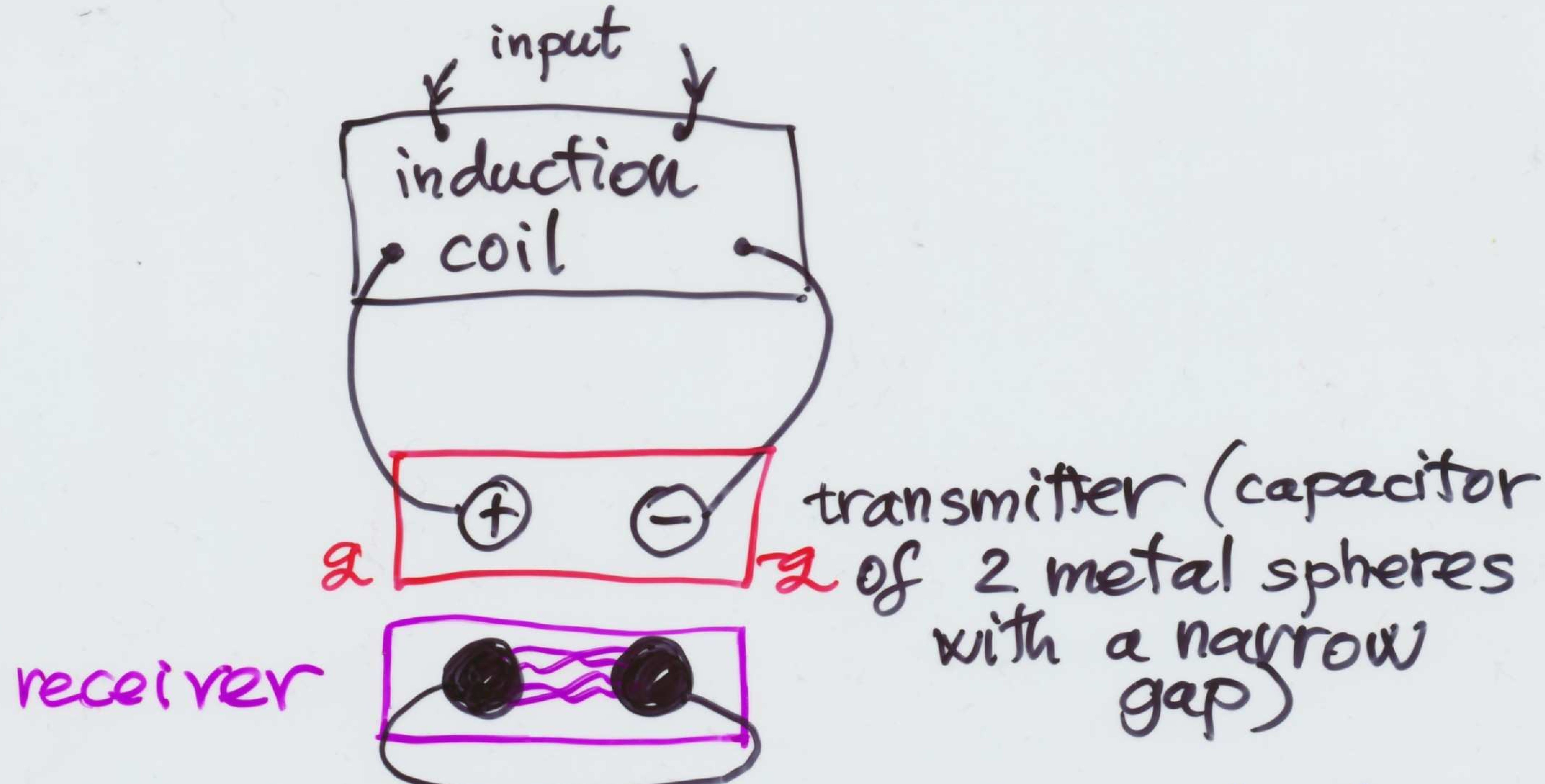
Hertz's Discoveries

Heinrich R. Hertz (in 1888) generated & detected EM waves in a lab for the first time using LC circuit.



