Ch. 14 (14.1 – 14.5 & 13.7: Sound Waves)
Superposition & Standing Waves

The principle of superposition

two traveling waves in a medium combine

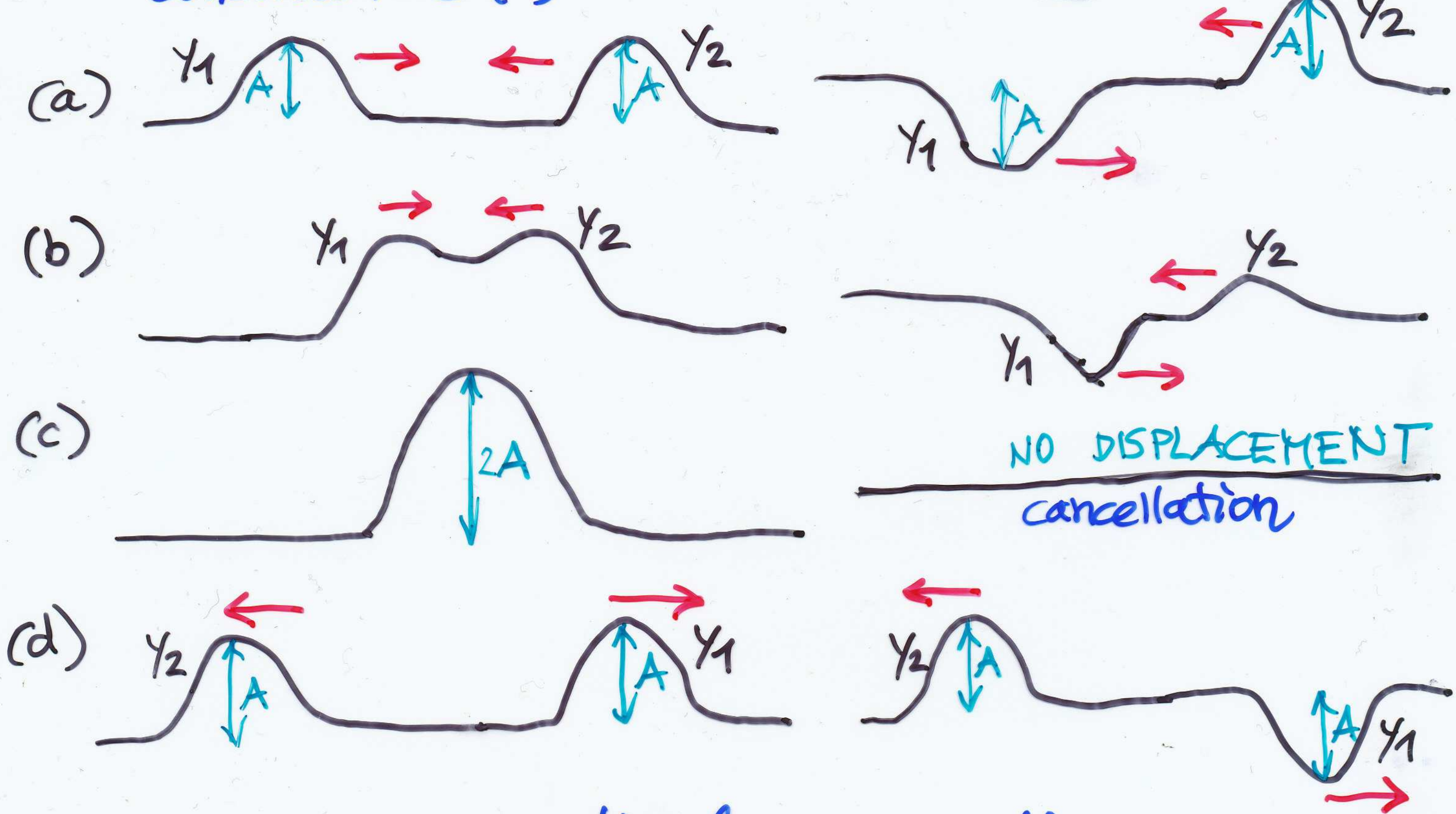
⇓

the position of any element in the medium is THE SUM of positions due to individual waves

INTERFERENCE

constructive (+)

destructive (−)



what is a mathematical description of interference?

Interference of Waves

Two sinusoidal waves with same amplitude & wavelength traveling in the same direction in a medium:

$$y_1 = A\cdot\sin(kx-\omega t) \;\&\; y_2 = A\cdot\sin(kx-\omega t+\phi)$$

According to the principle of superposition:

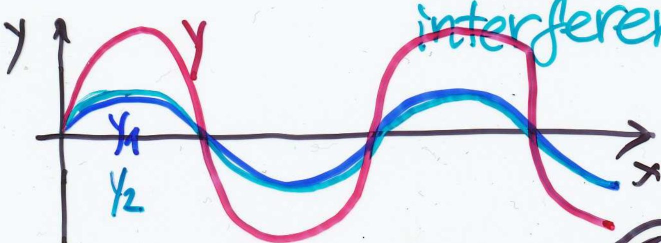
$$y = y_1 + y_2 = A\left[\sin(kx-\omega t) + \sin(kx-\omega t+\phi)\right] =$$

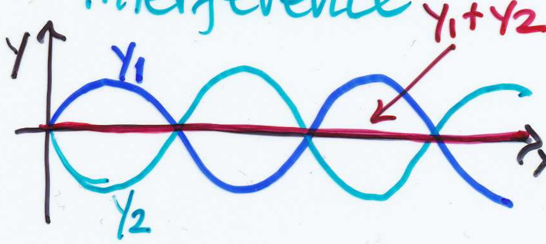
$$= \underbrace{2A\cos\frac{\phi}{2}}_{\text{amplitude}}\ \underbrace{\sin\left(kx-\omega t+\frac{\phi}{2}\right)}_{\text{sinusoidal wave with the same }\lambda}$$

