WELCOME TO PHYS 201

Winter Quarter '09

Brigita Urbanc

brigita@drexel.edu

www.physics.drexel.edu/~wking/course/index.shtml

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download syllabus

Recitations & Homework:

Tim Jones tdj28@drexel.edu

Trevor King wking@drexel.edu

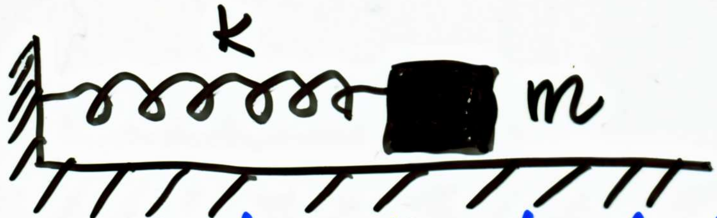
Labs:

Rachael Kratzer rmk55@...

Pradeesh Nair prn25@...

Oscillatory Motion

(1)

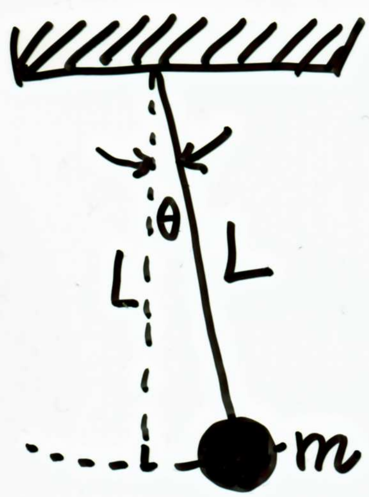


a particle attached to a string

↓

simple harmonic oscillator

(2)



a simple pendulum

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at small $\theta \ll$ "1"

$\Rightarrow$ simple harmonic oscillator

(3)

a physical pendulum

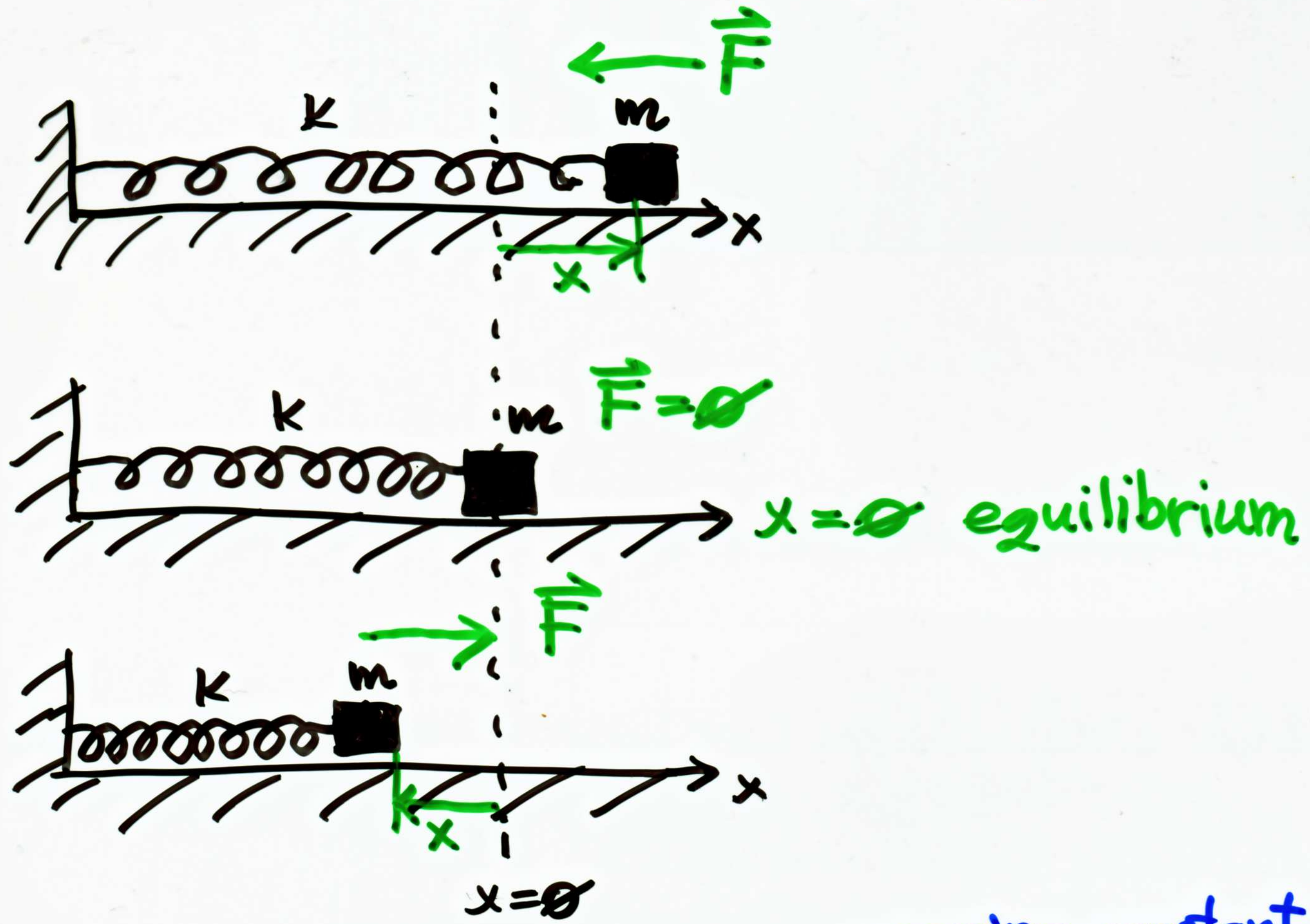
at small $\theta \Rightarrow$

simple harmonic motion

(4) damped oscillations

(5) forced oscillations & resonance

(1) A Particle Attached to a Spring



Hooke's law $F = -kx$ ← spring constant

linear restoring force

How do we mathematically describe the motion?