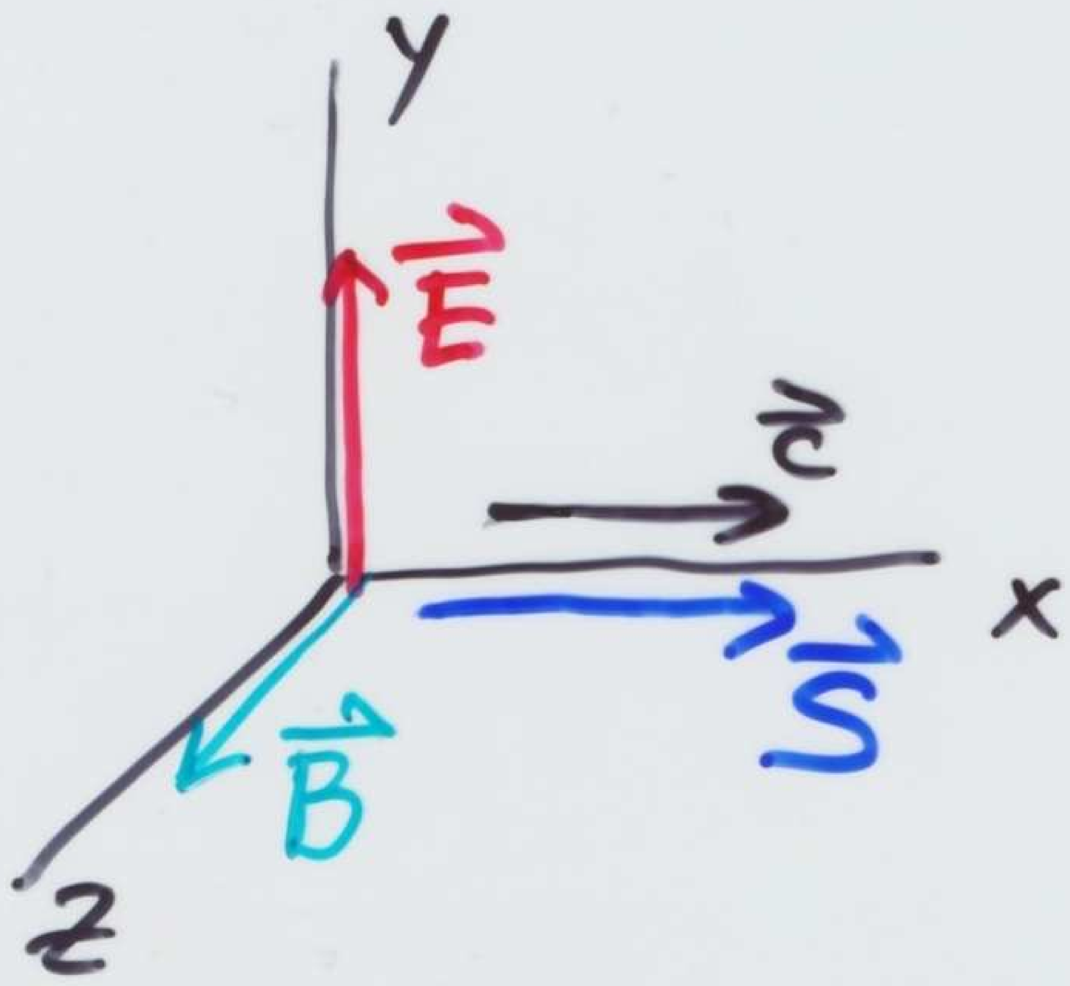
→ LC oscillations initiated by short voltage pulses

→ $f = \frac{1}{2\pi\sqrt{LC}} \sim$ 100 MHz $(10^8 s^{-1})$

→ transmitter produces EM waves

→ receiver, several meters away, receives energy from transmitter if the resonance f_R of receiver is matched to f of transmitter