$\sin a + \sin b =$
$= 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$

$\phi = 0, 2\pi, \ldots$
max amplitude $2A$
constructive interference

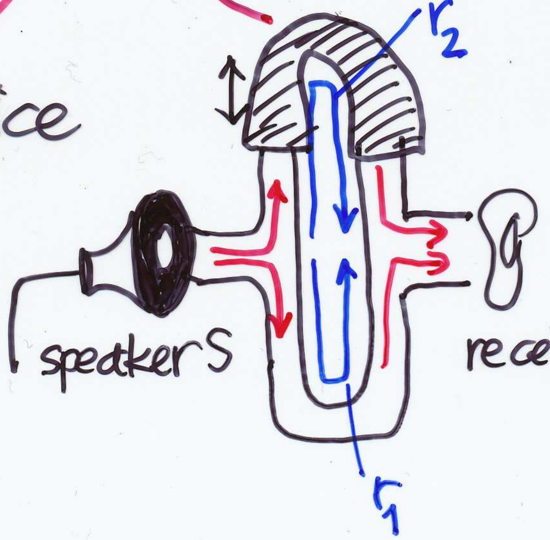
$\phi = \pi, 3\pi, 5\pi, \ldots$
amplitude zero
destructive interference





A simple device

$$\Delta r = \frac{\phi}{2\pi}\lambda$$



path lengths r_1 & r_2

→ constructive
$|r_1 - r_2| = 0, \lambda, 2\lambda, \ldots$

→ destructive
$|r_1 - r_2| = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda \ldots$

Quiz: Two waves interfere destructively. Which statement is correct?

(a) The energy of two initial waves is lost.
1

(b) The amplitude of the resulting wave is smaller than the amplitudes of the two initial waves.
18

(c) The amplitude of the resulting wave is greater than the amplitude of the initial wave.
0

(d) Interference is associated with an energy gain.
0

Standing Waves

Two transverse sinusoidal waves with the same amplitude, frequency, & wavelength travel in OPPOSITE directions in a medium:

$y_1 = A\sin(kx-\omega t)$ & $y_2 = A\cdot\sin(kx+\omega t)$

L → R   L ← R

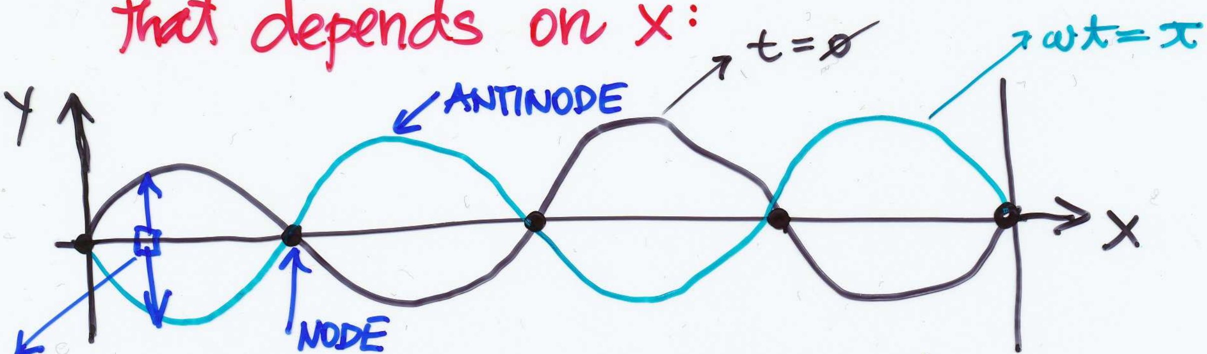
According to superposition principle:

$$y = y_1 + y_2 = A\cdot[\sin(kx-\omega t) + \sin(kx+\omega t)] =$$

$$= \underline{2A\sin(kx)\cdot\cos(\omega t)}$$

The result is NOT a traveling wave because it lacks the form $f(kx-\omega t)$ or $g(x-vt)$.

⇒ SIMPLE HARMONIC MOTION with amplitude that depends on x:



Every element of the medium vibrates in simple harmonic motion with the same angular freq. ω

The maximum amplitude (ANTINODE):

$$\sin(kx) = 1 \Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

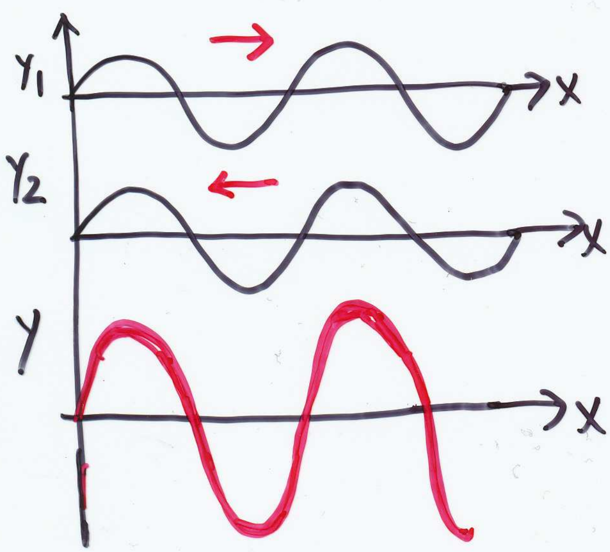
$$x = \frac{1}{4}\lambda, \frac{3}{4}\lambda, \frac{5}{4}\lambda, \ldots$$

The minimal amplitude (NODE):

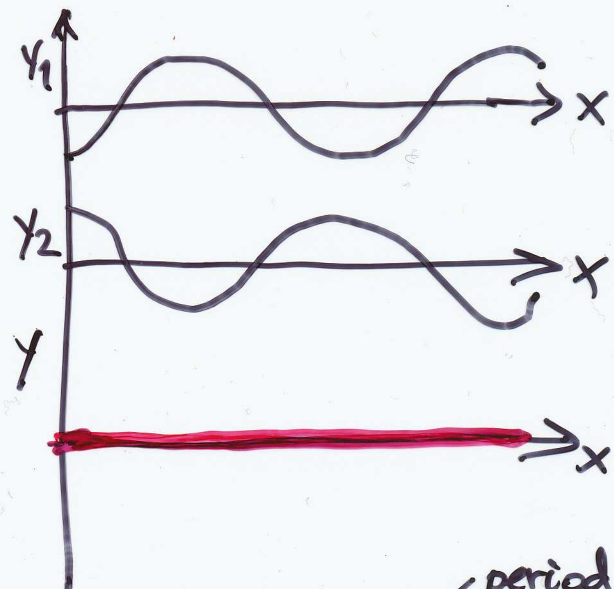
$$\sin(kx) = 0 \Rightarrow kx = 0, \pi, 2\pi, \ldots$$

$$x = \tfrac{1}{2}\lambda, \lambda, \tfrac{3}{2}\lambda, \ldots$$

Adjacent nodes are separated by $\frac{1}{2}\lambda$ & adjacent antinodes — " — $\frac{1}{2}\lambda$.



