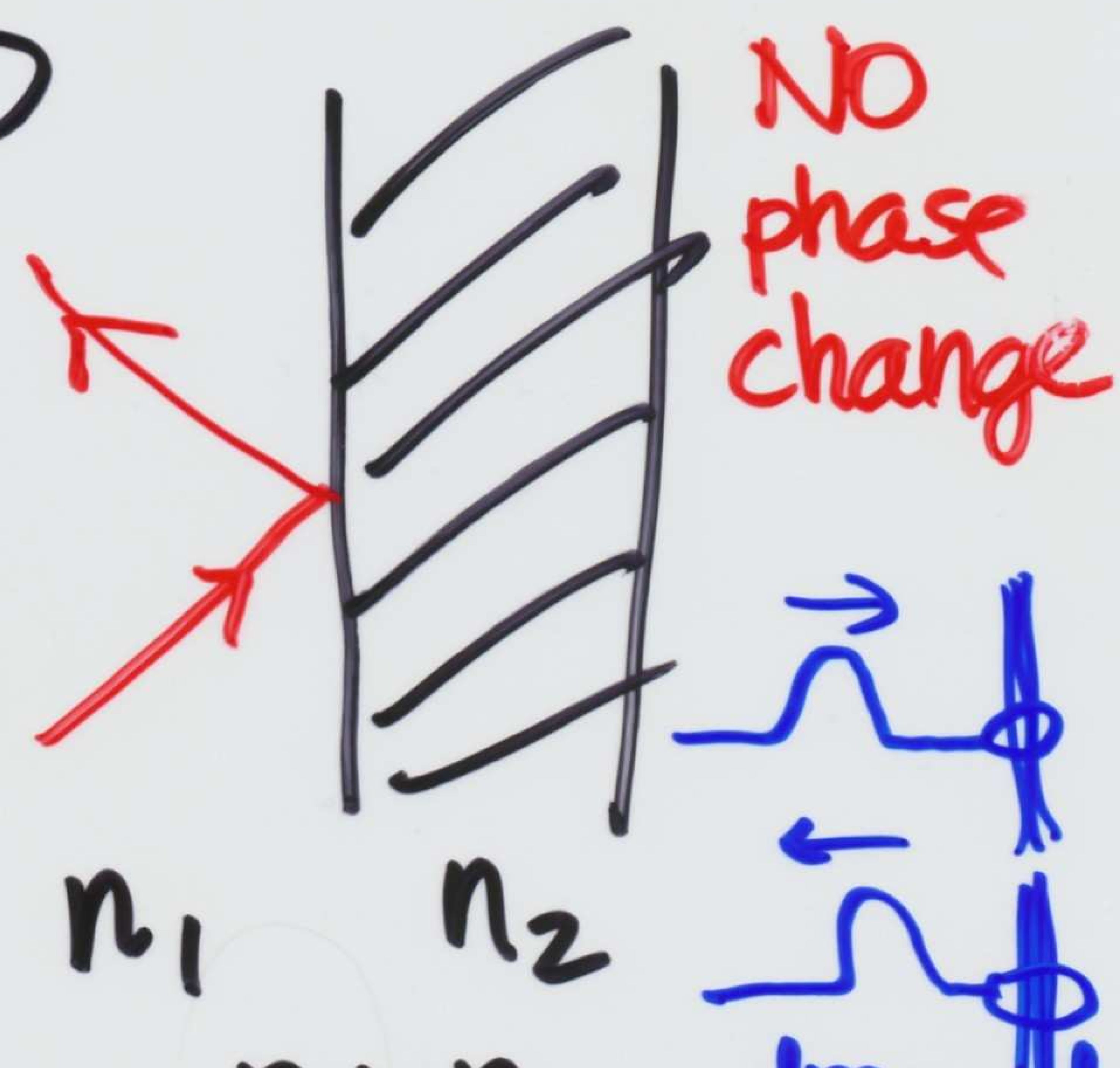
because coherent sources S & S' differ in phase by 180° (π)

(A)



$n_1 < n_2$

(B)



$n_1 > n_2$

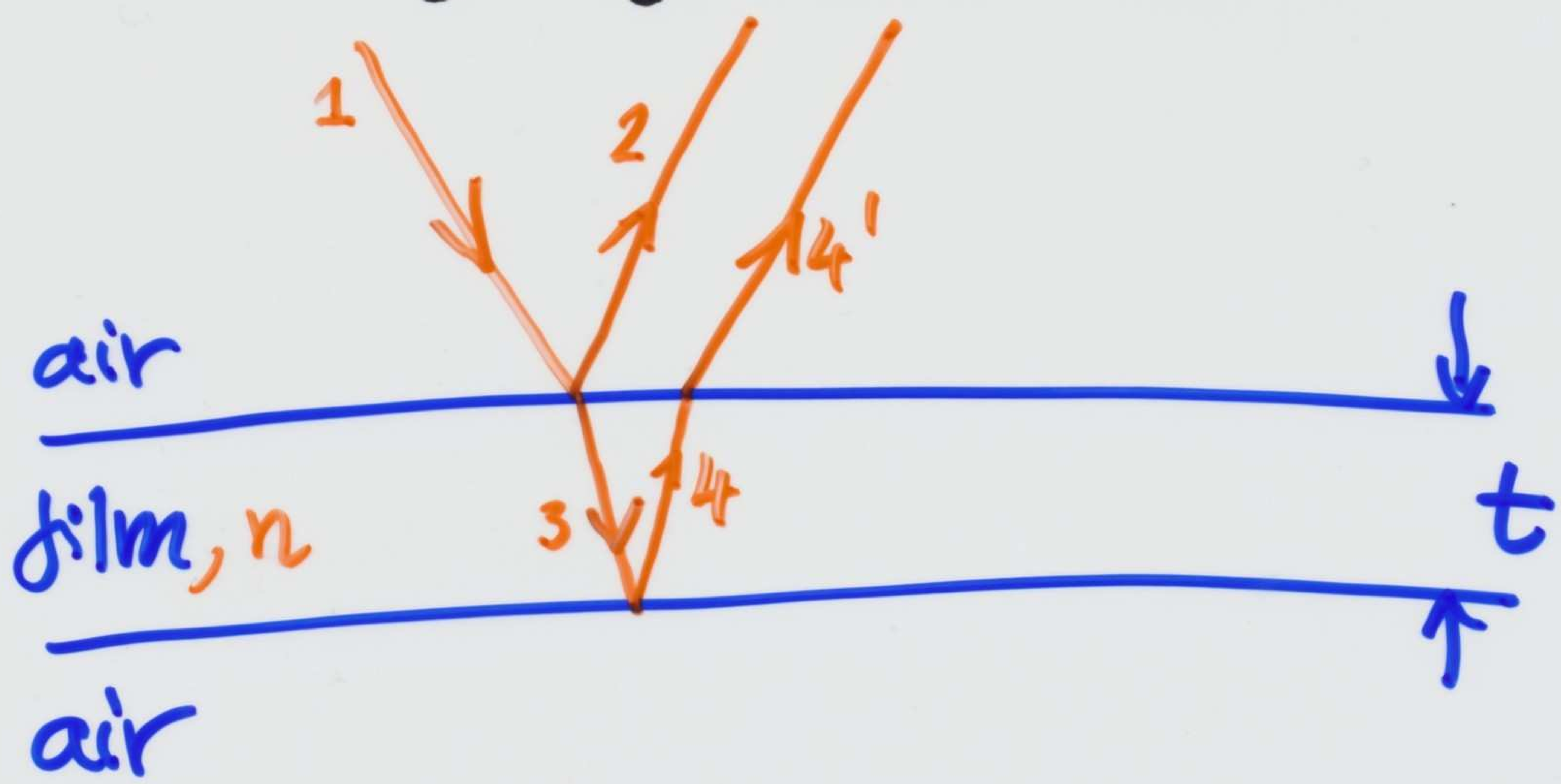
n_1, n_2 ... index of refraction

Interference in thin films

→ oil film on water

→ soap bubble reflections

Consider a film of uniform thickness t & index of refraction n



→ ray 1: reflected ray 2 undergoes a phase change 180° ($n > 1$, for air $n_{air} = 1$)

→ ray 3: reflected ray 4 (& 4') ⇒ no phase change $n_{air} < n$

→ inside the film $\lambda_n = \frac{\lambda}{n}$ (changed wavelength)

Rule for the constructive interference:

$$2t = \left(m + \frac{1}{2}\right)\lambda_n \quad (m = 0, 1, 2, \ldots)$$

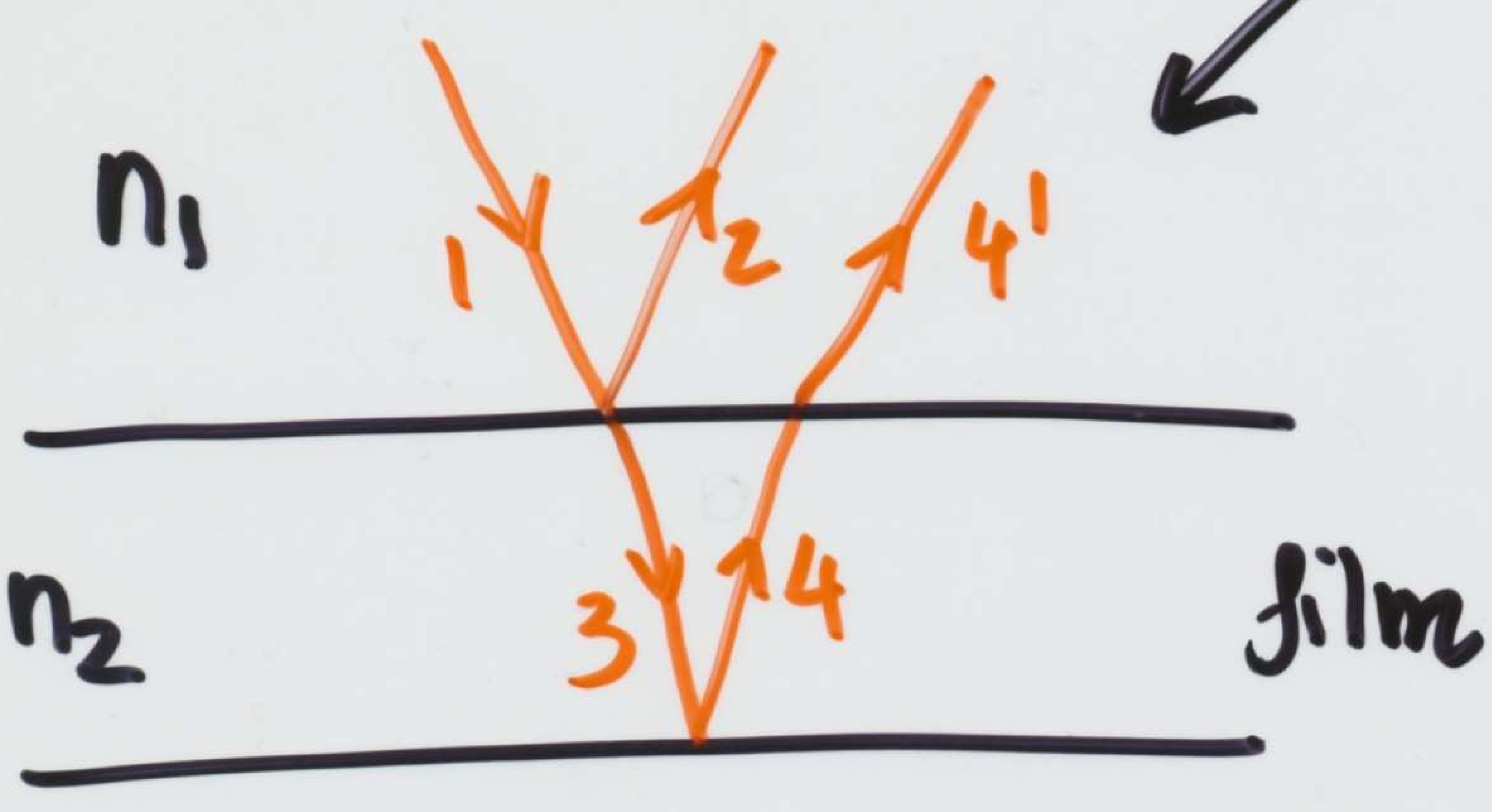
OR

$$\boxed{2nt = \left(m + \frac{1}{2}\right)\lambda} \quad (m = 0, 1, 2, 3, \ldots)$$