Energy carried by EM waves



$\vec{S}$ ... Poynting vector

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$\vec{S}$ describes the rate of energy flow in an EM wave $\equiv$ the rate of energy flow per unit surface area perpendicular to the direction of the EM wave

$$\vec{S} \text{ units } \left[\frac{J}{s\, m^2} = \frac{W}{m^2}\right]$$

Example : $|\vec{S}|$ for a plane EM wave

$$S = |\vec{S}| = \frac{1}{\mu_0} EB \underset{B = \frac{E}{c}}{=} \frac{1}{\mu_0 c} E^2 \underset{E = cB}{=} \frac{c}{\mu_0} B^2$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \rightarrow c\mu_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

→ at any time : $S = \frac{1}{\mu_0 c} E^2 = \frac{c}{\mu_0} B^2$

→ time average of S over one or more cycles is INTENSITY I

$$I = S_{avg} = \frac{1}{2}\frac{1}{\mu_0 c} E_{max}^2 = \frac{1}{2}\frac{c}{\mu_0} B_{max}^2$$

$$E(x,t) = E_{max}\cos(kx - \omega t)$$
$$B(x,t) = B_{max}\cos(kx - \omega t)$$

$$\langle \cos^2(kx - \omega t)\rangle_{\text{one cycle}} = \frac{1}{2}$$

→ energy per unit volume u_E & u_B

$$\underbrace{\left.\begin{aligned} u_E &= \frac{1}{2}\varepsilon_0 E^2 \\ u_B &= \frac{1}{2\mu_0} B^2 \end{aligned}\right\}}_{\text{TOTAL } u = u_E + u_B = \varepsilon_0 E^2 = \frac{1}{\mu_0}B^2}$$

at any instant t
$u_E = u_B$

→ average energy per unit volume

$$u_{avg} = \langle u_E\rangle_{cycle} + \langle u_B\rangle_{cycle}$$

$$u_{avg} = \frac{1}{2}\varepsilon_0 E_{max}^2 = \frac{1}{2\mu_0} B_{max}^2$$

→ relationship between intensity I and the average energy per unit volume (av. energy density)

$$\boxed{I = S_{avg} = c\, u_{avg}}$$

Quiz: Which quantity varies with time?

1 (a) intensity of an EM wave I

0 (b) average energy density u_{avg}

~~(c)~~ magnitude of the Poynting vector $|\vec{S}|$

Momentum and Radiation Pressure

EM waves carry linear momentum & energy ⇒ EM wave that hits a surface exerts pressure on the surface

EM wave hits the surface at normal incidence, gets absorbed:

$$p = \frac{U}{c} \quad \text{(complete absorption)}$$

U ← total energy to a surface in time Δt

momentum → p

→ pressure P

$$\overset{\text{pressure}}{P} = \frac{F}{A} = \frac{1}{A}\frac{d\overset{\text{momentum}}{p}}{dt} = \frac{1}{cA}\frac{dU}{dt}$$