(A) $t = 0$



(B) $t = \frac{1}{4}T$ ← period $T = \frac{1}{f} = \frac{2\pi}{\omega}$

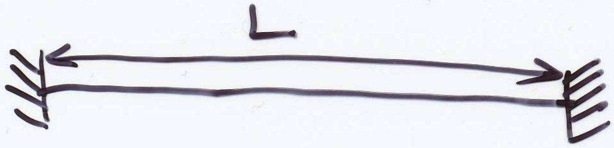
Quiz: If an element of the string is moving up, its velocity is positive (if down ⇒ negative). For which of the two figures (A) and (B) the instantaneous velocity of all elements on the string is equal to zero?

3 ~~(A)~~

(B)

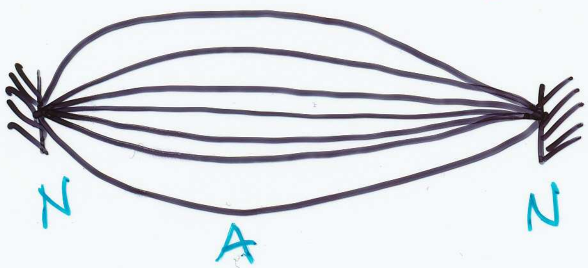
Standing waves in strings

A string stretched between two rigid supports ⇒ incoming wave (L → R) will interfere with the reflected wave (R → L) to form a standing wave:



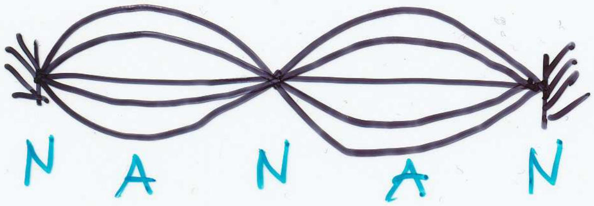
model for all string instruments

Normal modes of a string: $\lambda_n = \frac{2L}{n}$



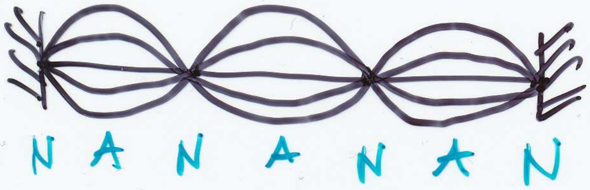
$n=1, \ L = \frac{1}{2}\lambda_1 \Rightarrow \lambda_1 = 2L$

2 nodes at the two ends
1 antinode at the center



$n=2, \ L = \lambda_2 \Rightarrow \lambda_2 = L$

3 nodes & 2 antinodes



$n=3, \ L = \frac{3}{2}\lambda_3 \Rightarrow \lambda_3 = \frac{2}{3}L$

Wave lengths are discrete/quantified and determined by the length L of the string & boundary conditions.

Frequency of normal modes:

$$\boxed{f_n = \frac{v}{\lambda_n} = \frac{n}{2L} v = \frac{n}{2L}\sqrt{\frac{T}{\mu}}}$$

v only depends on T & μ of the string

Fundamental frequency f_1:

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Frequencies of higher modes are INTEGRAL MULTIPLES of the fundamental frequency f_1:

$$f_n = n f_1 \quad (2f_1,\ 3f_1,\ 4f_1,\ \ldots)$$

harmonics!

f_1 .... first harmonic

$f_2 = 2f_1$ ... second harmonic ....

Quiz: A standing wave is set up on the string which is fixed at both ends. Which statement is true?

(a) The center of the string is always an
0 antinode.

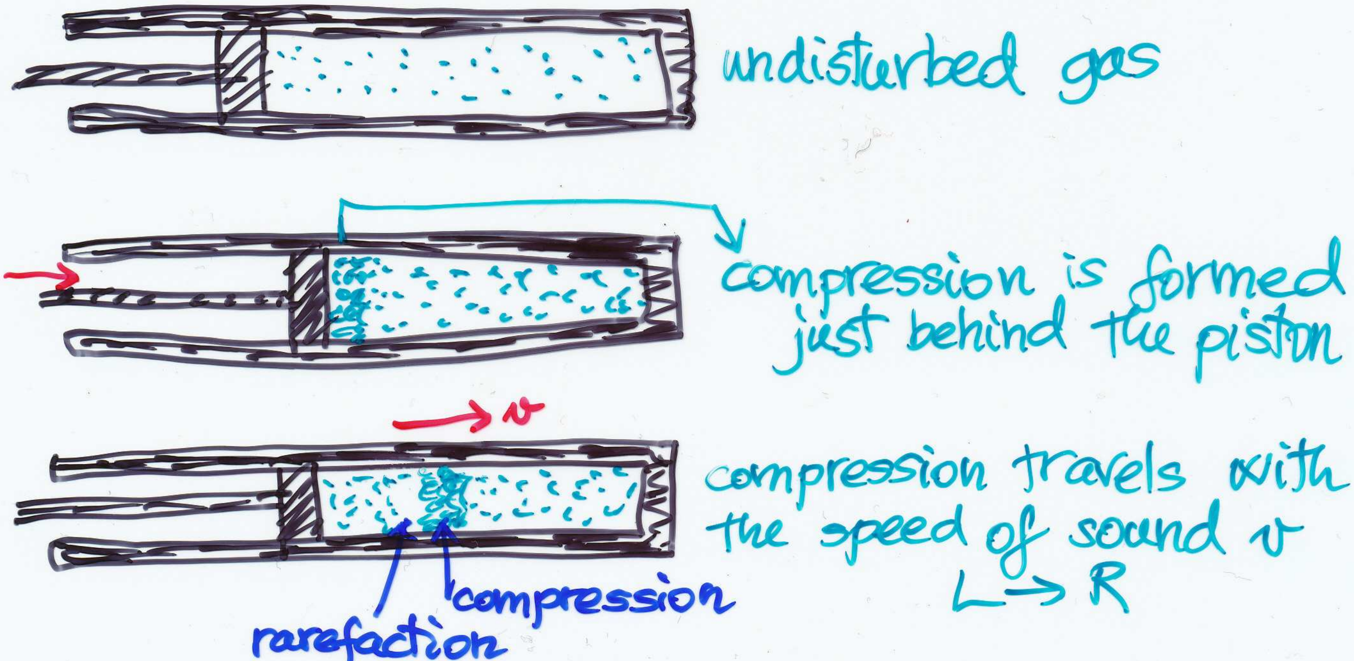
(b) The number of nodes is equal to the
12 number of antinodes plus 1.