Rule for the destructive interference:

$$\boxed{2nt = m\lambda} \quad (m = 0, 1, 2, 3, \ldots)$$

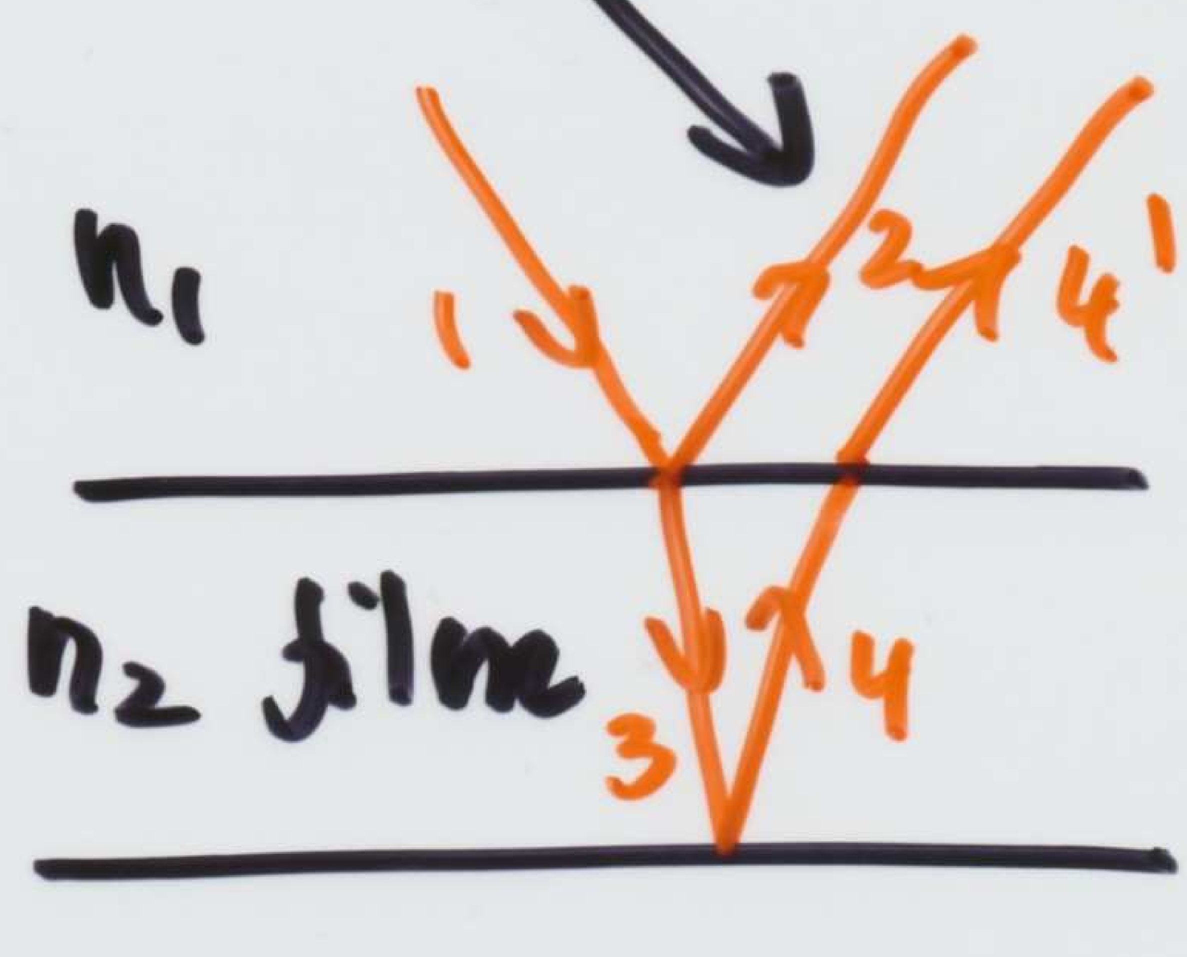
BUT if $n_1 < n_2 < n_3$ or $n_1 > n_2 > n_3$



n_3

1 → 2 180°

3 → 4 180°



n_3

1 → 2 NO

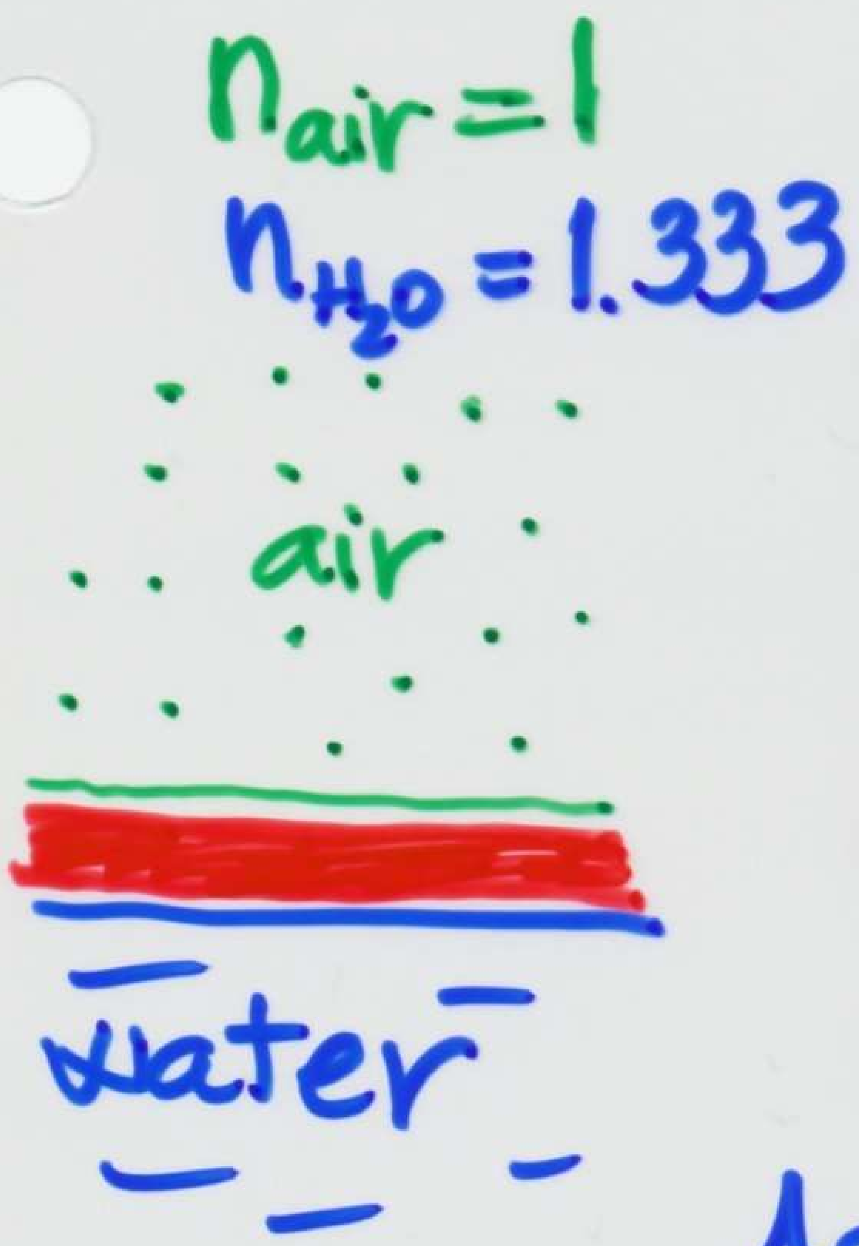
3 → 4 NO

constructive: $2n_2t = m\lambda$

destructive: $2n_2t = \left(m + \frac{1}{2}\right)\lambda$

⇒ REVERSED criteria

Quiz: You spill two liquids onto water. They both form thin films on the water surface but film 1 appears dark & film 2 appears bright in reflected light. Explain!



Assume: $n_{film1} > n_{air}$ & $n_{film2} > n_{air}$

(a) $n_{film1} > n_{H_2O}$ & $n_{film2} > n_{H_2O}$

(b) $n_{film1} < n_{H_2O}$ & $n_{film2} < n_{H_2O}$

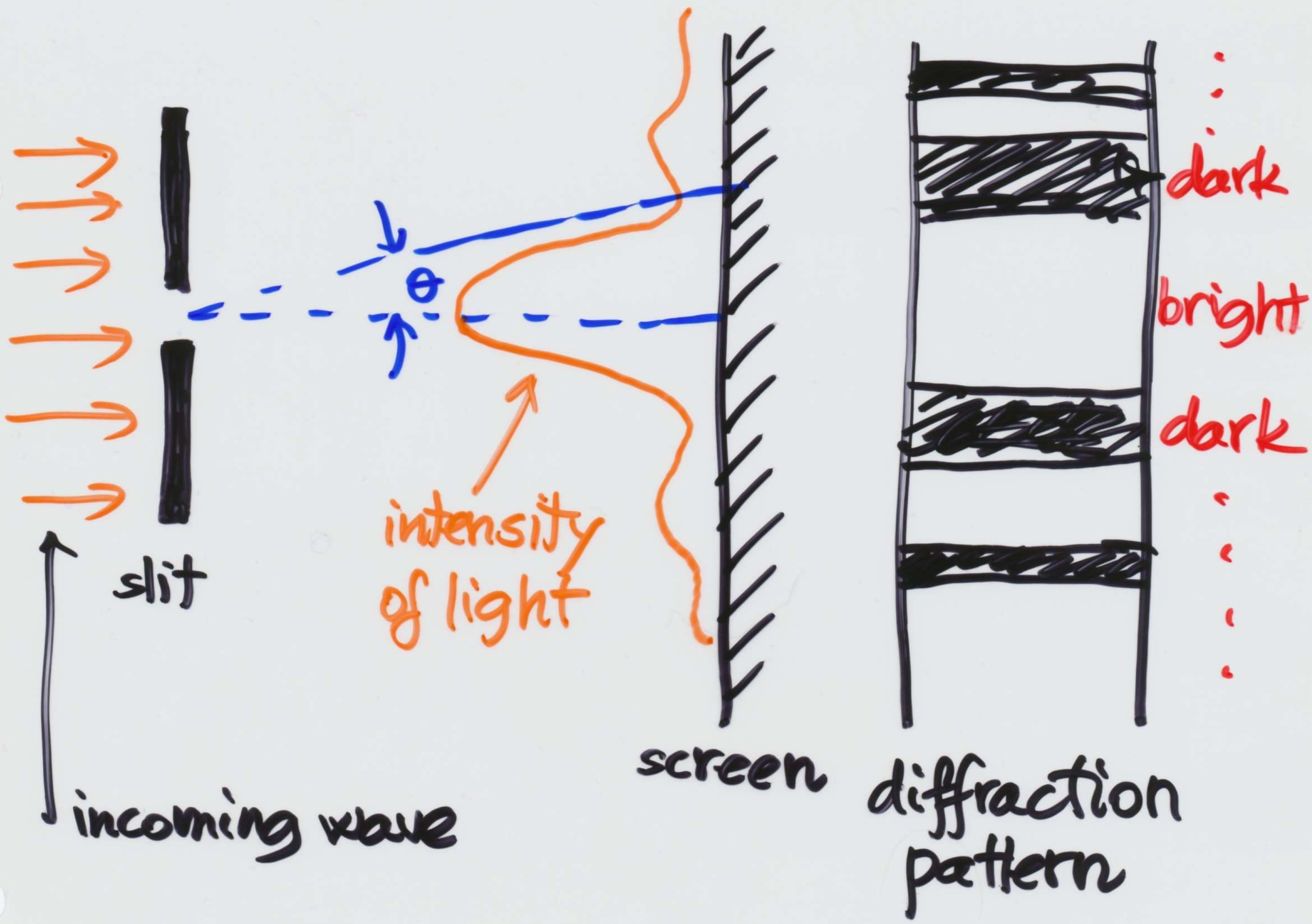
(c) $n_{film1} < n_{H_2O}$ & $n_{film2} > n_{H_2O}$