$$\frac{1}{A}\frac{dU}{dt} \overset{\text{def.}}{=} S$$

S ← the magnitude of the Poyting vector

⇓

$$\boxed{P = \frac{S}{c}}$$

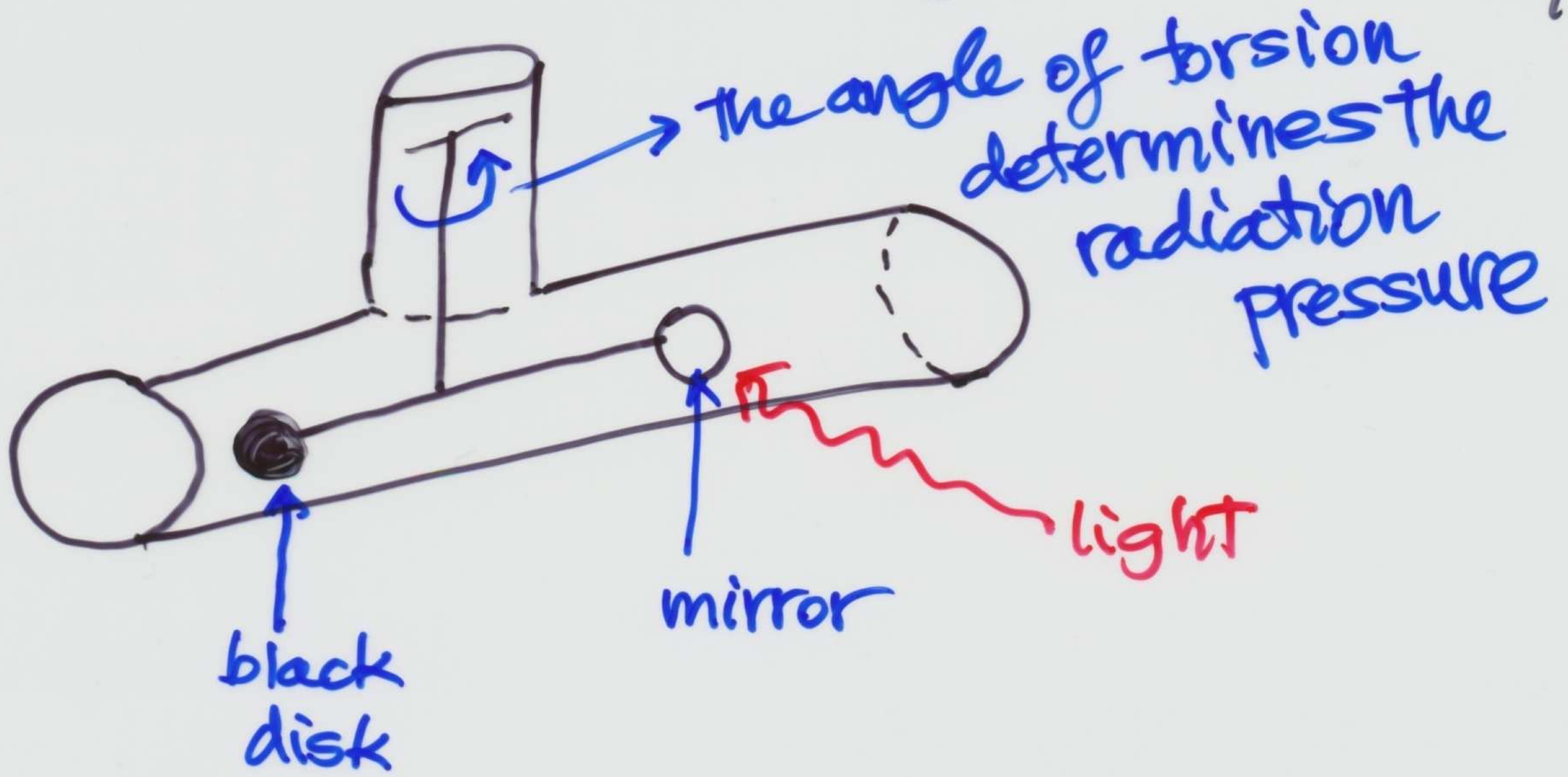
for complete absorption (black body)

pressure → P

$$\boxed{P = \frac{2S}{c}}$$

for complete reflection

Quiz: Consider apparatus for measuring radiation pressure which is small even for direct sunlight ~ 5×10^{-6} N/m^2



What happens if we replace the black disk with a radius R by a disk with a radius 2R? Which quantity remains the same?