0 (c) The wavelength λ is equal to the length L divided by n, an integer.

(d) The number of nodes is equal to the
1 number of antinodes.

Sound Waves (13.7)

Example of longitudinal waves $\Rightarrow$ elements of medium undergo displacements parallel to the direction of wave motion.



Mathematical description:

$s(x,t)$ .... a position of a small element of air (medium)

$\Delta P(x,t)$ .... a variation in pressure (compression...)

$$s(x,t) = \underbrace{s_{max}}_{\text{displacement amplitude}} \cdot \sin(kx - \omega t)$$

$$\Delta P(x,t) = \underbrace{\Delta P_{max}}_{\text{maximum change in pressure}} \cdot \cos(kx - \omega t)$$

ΔP_{max} & s_{max} are related:

$$\Delta P_{max} = \rho v \omega s_{max}$$

Pressure variations in a sound wave $\Rightarrow$ oscillating force on the eardrum $\Rightarrow$ HEARING

Pressure wave is 90° out of phase with the displacement wave [$90° = \frac{\pi}{2}$]: displacement is zero when $\Delta P = max$ and $\Delta P = 0$ when $s = s_{max}$.

The speed of a sound wave in air depends on air temperature:

$$v = 331\ m/s + (0.6\ m/s \cdot °C) \cdot T[°C]$$

$331\ ms^{-1}$ ... speed of sound at $T = 0°C$

Quiz: The wavelength of the sound is reduced by a factor of 2, $\lambda_N = \frac{1}{2}\lambda$. What happens to the frequency f and speed v ?

(a) $f_N = f$ and $v_N = \frac{1}{2}v$ because $v_N = f_N \cdot \lambda_N$ such that $\frac{1}{2}v = f\frac{1}{2}\lambda$
$\Rightarrow v = f \cdot \lambda$

(b) $f_N = \frac{1}{2}f$ and $v_N = v$

(c) $f_N = 2f$ and $v_N = v$

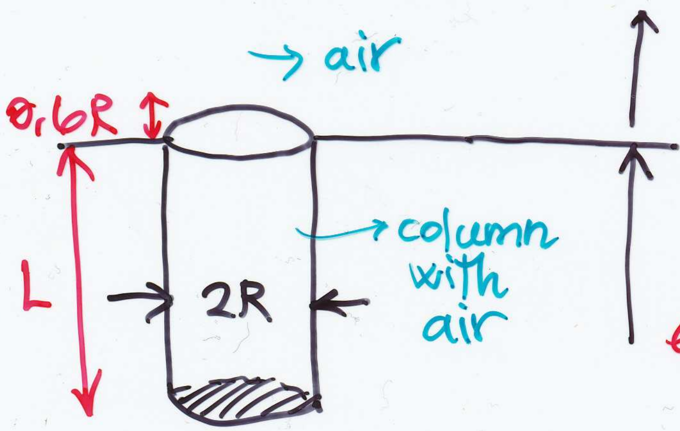
Standing Waves in Air Columns

compressions ⇒ pressure antinodes

Instruments like brasses or woodwinds produce music using a column of air as a result of interference between two longitudinal sound wave traveling in opposite directions.

→ The closed end of the column corresponds to a displacement node and a pressure anti-node.

→ The open end of the column corresponds to a displacement antinode & a pressure node.



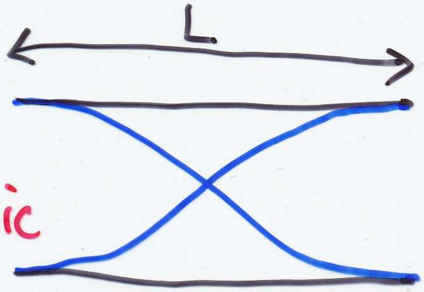
change in the character of the medium

end correction

$$\underbrace{L}_{\text{physical length}} \to \underbrace{L + 0.6R}_{\text{acoustical length}}$$

Air column open at both ends

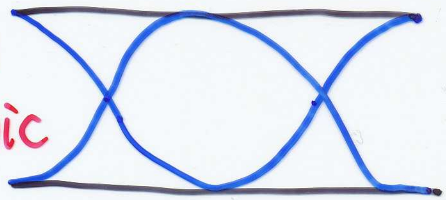
first harmonic



$L = \frac{1}{2}\lambda_1 \Rightarrow \lambda_1 = 2L$

$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$

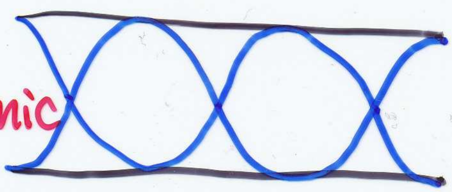
second harmonic



$L = \lambda_2 \Rightarrow \lambda_2 = L$

$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2f_1$

third harmonic



$L = \frac{3}{2}\lambda_3 \Rightarrow \lambda_3 = \frac{2}{3}L$

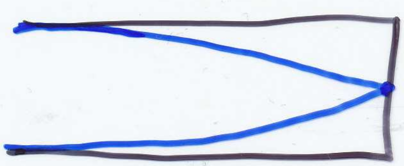
$f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3f_1$