(d) $n_{film2} < n_{H_2O}$ & $n_{film1} > n_{H_2O}$

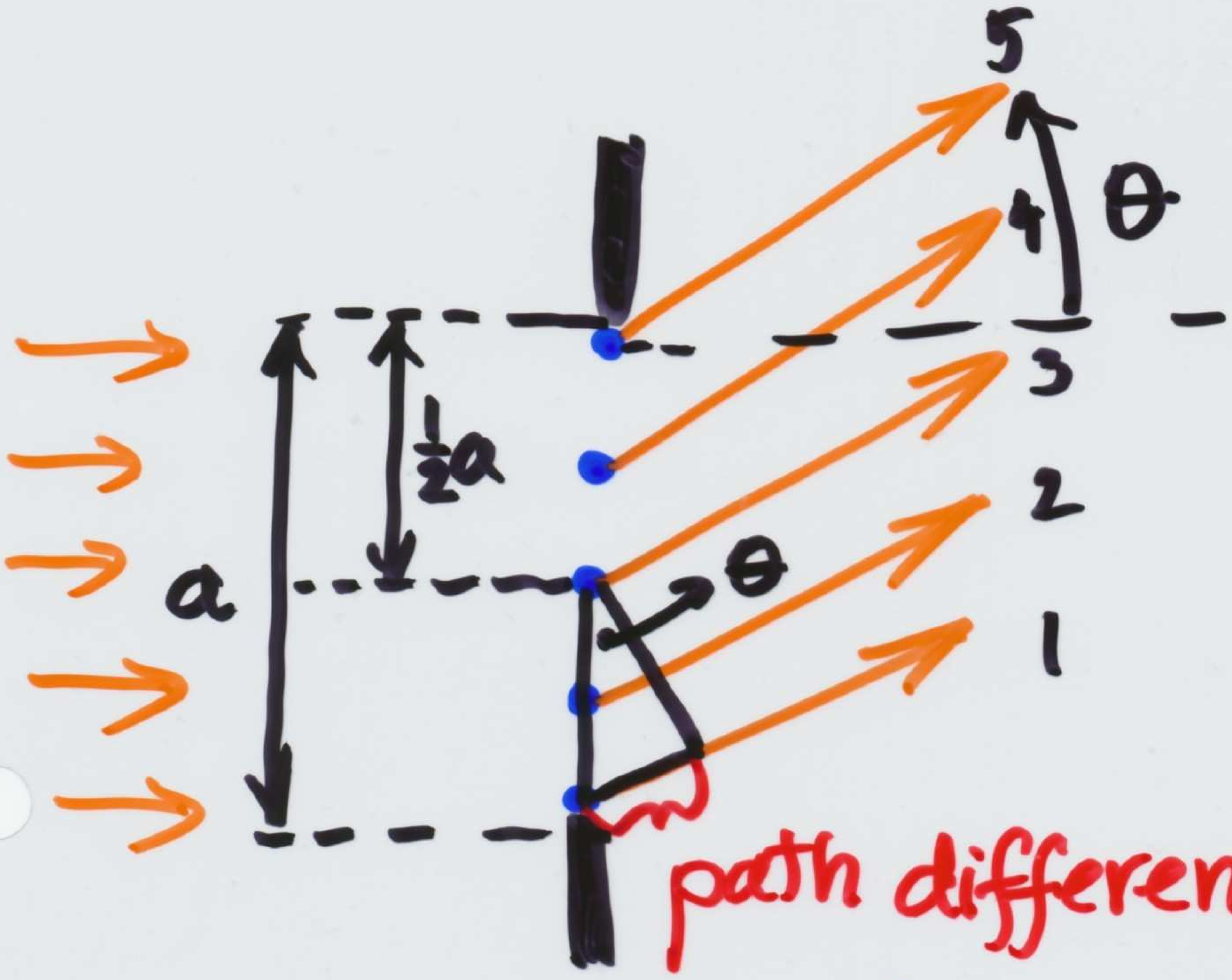
Diffraction Patterns

→ EM waves pass through small openings & around obstacles & by sharp edges

Consider light passing through a narrow opening, slit, & projected onto a screen:



Mathematical model for single-slit Frauenhofer diffraction pattern



path difference 1, 3, 5 & 2, 4

$\frac{1}{2} a \sin\theta$

Huygens principle: each portion of the slit acts as a source of waves

⇓

interference

Where is the first dark band?

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$ (phase change π, 180°)

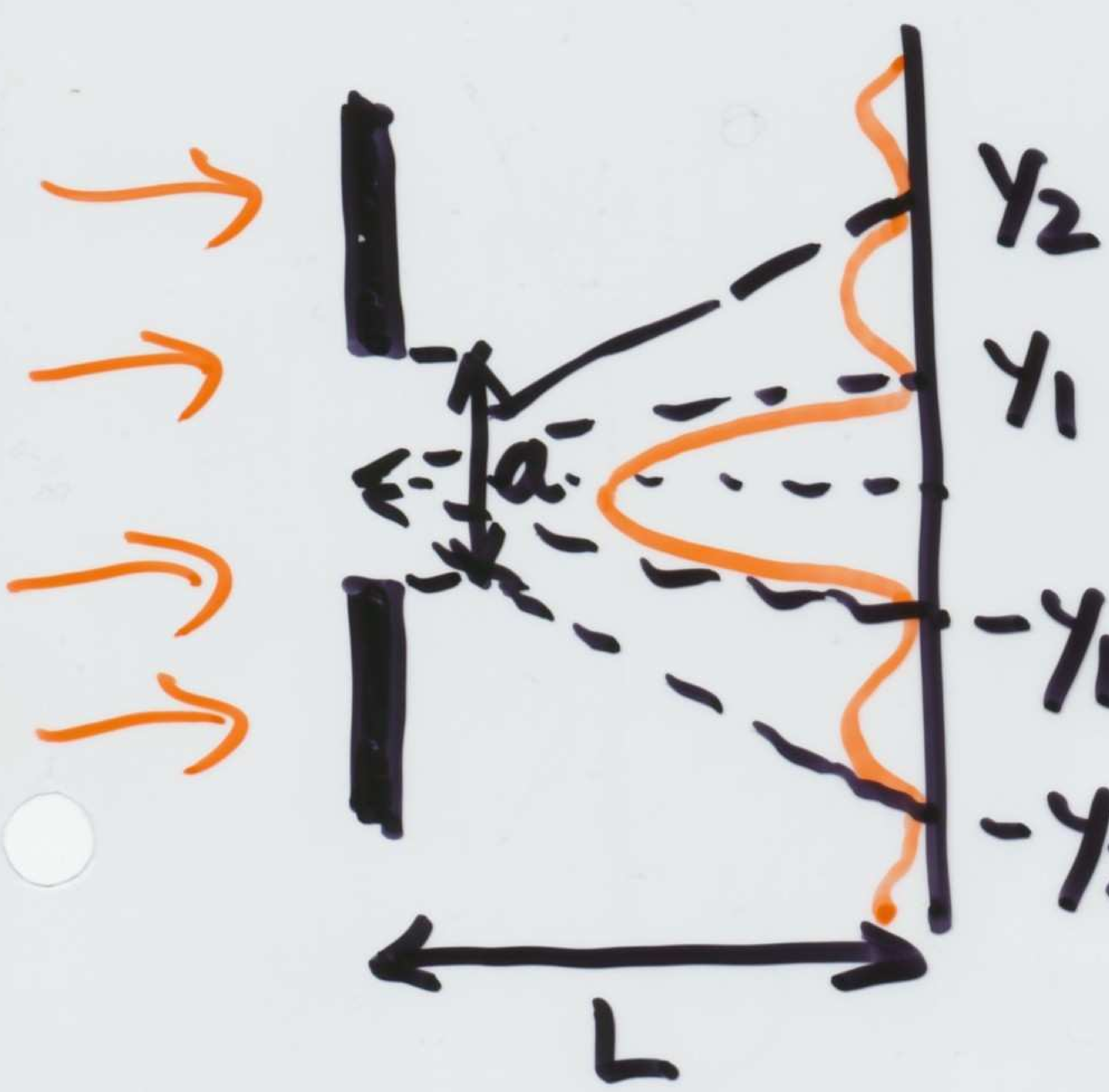
$$\Downarrow$$

$$\sin\theta = \frac{\lambda}{a}$$

General condition for destructive interference

$$\sin\theta_{dark} = m\frac{\lambda}{a} \quad m = \pm 1, \pm 2, \pm 3, \ldots$$

(but NOT $m = 0$)



$\sin\theta_2 = \frac{2\lambda}{a}$ $\quad y_2 = L\tan\theta_2$

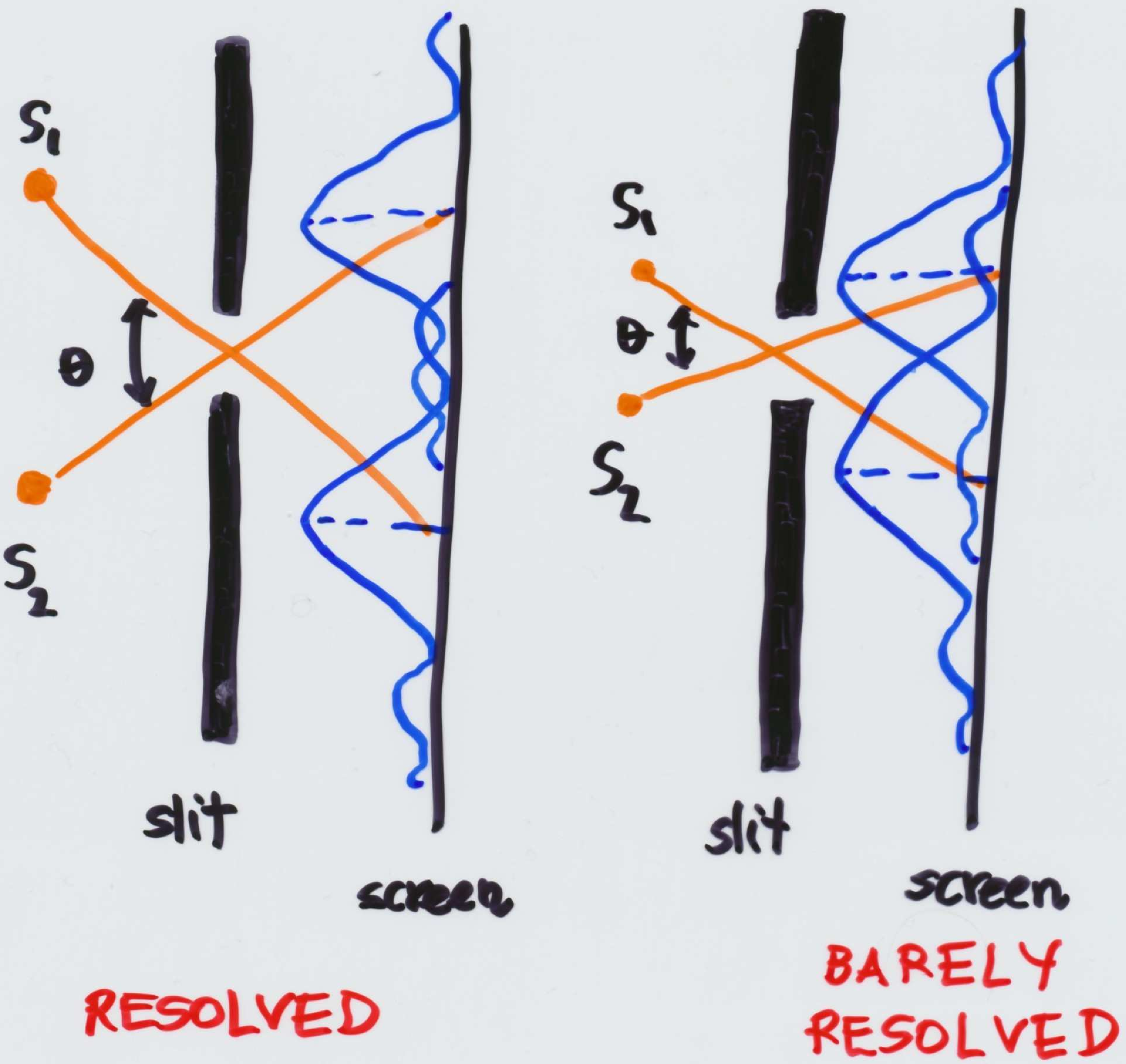
$\sin\theta_1 = \lambda/a$ $\quad y_1 = L\tan\theta_1$

$\sin\theta_{-1} = -\frac{\lambda}{a}$ $\quad y_{-1} = L\tan\theta_{-1}$

$\sin\theta_{-2} = -\frac{2\lambda}{a}$ $\quad y_{-2} = L\tan\theta_{-2}$

Resolution of Single-Slit & Circular Apertures

When an instrument or your eye can distinguish between two objects (light sources) ⇒ the two objects are RESOLVED



Rayleigh's criterion: The central maximum of the diffraction pattern of one source falls on the first minimum of the diffraction pattern of the other source