1 ~~(a)~~ radiation pressure on the disk

3 (b) radiation force on the disk

6 (c) radiation momentum delivered to the disk in a time interval Δt

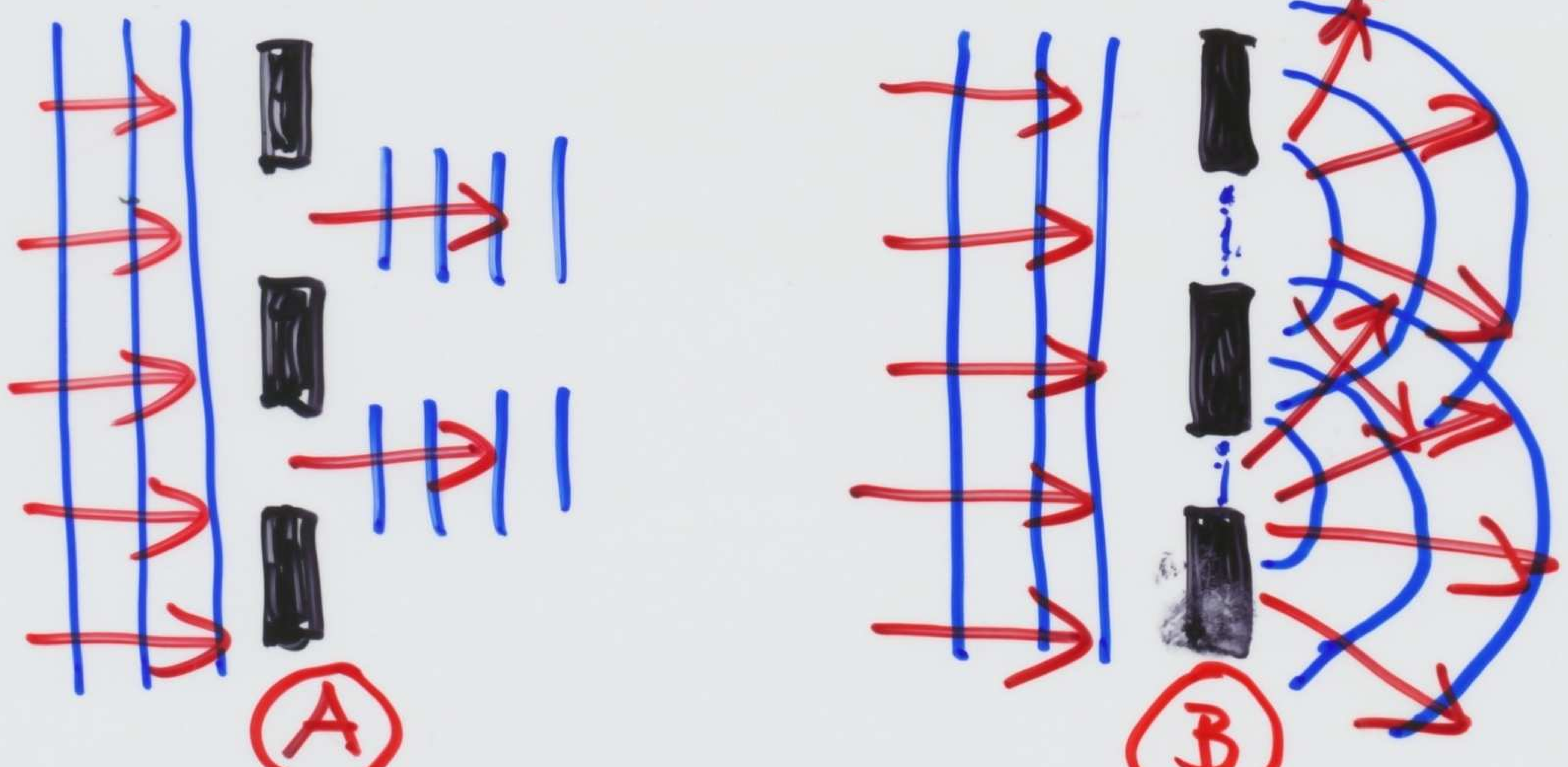
Ch. 27: Wave Optics

Wave optics = diffraction & interference
wave nature of light & principle of superposition
(addition of $\vec{E}$ & $\vec{B}$ in the EM wave)