$\boxed{f_n = n\frac{v}{2L}}$

$n = 1, 2, 3, 4, \ldots$

Air column closed at one end

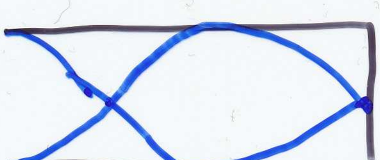
first



$L = \frac{1}{4}\lambda_1 \Rightarrow \lambda_1 = 4L$

$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$

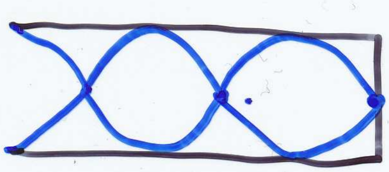
third



$L = \frac{3}{4}\lambda_3 \Rightarrow \lambda_3 = \frac{4}{3}L$

$f_3 = \frac{v}{\lambda_3} = \frac{3v}{4L} = 3f_1$

fifth



$L = \frac{5}{4}\lambda_5 \Rightarrow \lambda_5 = \frac{4}{5}L$

$f_5 = 5f_1$

$\boxed{f_n = n\frac{v}{4L}}$

$n = 1, 3, 5, 7, \ldots$

Quiz: Standing waves in a pipe are excited at a fundamental frequency f, at a room temperature. Someone heats up the room with a pipe. What happens with the frequency f? Consider that $v = \lambda \cdot f$ (v is the speed of sound)

(a) f increases (the pipe goes sharp)

(b) f decreases (the pipe goes flat)

Quiz: The pipe is open at both ends and is excited at a fundamental frequency f_{open}. Someone closes the pipe at one end, and the pipe is excited at a fundamental frequency f_{closed}. Which is correct?

(a) $f_{closed} = f_{open}$

(b) $f_{closed} = \frac{1}{2} f_{open}$

(c) $f_{closed} = 2 f_{open}$

Ch. 24: Electromagnetic Waves

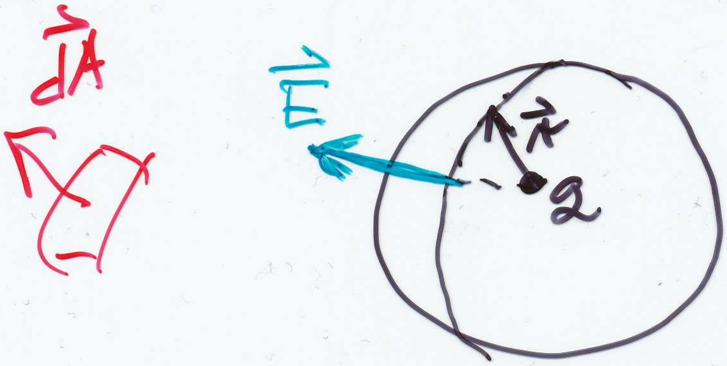
→ visible light
→ infrared waves from Earth's surface
→ microwaves
→ radio-frequency waves
⇒ prediction of Maxwell's equations

Maxwell's Equations (4 equations & Lorentz force law)

(1) Gauss Law for electricity

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$ integral around CLOSED surface

simple example: spherical surface A around a point charge
⇒ radial electric field E