⇓

the two sources are just RESOLVED

Minimum angular separation θ_{min}:

$$\sin\theta_{min} = \frac{\lambda}{a}$$

λ ← wavelength of EM wave (light)

a ← width of a slit

$\lambda \ll a$ (most situations)

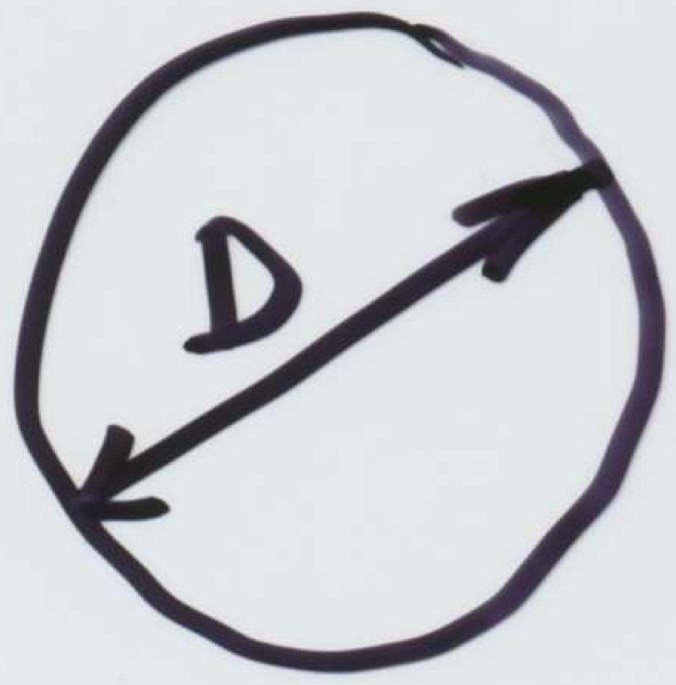
⇓ $\sin\theta \approx \theta$

$$\theta_{min} = \frac{\lambda}{a}$$ [in radians!]

The angle between 2 sources, S_1 & S_2, has to be > θ_{min}:

$\theta > \theta_{min} \Rightarrow$ RESOLVED.

Circular aperture of diameter D



$$\theta_{min} = \underline{1.22}\,\frac{\lambda}{D}$$

↑ for circular apertures

For example:

D ... diameter of a pupil

OR

diameter of the astronomical telescope

Quiz: You are observing a binary star (2 stars close by). Which λ of light you should choose to MAXIMIZE resolution?

(a) red (b) yellow (c) green

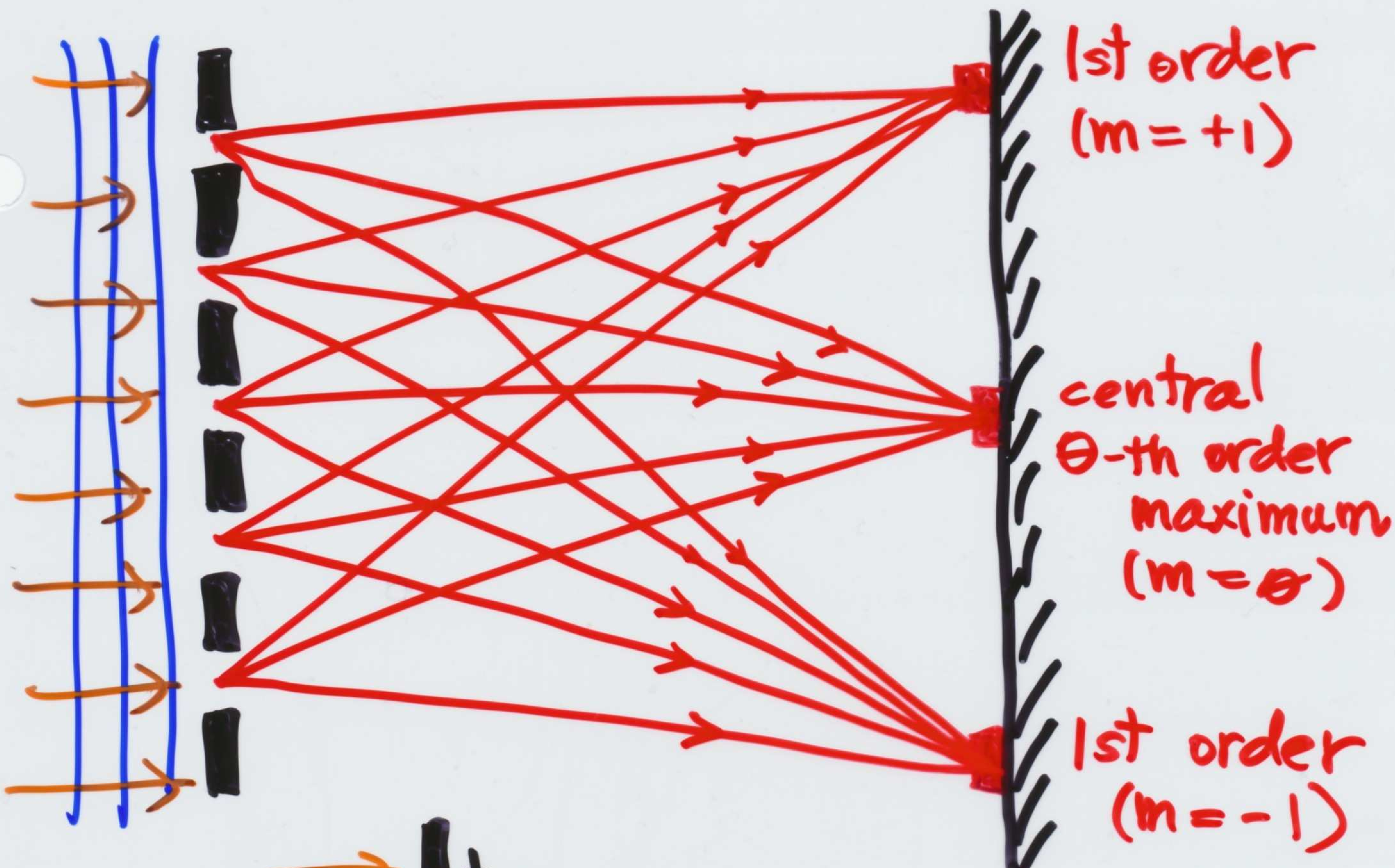
~~(d)~~ blue

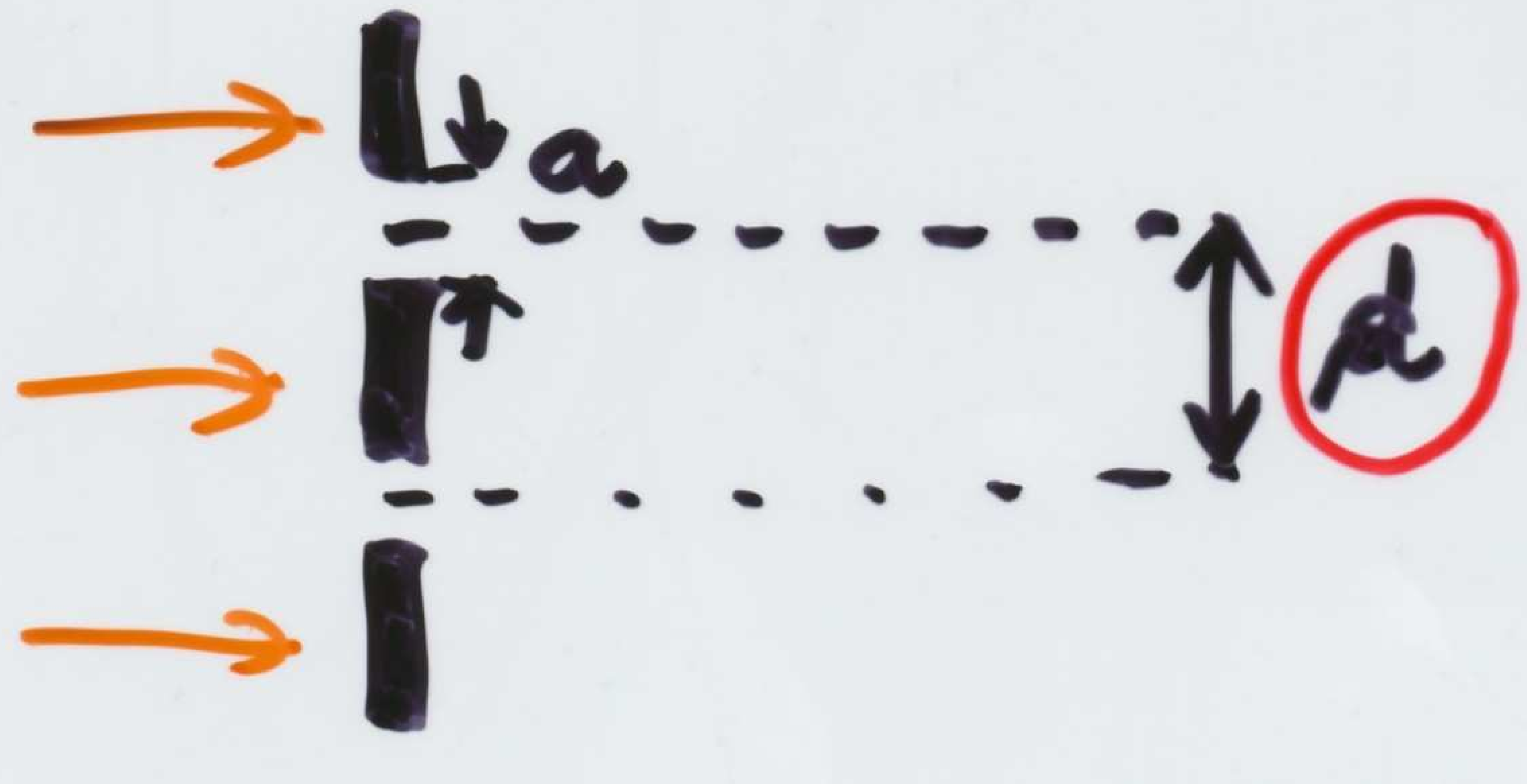
The Diffraction Grating = device for analysing light sources consists of large # of equally spaced slits, e.g. 5000 slits per cm

↓

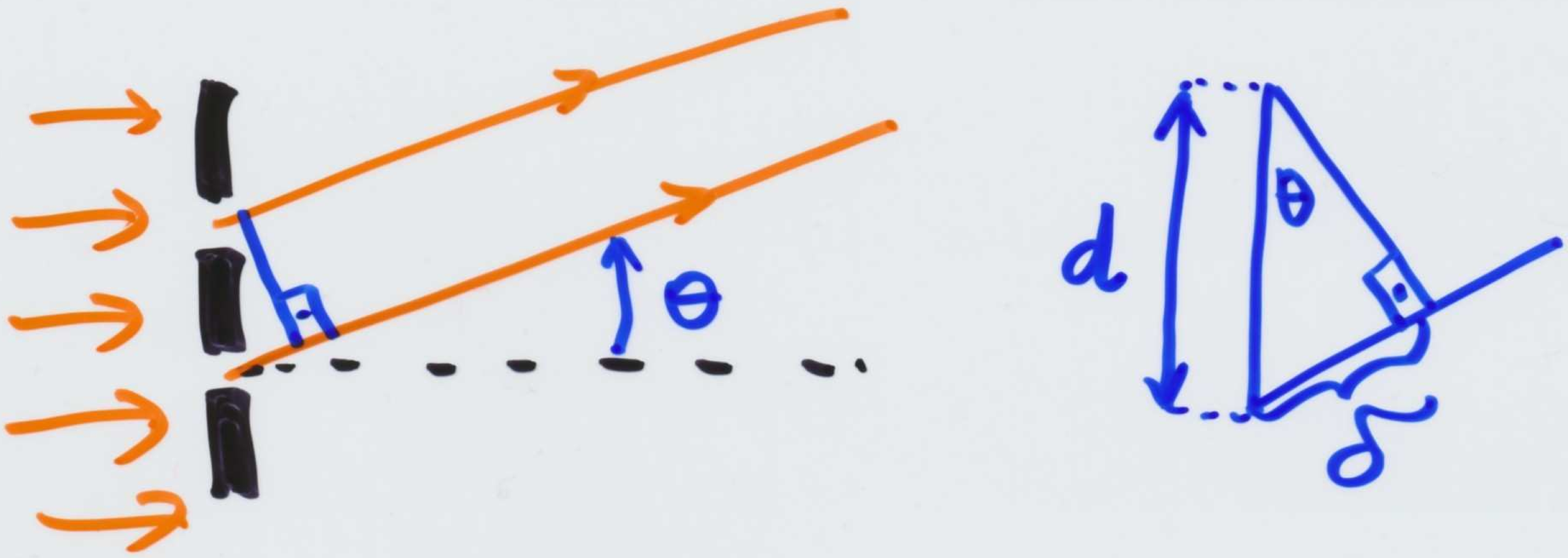
slit spacing $d = \frac{10^{-2}\,m}{5000} = 2 \times 10^{-6}\,m$