Conditions for interference

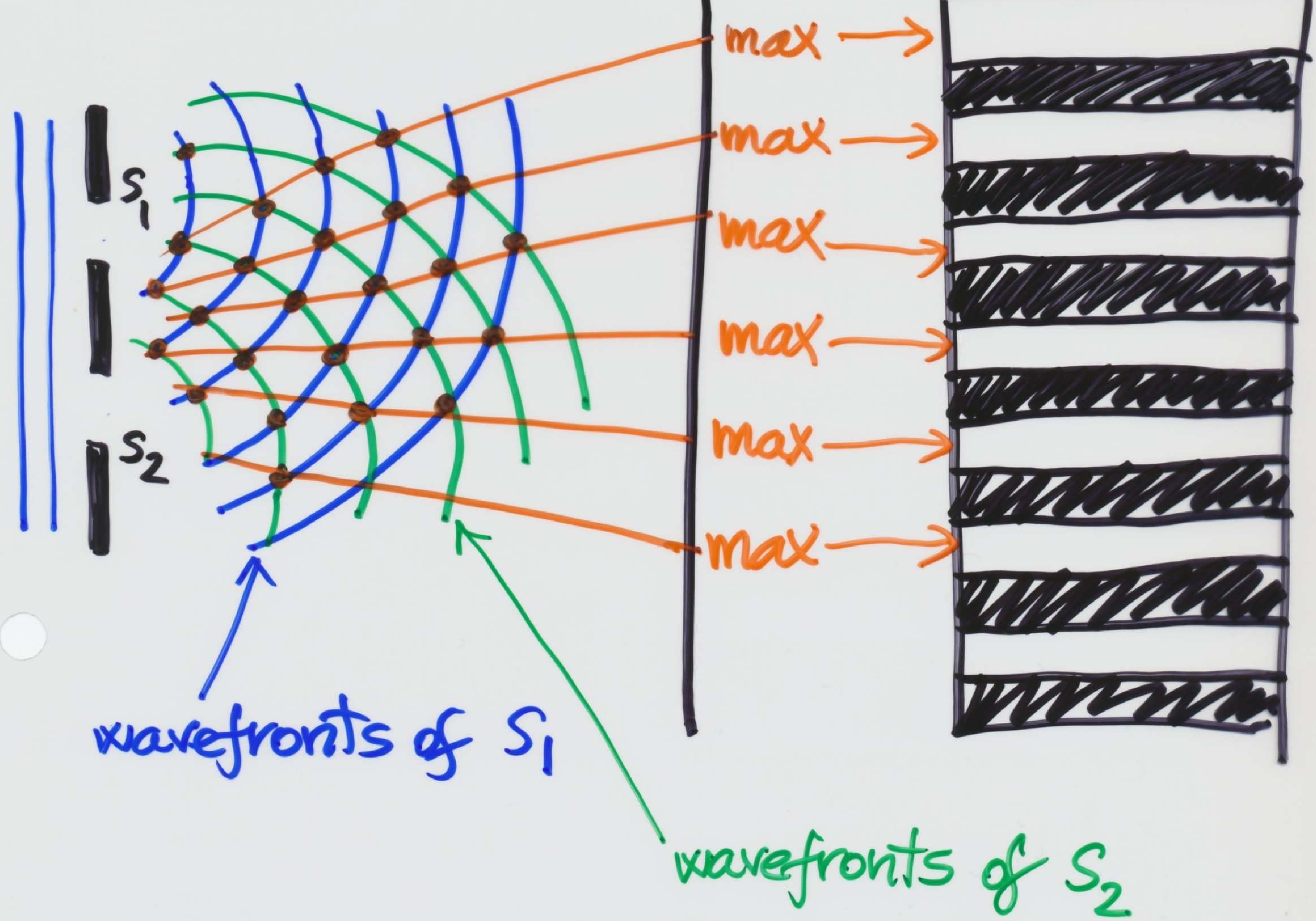
→ at least 2 sources of waves with IDENTICAL wavelengths
→ individual waves must maintain A CONSTANT PHASE relationship
≡ COHERENT WAVES
[ordinary light sources → random changes in $\Delta t \sim 10^{-8}$ s ⇒ INCOHERENT]