$$E \cdot 4\pi r^2 = q/\varepsilon_0$$

$$\boxed{E(r) = \frac{q}{4\pi r^2 \varepsilon_0}}$$

(2) Gauss law for magnetism

$$\oint \vec{B} \cdot d\vec{A} = 0$$ there is NO magnetic "charges" or monopoles

flux of magnetic field density through a CLOSED surface

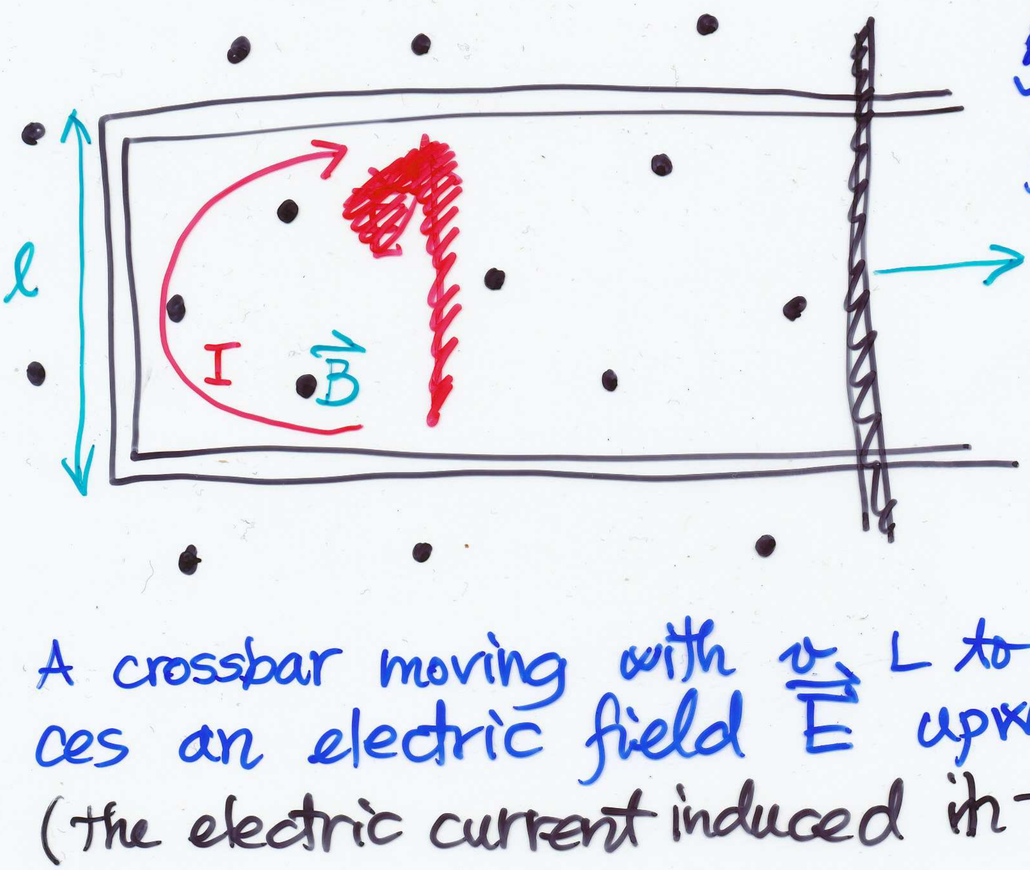
(3) Faraday's law of induction

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

line integral of $\vec{E}$ around a CLOSED path

time derivative of the magnetic flux Φ_B through the surface (ANY) enclosed by the path

Example: consider homogeneous magnetic field $\vec{B}$ pointing out of the transparency & a U-shape wire with a movable crossbar $\perp$ to $\vec{B}$



$$\underbrace{\ell E}_{\oint \vec{E}\cdot d\vec{s}} = \underbrace{B\ell v}_{\frac{d\Phi_B}{dt}}$$

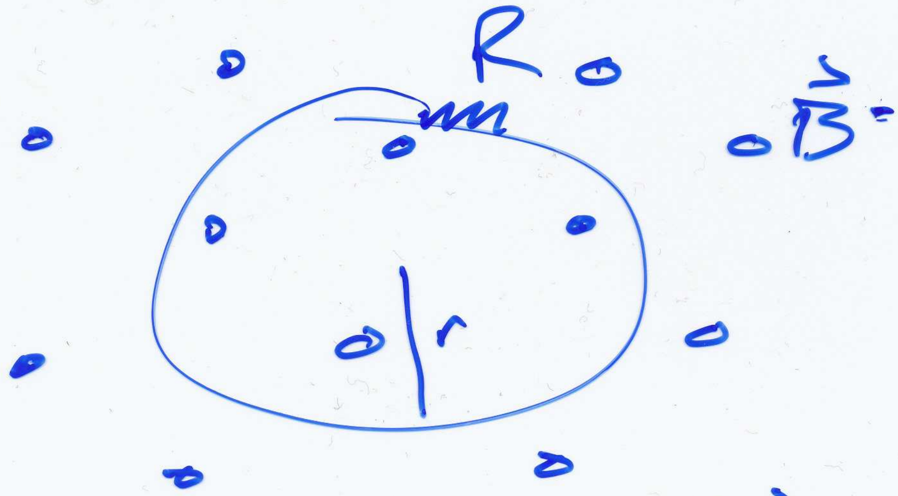
$$\Downarrow$$

$$I = \frac{B\ell v}{R}$$

$$\left(E = \frac{V}{\ell} = \frac{RI}{\ell}\right)$$

A crossbar moving with $\vec{v}$ L to R, produces an electric field $\vec{E}$ upwards (the electric current induced in the wire)

$$\oint \vec{E} \cdot \vec{ds} = -\frac{\partial \Phi_B}{\partial t}$$



$$\Phi_B = \oint \vec{B} \cdot \vec{dA} = B \oint d\vec{A} = B \pi r^2$$

$$IR = \mathcal{E} = \oint \vec{E} \cdot ds = -\frac{\partial \Phi_B}{\partial t} = -\pi r^2 \frac{\partial B}{\partial t}$$

$$I = \frac{-\pi r^2 \frac{\partial B}{\partial t}}{R}$$

(4) Generalized Ampere's law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \underbrace{\frac{d\Phi_E}{dt}} + \mu_0 I$$

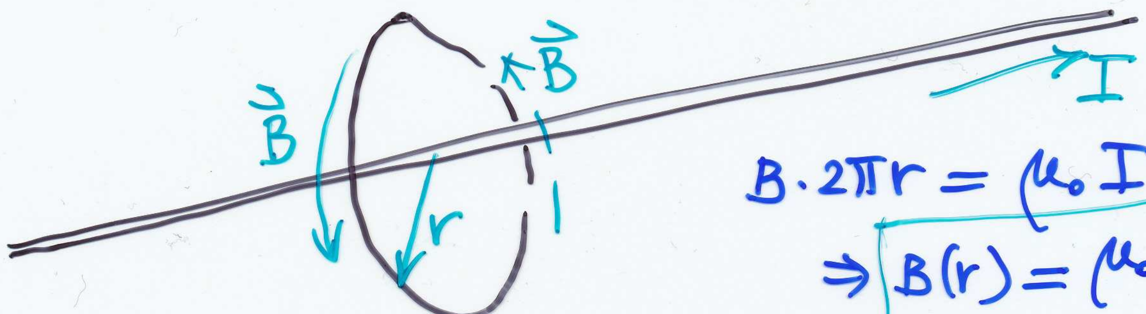
line integral of $\vec{B}$ around a closed path

displacement current through ANY surface enclosed by the path

current through ANY surface enclosed by the path

Φ_E ... electric flux

Example (without the displacement current): magnetic field outside a LONG wire with electric current I



$$B \cdot 2\pi r = \mu_0 I$$

$$\Rightarrow \boxed{B(r) = \mu_0 \frac{I}{2\pi r}}$$

Electric current through the wire produces circular magnetic field in the plane perpendicular to the wire.