$= \underline{\underline{2\,\mu m}}$





Warning : here the size of the slit a is not the parameter which determine the maxima of the diffraction → interference pattern but rather the spacing d BETWEEN slits



δ... path difference

$$\delta = d \cdot \sin\theta$$

Condition for constructive interference:

$$d \cdot \sin\theta_{bright} = m \cdot \lambda$$

$m = 0, \pm 1, \pm 2, \ldots$

Quiz: Red light of $\lambda = 700nm$ is used to form interference pattern on a screen a distance L away. The spacing between slits is d. We mark the bright maxima on the screen, then replace the red light source with UV source with $\lambda_1 = \frac{1}{2}\lambda =$ $= 350nm$. If we want to match the marks on the screen with the new bright maxima of λ_1 source, we have to:

(a) move the screen to 2L

(b) —— " —— to $\frac{1}{2}L$

(c) double the distance $d_1 = 2d$ between the slits

~~(d)~~ half the distance $d_1 = \frac{1}{2}d$ between the slits

Diffraction of X-rays by Crystals

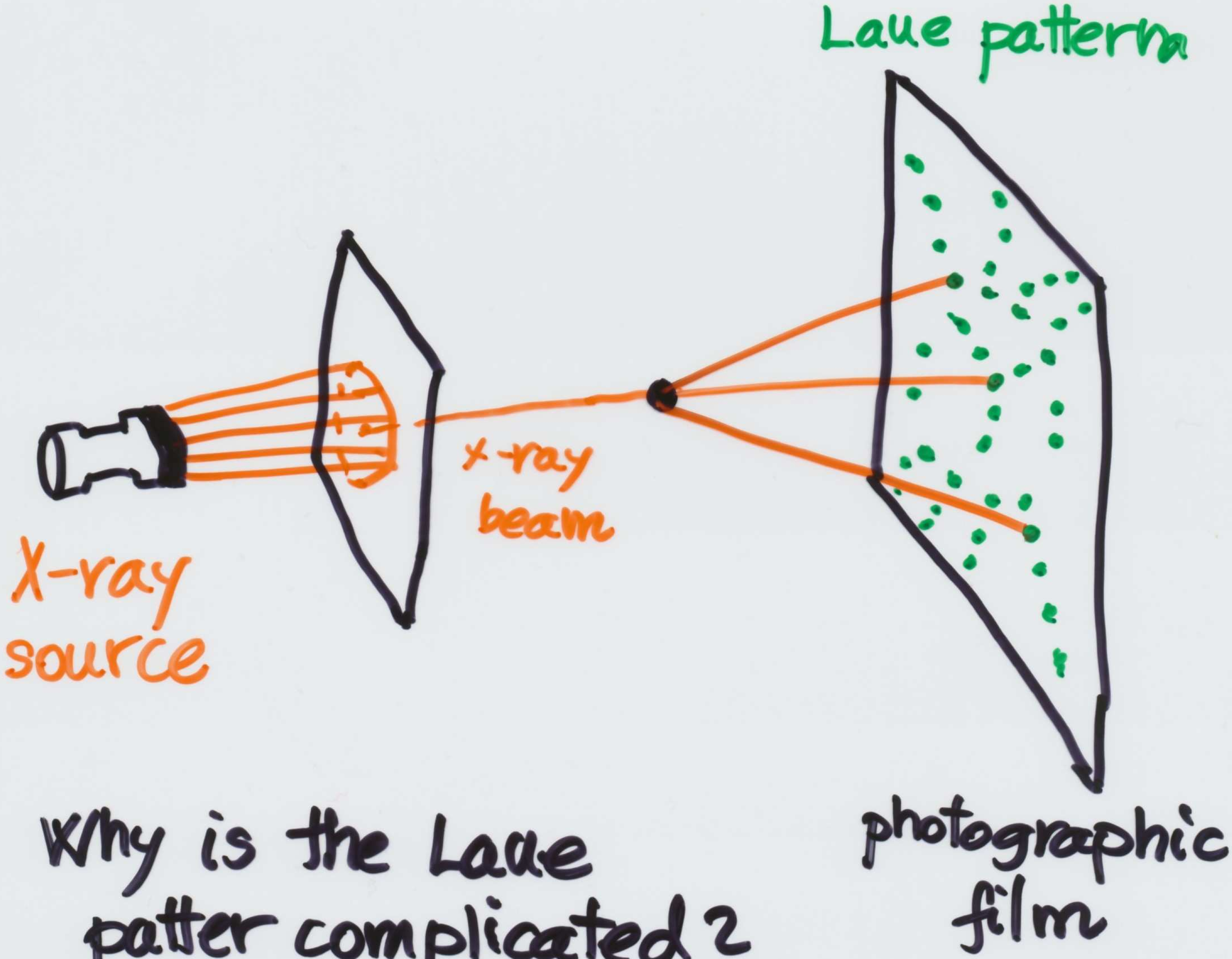
The wavelength λ of any EM wave can be determined if a grating of the spacing $\sim \lambda$ is available

X-rays $\lambda \approx 10^{-10}$ m = 0.1 nm

10^{-10} m is the order of inter-atom spacing in a crystal.

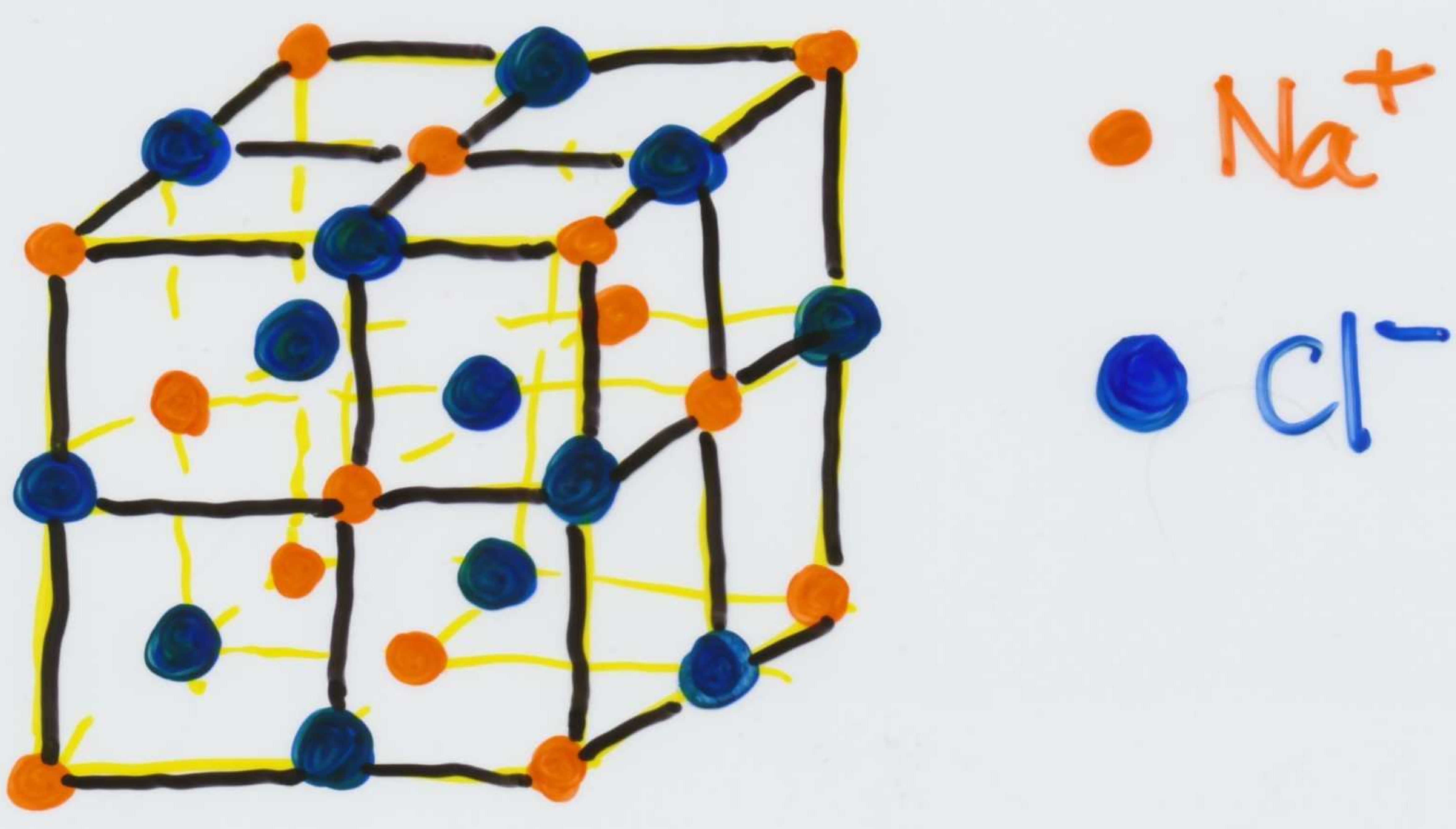
Max von Laue suggested that a crystal can act as a 3D grating for X-rays and yield a diffraction pattern.

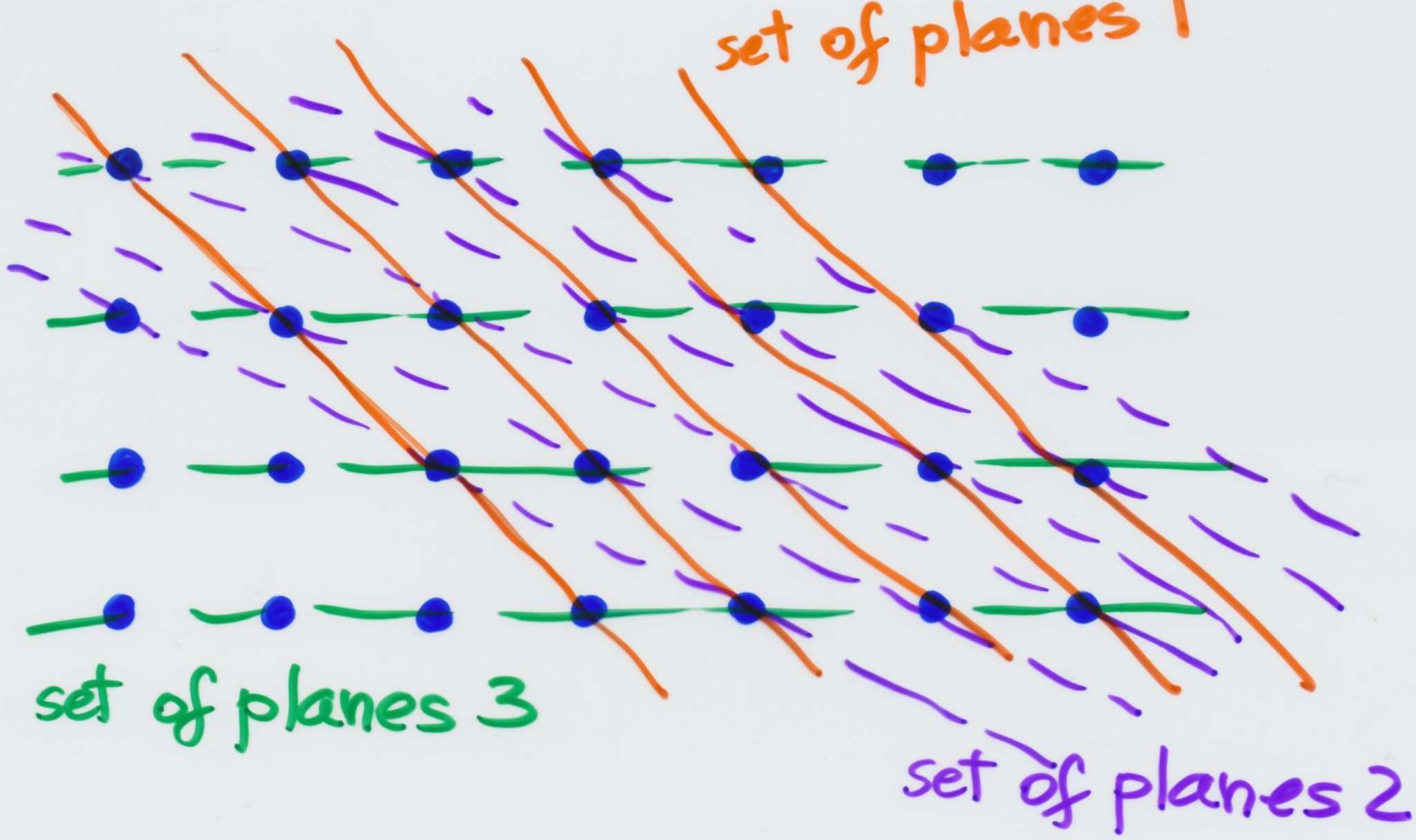
X-ray diffraction nowadays
$\Rightarrow$ elucidates structure of matter