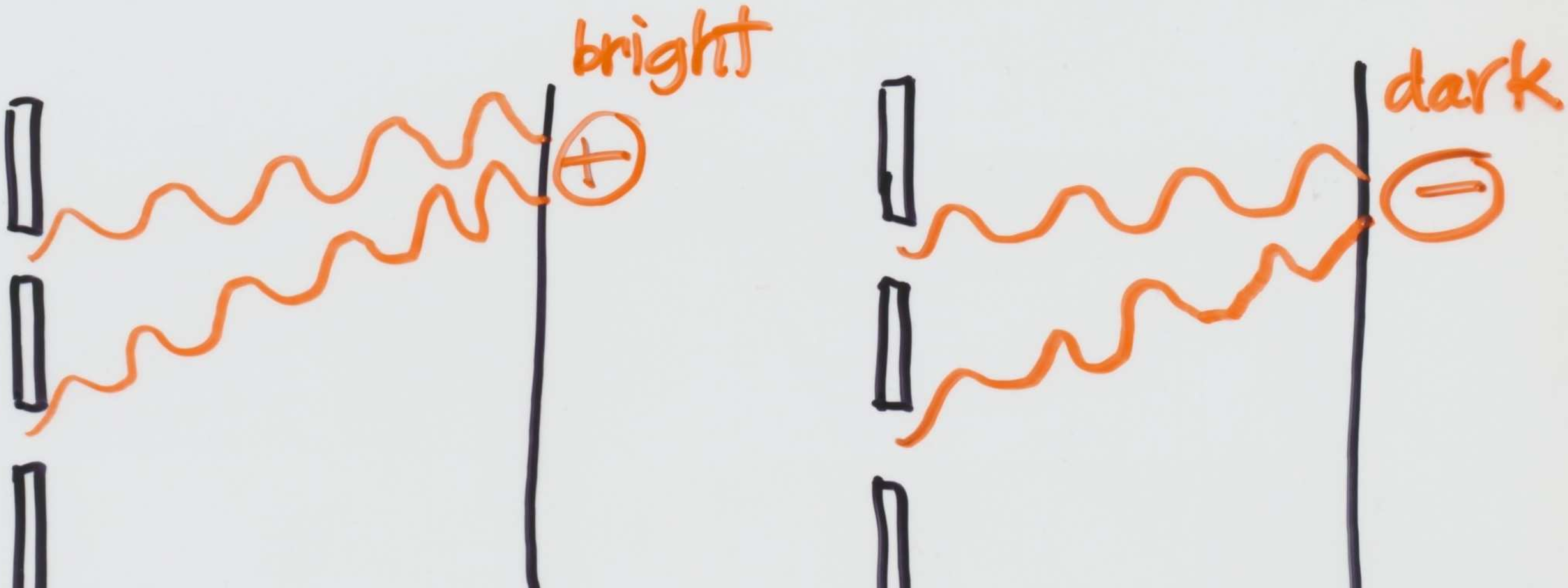
Young's double-slit experiment



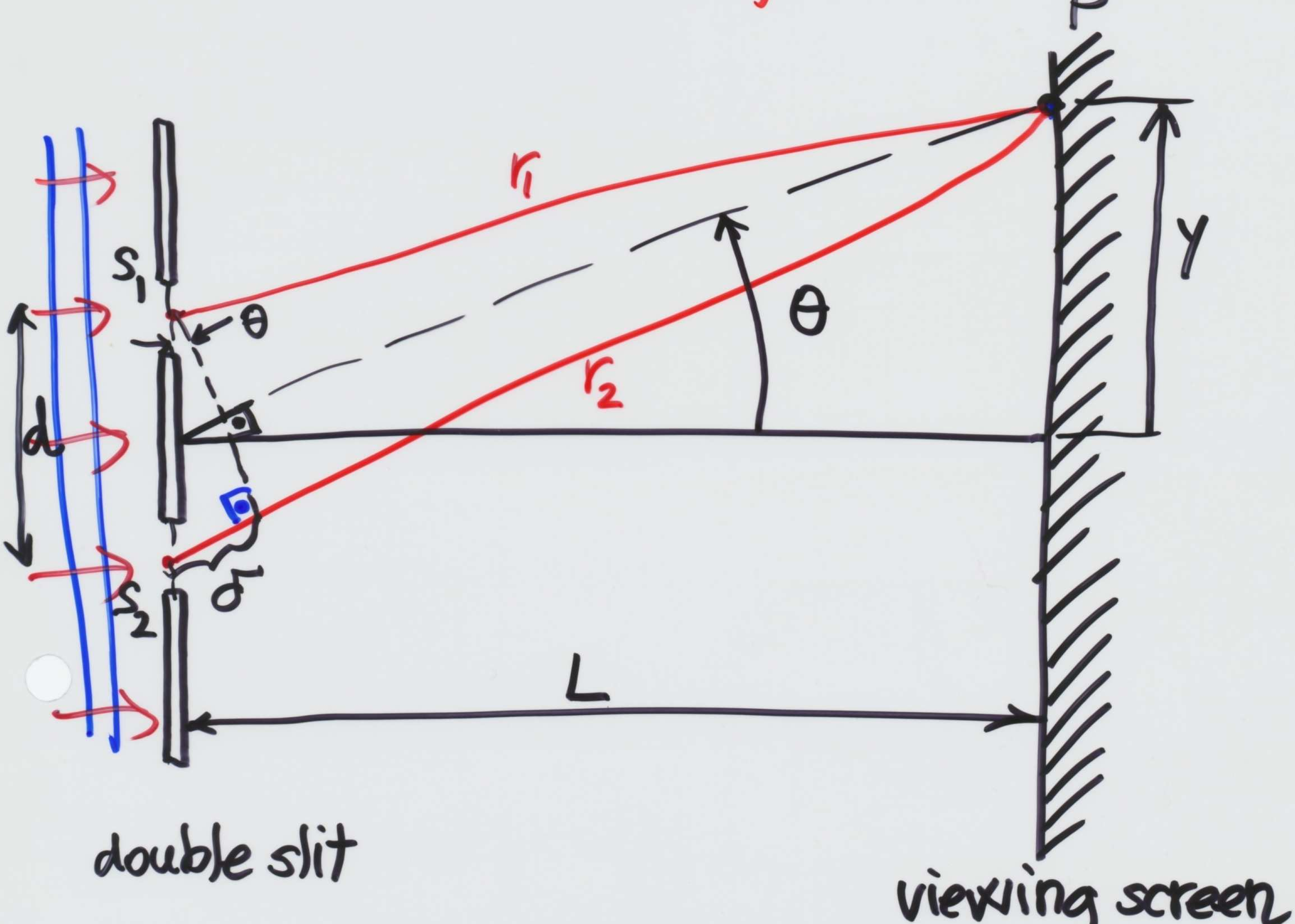
Ⓐ NO diffraction
Ⓑ diffraction

diffraction ⇒ interference ⇒ bright & dark parallel bands (= FRINGES)





Conditions for constructive & destructive interference



d .... distance between the slits

L ... distance between the slits & screen

r_1 & r_2 .... paths of waves from S_1 & S_2

$\delta = r_2 - r_1$ ... path difference

$L >> d \Rightarrow$ paths are parallel

$$\delta = r_2 - r_1 = d \sin\theta$$

→ constructive interference

$$d \cdot \sin\theta_{bright} = m\lambda$$

$$m = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$$

→ destructive interference

$$d \cdot \sin\theta_{dark} = (m + \tfrac{1}{2})\lambda$$

$$m = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$$

→ displacement y on the screen

$$y = L \tan\theta$$

→ small angles : $\sin\theta \approx \tan\theta \approx \theta$

$$y_{bright} \approx L \frac{m\lambda}{d}$$

$$m = \{0, \pm 1, \pm 2, \ldots\}$$

Quiz: Which of the following will result in narrower fringes closer together?

(a) increasing the wavelength of light λ

(b) increasing the screen distance L

(c) increasing the slit spacing d

Example: laser light (coherent)

$d = 0.030$ mm
$L = 1.2$ m

$\left.\begin{matrix} \\ \\ \end{matrix}\right\} d \ll L$

2^{nd} order fringe $(m=2)$
$y_2 = 5.1$ cm

(a) $\lambda = ?$

(b) $y_1 - y_0 = ?$

(a) $y_2 \doteq L\left(\frac{2\lambda}{d}\right)$

$\Rightarrow \lambda = \frac{1}{2}\frac{d\,y_2}{L}$

$\approx \underline{640\text{ nm}}$

(b) $y_2 - y_0 = 5.1$ cm

$y_1 - y_0 \approx \frac{1}{2}(y_2 - y_0)$

$\approx \underline{2.6\text{ cm}}$