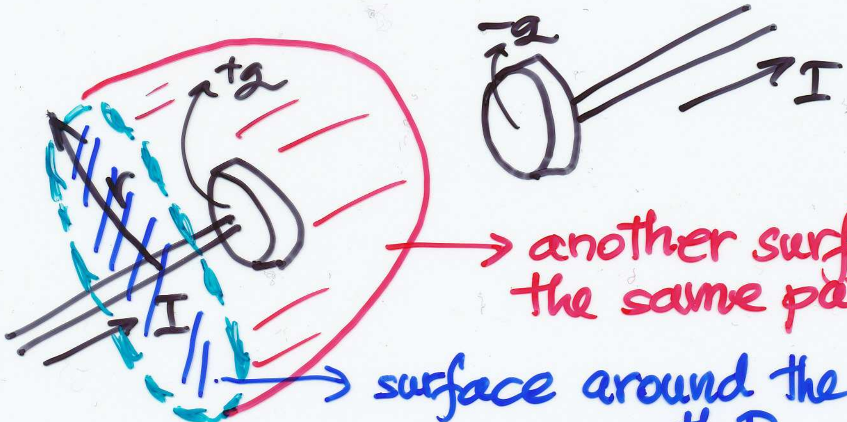
Example with the DISPLACEMENT current

$$\boxed{I_d = \varepsilon_0 \frac{d\Phi_E}{dt}}$$

What is the displacement current?

Consider a charging capacitor

– – – – path $d\vec{s} \to P$



another surface around the same path P, S_2

surface around the enclosed path P, S_1

(1) choose S_1

$$\oint \vec{B} \cdot d\vec{s} = B \cdot 2\pi r = \mu_0 I + \mu_0 I_d$$

$I_d = 0$ (no displacement current)

(2) choose S_2

$$\oint \vec{B} \cdot d\vec{s} = B \cdot 2\pi r = \mu_0 I_d + \mu_0 I$$

($I = 0 \Rightarrow$ no current between the two capacitor plates)

The generalized Ampère's law states that (1) & (2) give the same result.

$\Downarrow$

Calculate the displacement current I_d :

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt} \qquad \phi_E = \int_{S_2} \vec{E} \cdot d\vec{A} \Rightarrow$$

$\uparrow$ homogeneous electric field

$$\Rightarrow \phi_E = E \cdot A = \frac{q}{\varepsilon_0}$$

$\Downarrow$

$$\underline{\underline{I_d}} = \varepsilon_0 \frac{d\phi_E}{dt} = \frac{dq}{dt} \Rightarrow \underline{\underline{\frac{dq}{dt} = I}}$$

We showed that in this example the displacement current is the same as the current through the wire.

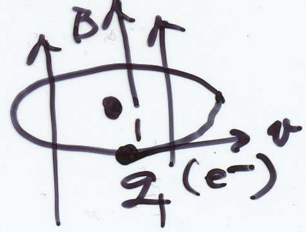
The Ampère's law in generalized form states that magnetic fields are produced by conduction & changing electric field currents.

(5) Lorentz force law on a particle of charge q

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

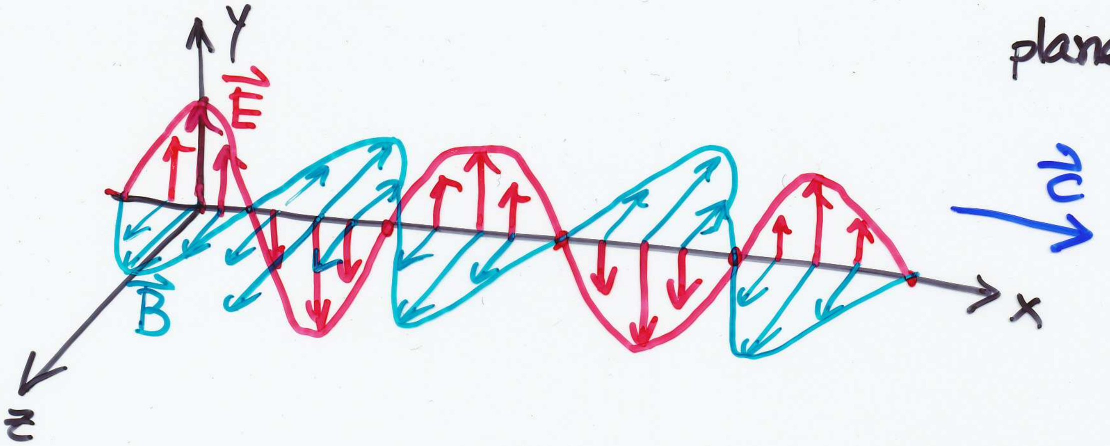
$\swarrow$ $\vec{B} = 0, \vec{E} \neq 0$ $\qquad$ $\searrow$ $\vec{E} = 0, \vec{B} \neq 0$

accelerated linear motion



Electromagnetic waves

plane wave

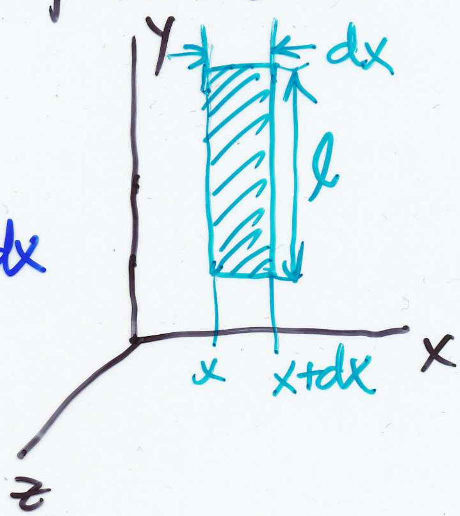


Linearly polarized waves : EM waves propagate in the direction of $\vec{E}\times\vec{B}$. $\vec{E}$ & $\vec{B}$ are equivalent of a displacement on a string of a transverse mechanical wave.

EM waves CAN propagate in vacuum (do not need any medium): no charges q & no currents I.