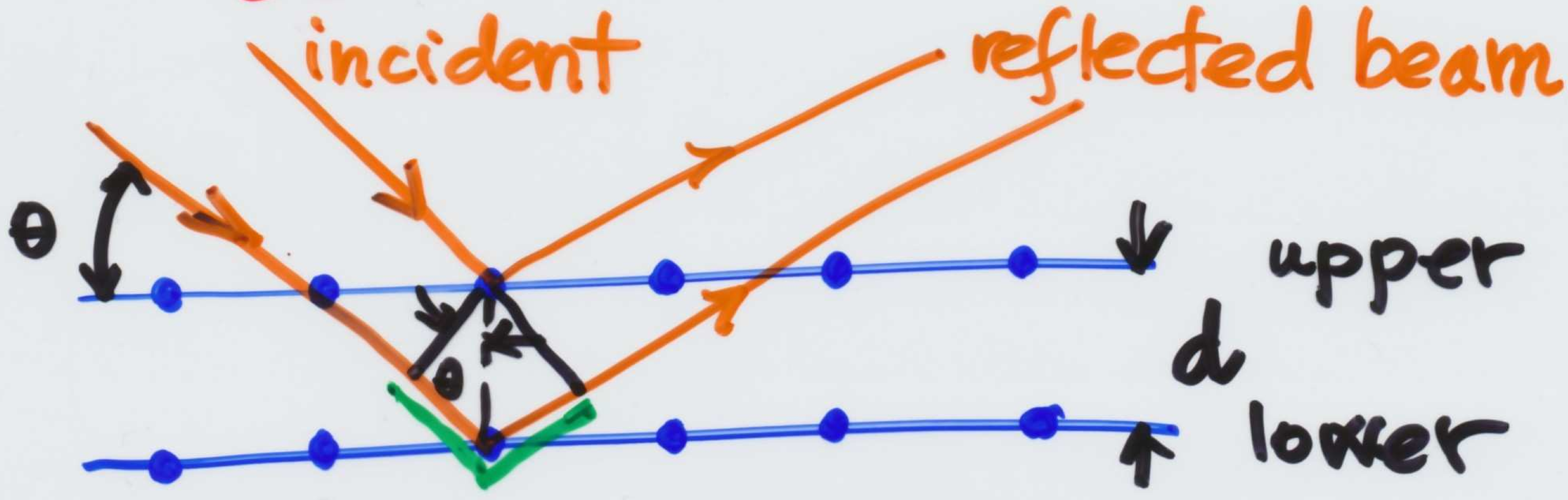
Why is the Laue patter complicated?

Consider 3D structure of a crystal, for example Na Cl:





Bragg's law



The path difference between the upper plane & lower plane reflection:

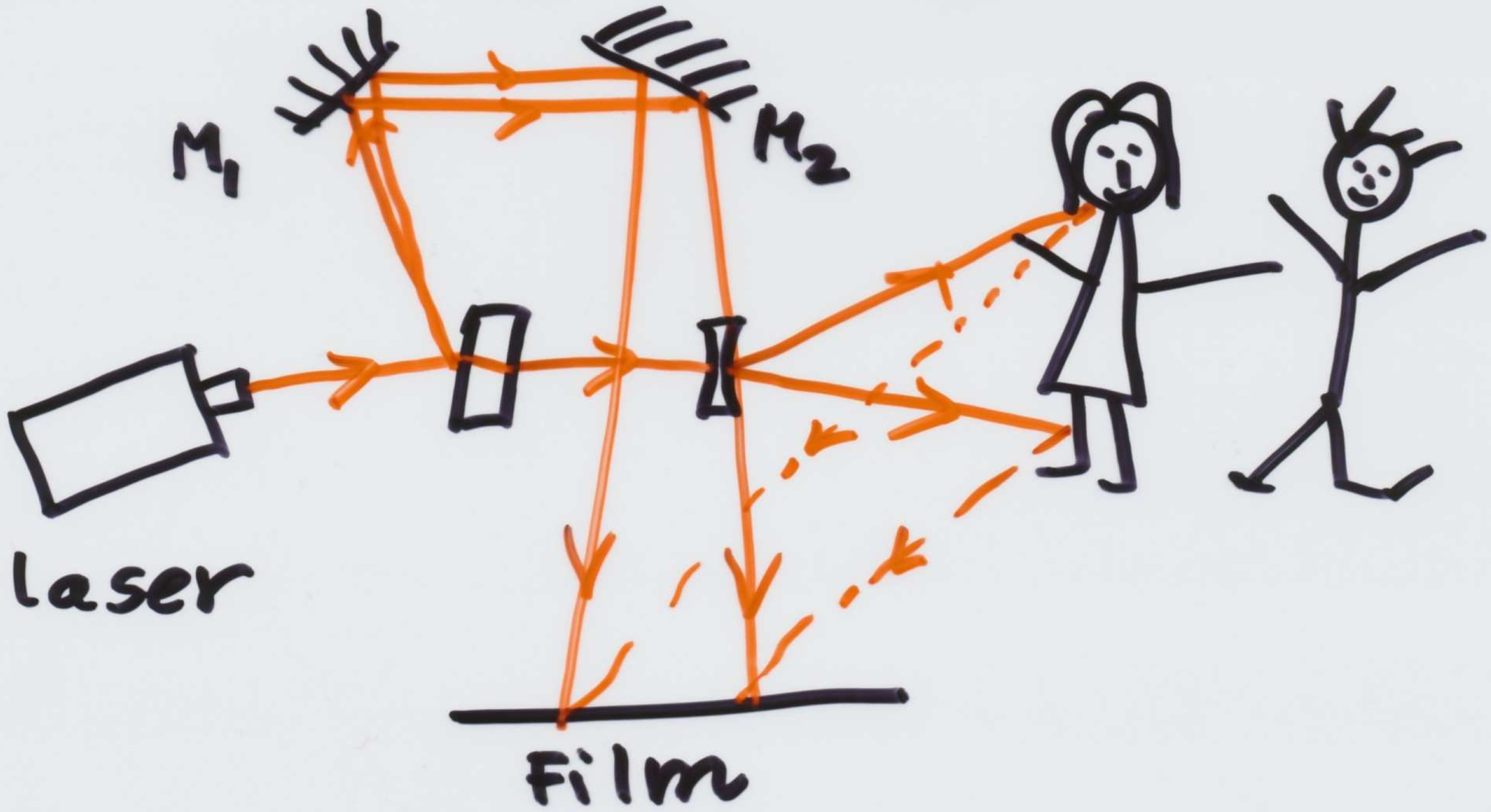
$$\boxed{2d\sin\theta = m\lambda}$$ Bragg's law

$m = 1, 2, 3, \ldots$

Holography

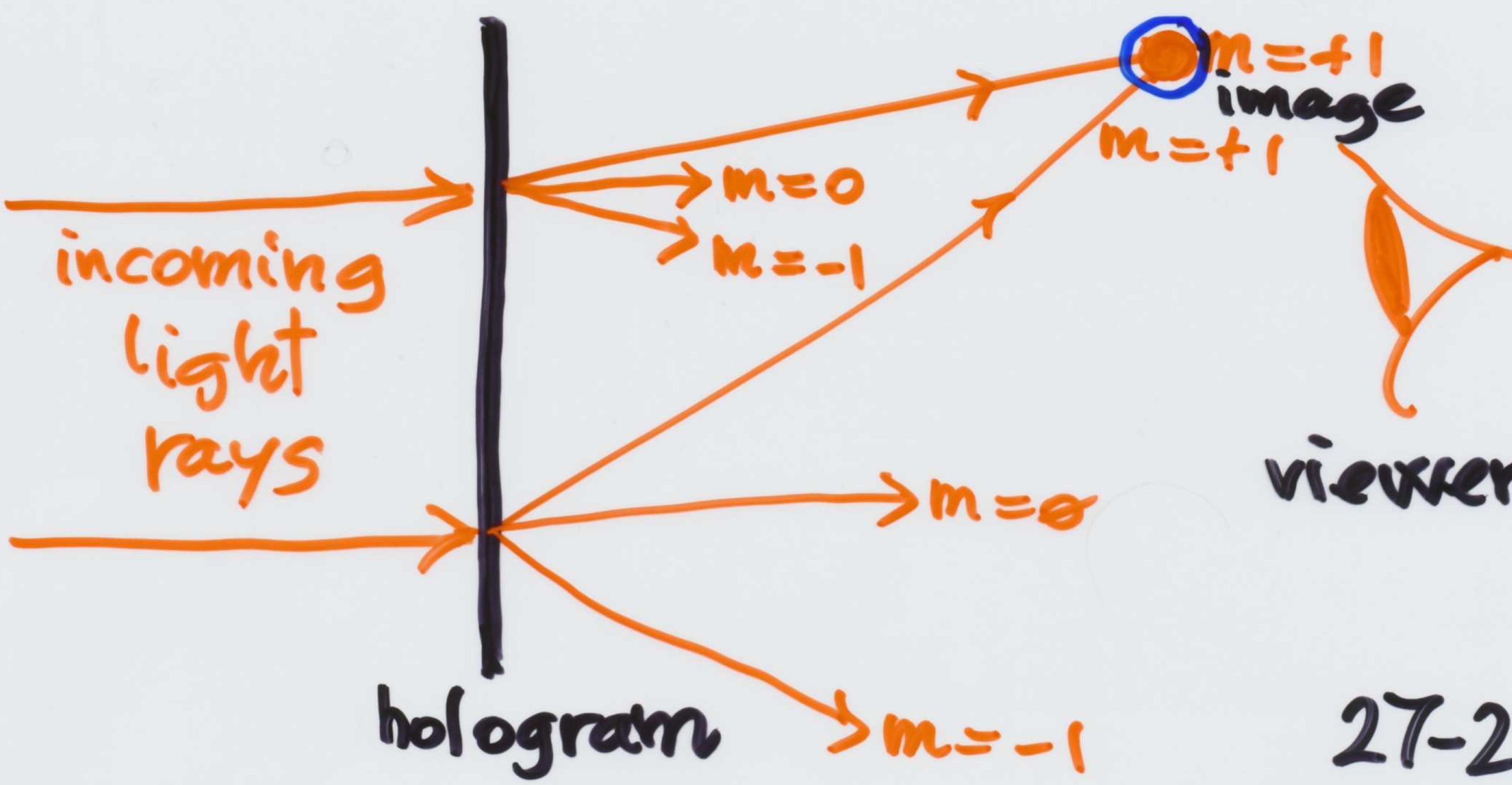
Dennis Gabor
Nobel Prize 1971

(A) Producing a hologram



(B) Viewing a hologram

hologram acts as a grating



Quantum Physics

Dual nature of light?