Consider Faraday's & Ampere's laws

(1) $\oint \vec{E}\cdot d\vec{s} = -\dfrac{d\Phi_B}{dt}$



L: $E(x+dx) = E(x) + \dfrac{dE}{dx}\cdot dx$

$\oint \vec{E}\cdot d\vec{s} = E(x+dx,t)\,\ell - $

$-E(x,t)\,\ell \approx \ell\,\dfrac{\partial E}{\partial x}\,dx$

R: $\dfrac{d\Phi_B}{dt} = \ell\cdot dx\,\dfrac{dB}{dt}\Big|_{x=const} = \ell\,\dfrac{\partial B}{\partial t}\cdot dx$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$ $$\frac{\partial^2 E}{\partial t \partial x} = -\frac{\partial^2 B}{\partial t^2}$$ $$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t}$$

(2) $\oint \vec{B} \cdot d\vec{s} = B(x,t)\,l - B(x+dx,t)\,l \approx$

$$\left.\begin{aligned} &\approx -l \frac{\partial B}{\partial x} dx \\ &\frac{d\Phi_E}{dt} = l \frac{\partial E}{\partial t} dx \end{aligned}\right\} \Rightarrow$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\frac{\partial B}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$$

(1) & (2) combined

$$\frac{\partial^2 B}{\partial x^2} = -\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial x \partial t}$$

$$\frac{\partial^2 B}{\partial t \partial x} = -\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\begin{aligned} \frac{\partial^2 B}{\partial x^2} &= \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \\ \frac{\partial^2 E}{\partial x^2} &= \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \end{aligned}}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

speed of ligth

These are wave equations with solutions:

$$E = E_{max} \cos(kx - \omega t) = E_{max} \cos[k(x - ct)]$$

$$B = B_{max} \cos(kx - \omega t) = B_{max} \cos[k(x - ct)]$$

$$k = \frac{2\pi}{\lambda} \quad \& \quad \omega = 2\pi f \quad \& \quad c = \lambda \cdot f$$

Important relationship :

$$\frac{E}{B} = c \quad \text{OR} \quad \frac{E_{max}}{B_{max}} = c$$

$c = 2.997... \times 10^8$ m/s

$$\frac{\partial E}{\partial x} = -kE_{max}\sin(k(x-ct))$$

$$\frac{\partial B}{\partial t} = +k B_{max} c \sin(k(x-ct))$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

Doppler Effect for Light

Because EM waves do not require any medium for propagation, ONLY relative speed between the source and observer is relevant to the detected frequency f' relative to f at rest:

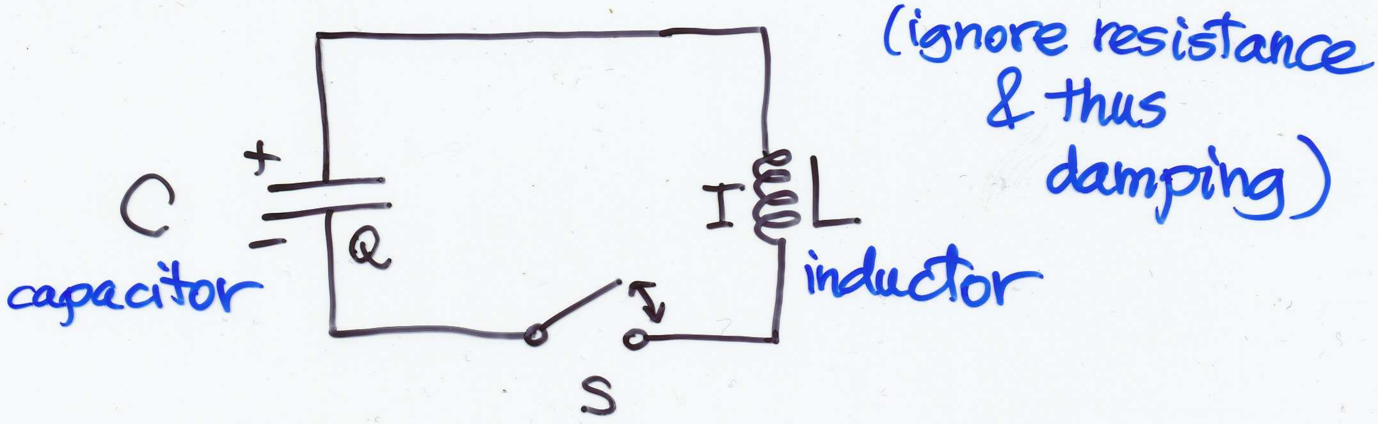
(a) $$f' = f\sqrt{\frac{c+v}{c-v}}$$ source & observer approaching

(b) $$f' = f\sqrt{\frac{c-v}{c+v}}$$ source & observer moving away from each other

$$\Rightarrow \lambda' = \lambda\sqrt{\frac{c+v}{c-v}} \Rightarrow \underline{\underline{\lambda' > \lambda}}$$

red shift effect for galaxies moving away from us

LC circuit as a simple harmonic oscillator



→ initial charge on capacitor Q_{max}

→ switch S is closed at $t=0$

⇓

total energy of the LC circuit at $t=0$

$$E = \frac{Q_{max}^2}{2C}$$ (maximal electric potential energy)

→ at $t>0$, the capacitor discharges

⇒ current I through L ⇒ the maximal electric potential energy transforms eventually to maximal electric current ⇒ induction energy

$$E = \frac{1}{2} L I_{max}^2$$

→ the process then reverses & repeats

Apply Kirchhoff's loop rule: the sum of potential differences ΔV across each element around closed circuit loop is zero:

$$\frac{Q}{C} + L\frac{dI}{dt} = 0 \quad \& \quad I = \frac{dQ}{dt}$$

$$\Rightarrow \frac{d^2Q}{dt^2} = -\frac{1}{LC}Q \quad \left(\frac{d^2x}{dt^2} = -\omega^2 x\right)$$

$$\Rightarrow \quad \omega = \frac{1}{\sqrt{LC}}$$ resonance frequency of the LC circuit

Quiz: A capacitor in an LC circuit is discharging. Which statement is correct?

(a) There is an electric current between the plates of capacitor.

(b) There is a static charge on the inductor.

(c) There is a constant electric field between the plates of the capacitor.

(d) There is an electric field between the C plates, varying in time.