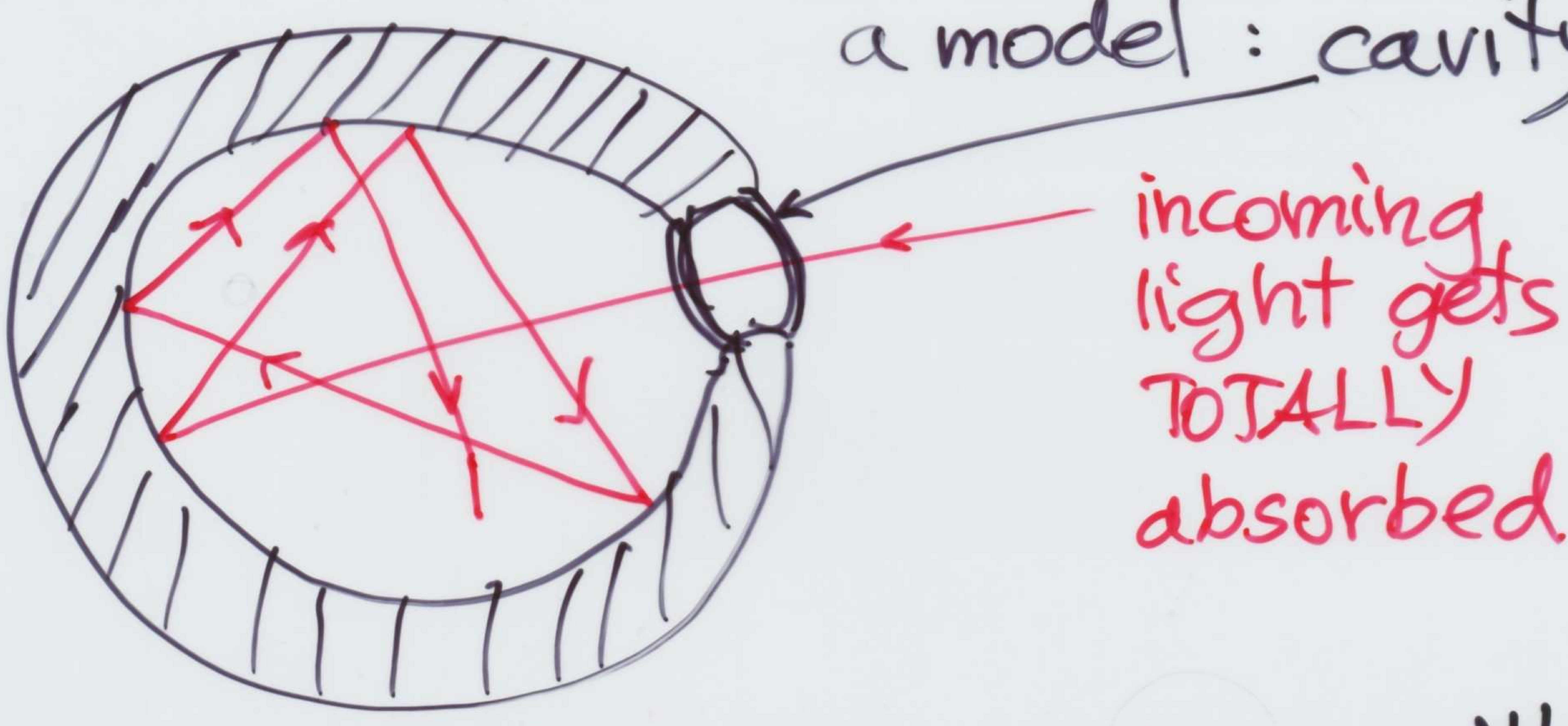
wave vs. particle

Blackbody Radiation & Planck's Theory

→ any object at temperature $T > 0$ emits energy called thermal radiation

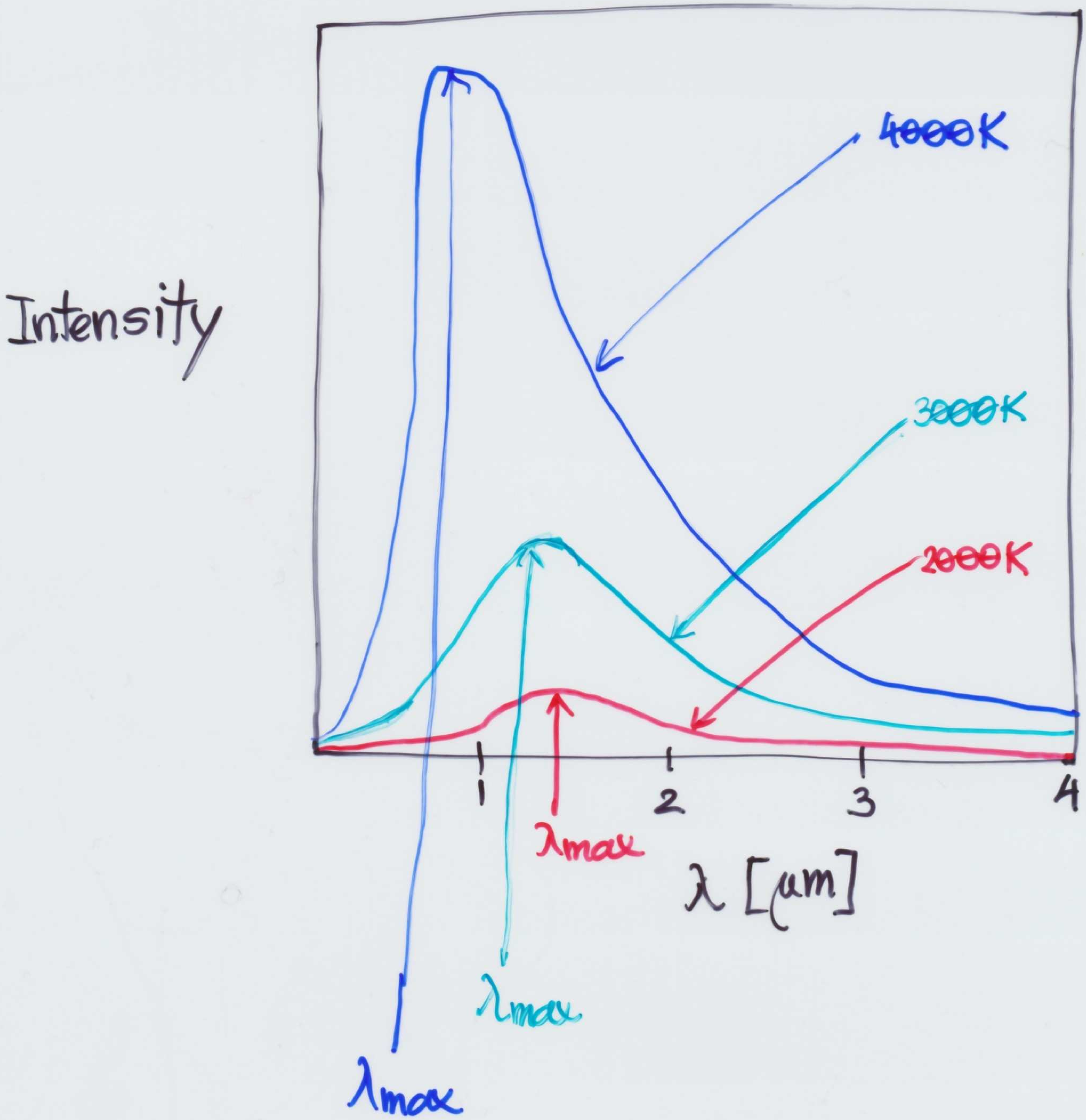
→ what is a black body?

a model: cavity



→ the characteristics of radiation emitted from the hole ⇒ temperature of the walls

→ distribution of different wavelengths of cavity (blackbody) radiation [exp. studied in late 19th century]:



Experimentally observed features of the distribution:

→ The total power of emitted radiation increases with temperature ⇒ Stefan's law

$$\mathcal{P} = \sigma A e T^4$$

- T ← temperature
- e ← emissivity (= 1 for blackbody)
- A ← area (e.g. surface of the black body)
- σ ← Stefan's constant $\sigma = 5.67 \times 10^{-8}\ W/m^2$

→ The peak of the wavelength distribution λ_{max} shifts to shorter wavelengths as the temperature increases ⇒ Wien's displacement law

$$\lambda_{max} T = 2.898 \times 10^{-3}\ m \cdot K$$

- λ_{max} ← peak
- T ← temperature

Theoretical explanation for blackbody radiation

→ classically:

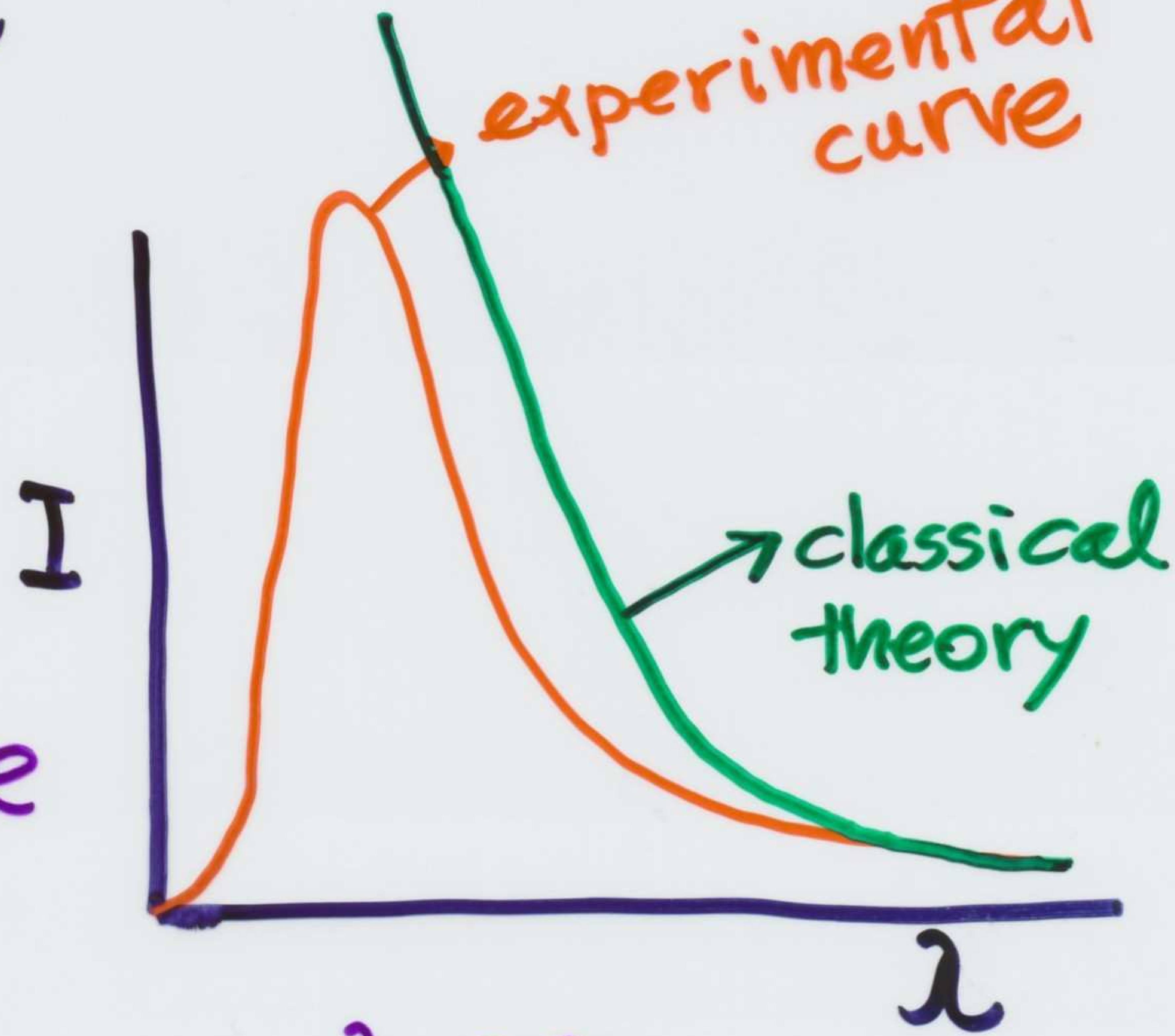
large λ → agree

small λ ⇒

ultraviolet catastrophe

$I \to \infty$ as $\lambda \to 0$

small λ ⇒ UV light



→ explanation by Max Planck:

onset of quantum physics

Planck imagined oscillators at the surface of b.b. (related to fluctuating charges in molecules) & made 2 assumptions

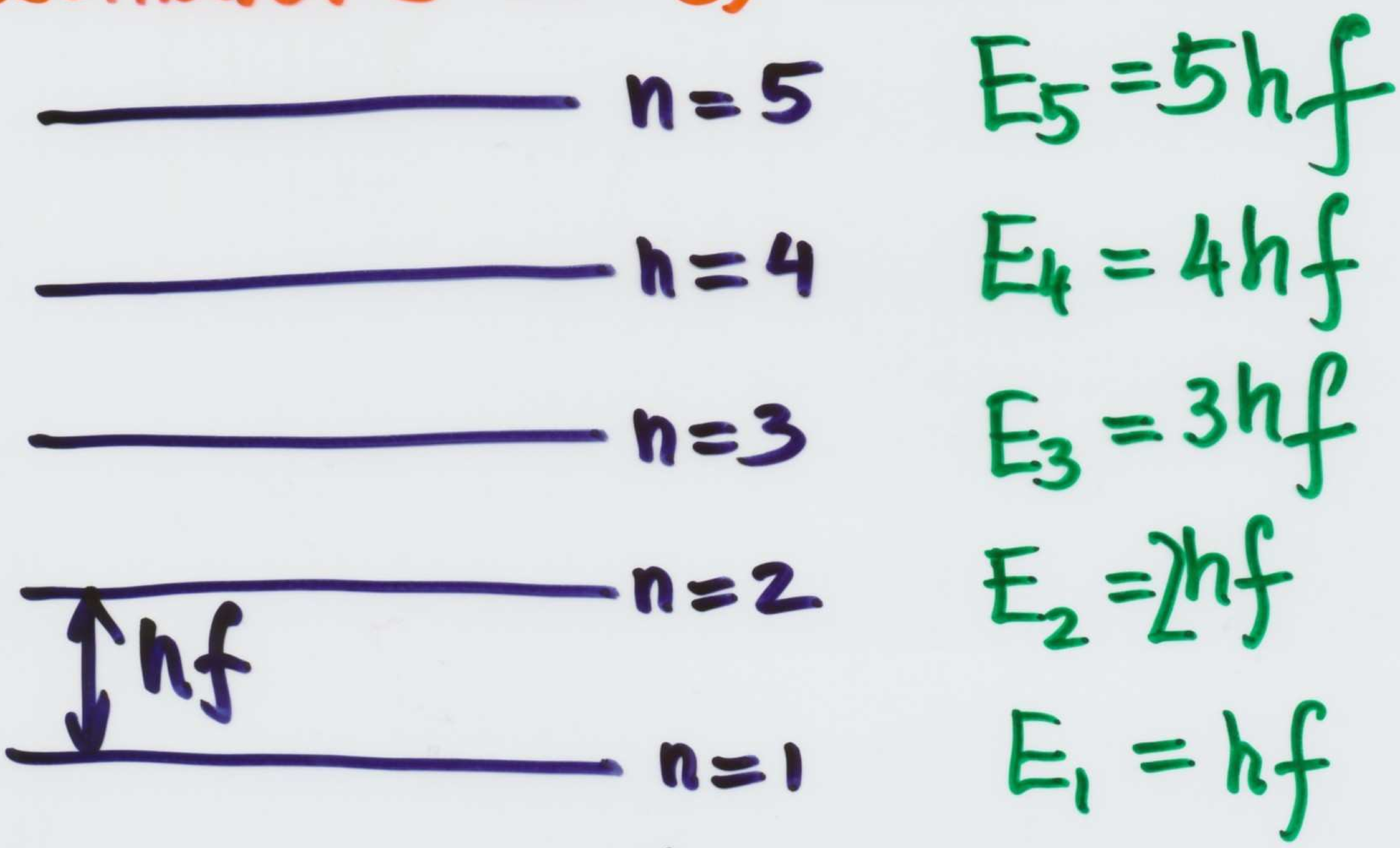
(1) → energy of the oscillator is quantized:

$$E_n = n \cdot h \cdot f$$

n: integer; h: Planck constant; f: frequency

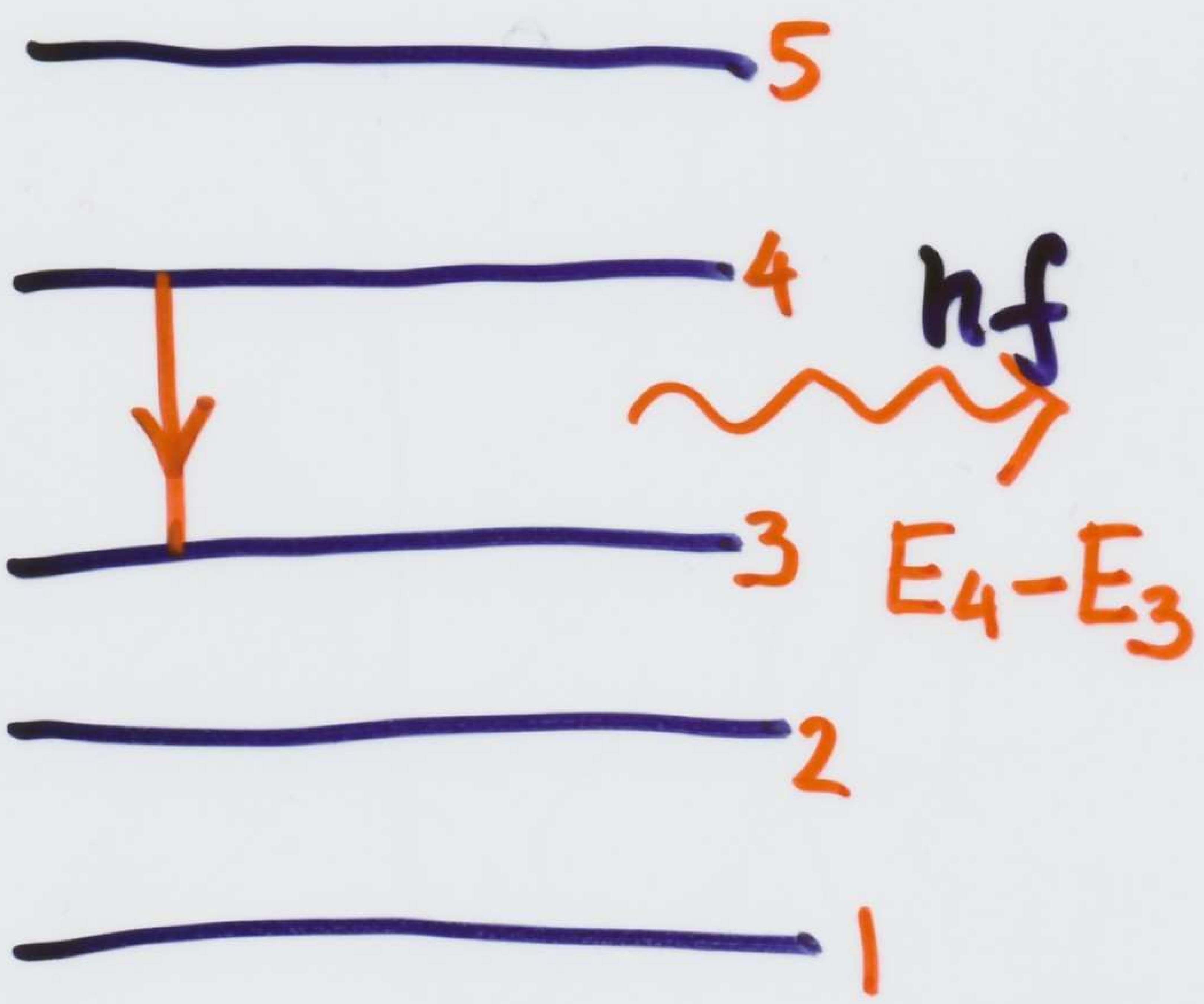
(2) → oscillators emit or absorb energy in discrete units, quanta of radiation

oscillator's energy states : quantum states

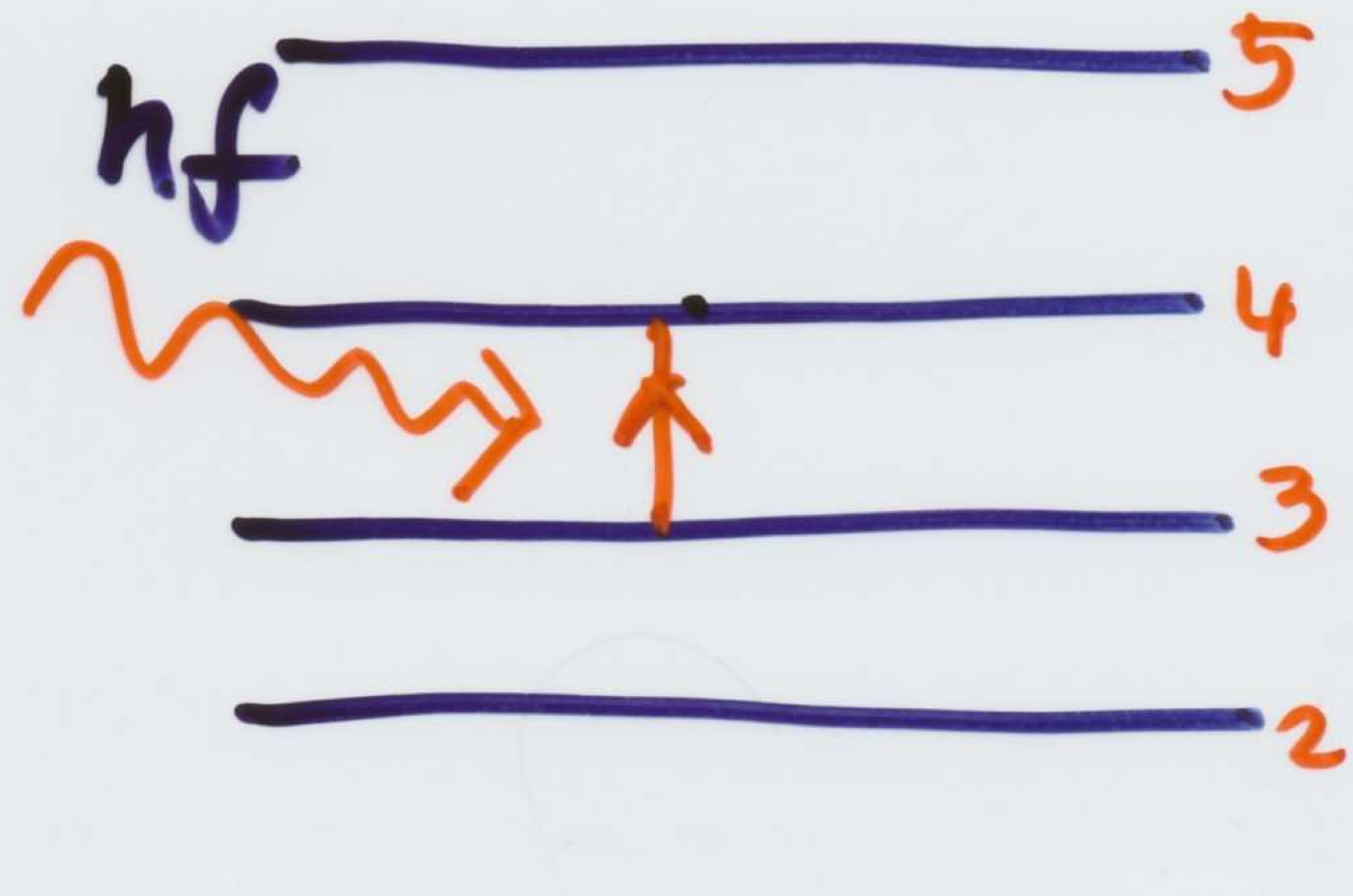


quantum transitions

EMISSION
4→3



ABSORPTION
3→4



Planck presented his theory which yielded the wavelength distribution in remarkable agreement with experimental curves.

Why do we not see quantum effects on a daily basis?

→ in a macroscopic world quantum jumps (say in energy) are so small that our senses percieve continuous behavior

→ quantum effect observed on the submicroscopic level of atoms & molecules

Quiz: In the constellation Orion, stars Rigel & Betelgeuse glow in blue & red light, respectively. Which one has a higher surface temperature?

EXAMPLE: Thermal radiation from the Human Body

The temperature of the surface of the human body is 35°C. Calculate λ_{max}, the peak wavelength, of the radiation emitted by the human body, & estimate the total power $\mathcal{P}$.

→ Wien's displacement law

$$\lambda_{max} T = 2.898 \times 10^{-3} \text{ m K}$$

→ $T_{HB} = 35°C = 308 K$ (273 + 35)

$$\lambda_{max} = \frac{2.898 \times 10^{-3} \text{ m } \cancel{K}}{308 \cancel{K}} =$$

$$= 9.41 \times 10^{-6} \text{ m} = \underline{\underline{9.41 \mu m}}$$

→ infrared region of spectrum

→ Stefan's law: $\mathcal{P} = \sigma A T^4$

→ area of HB: $A \doteq 2 m^2$

} ⇒ $\underline{\underline{\mathcal{P} \approx 10^3 W}}$

height → 2m × 0.3m (width) × 0.2m → depth