Apply the 2nd law of Newton

$$F = m \cdot a$$

$$-kx = m \cdot a \quad \& \quad a = d^2x/dt^2$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$

linear, 2^{nd} order differential eq., homogeneous

Solution:

$$\boxed{x(t) = A\cos(\omega t + \phi)}$$

A ... amplitude of oscillation

ϕ .... oscillation phase

$x(t)$ describes position of a particle in simple harmonic motion

$\boxed{\text{Check:}}$

$$\frac{dx(t)}{dt} = -A\cdot\omega\sin(\omega t + \phi)$$

$$\left.\begin{aligned}\frac{d^2x(t)}{dt^2} &= -A\omega^2\cos(\omega t + \phi)\\ \omega^2 x(t) &= +A\omega^2\cos(\omega t + \phi)\end{aligned}\right\} \oplus \Rightarrow 0$$

Angular frequency $\omega = \sqrt{\frac{k}{m}}$ [rad/s]

Frequency $f = \frac{\omega}{2\pi}$ [s^{-1}]

Period $T = \frac{2\pi}{\omega}$

(time needed for one full path of the oscillator)

2π angle (path)

ω ... angular velocity

Phase constant ϕ & amplitude A in

$$x(t) = A\cos(\omega t + \phi)$$

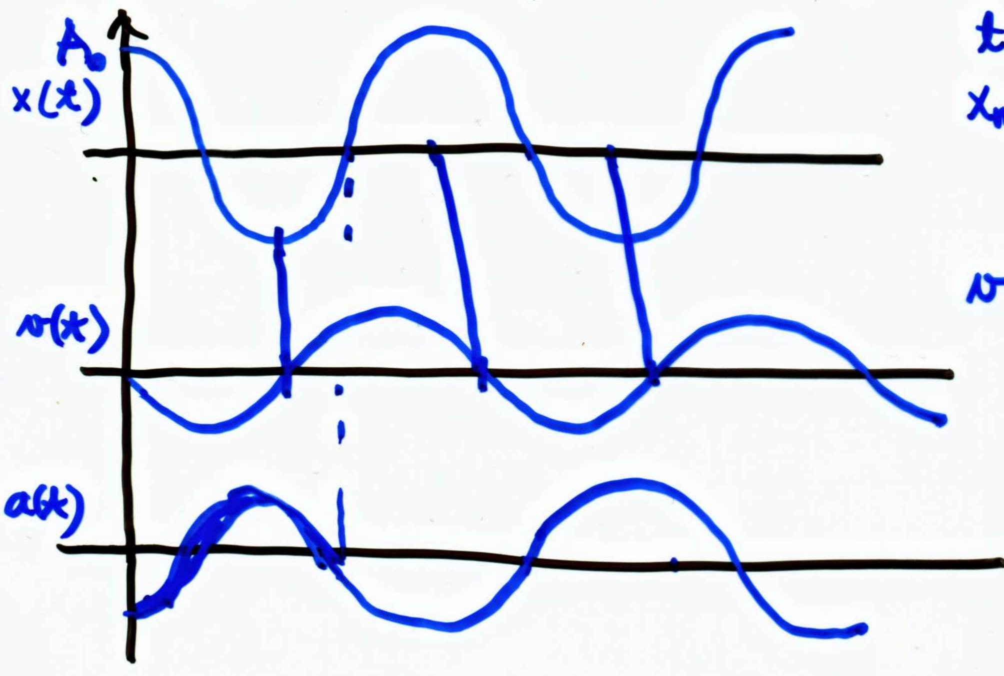
determined by INITIAL CONDITIONS

$$x(t=0) = A_0 \Rightarrow A_0 = A \cdot \cos\phi \Rightarrow \phi = 0, \; A = A_0$$

$$x(t) = A_0 \cos(\omega t)$$

$$v(t) = dx(t)/dt = -A_0 \sin(\omega t)\cdot\omega$$

$$a(t) = d^2x(t)/dt^2 = -A_0\cos(\omega t)\cdot\omega^2$$



$t=0$, $t=\frac{1}{4}T$

$x_{max}=A_0$, $x=0$

$v=0$, $v=-A_0\omega$

$a=-\omega^2A_0$, $a=0$

Quiz: A block on the end of the spring is pulled to position $x = A$ and released. When is the speed of the block the largest?

11 (a) at $x = 0$ (equilibrium)

1 (b) at $x = -A$ (the left end)

8 (c) at $x = A$ (the right end)

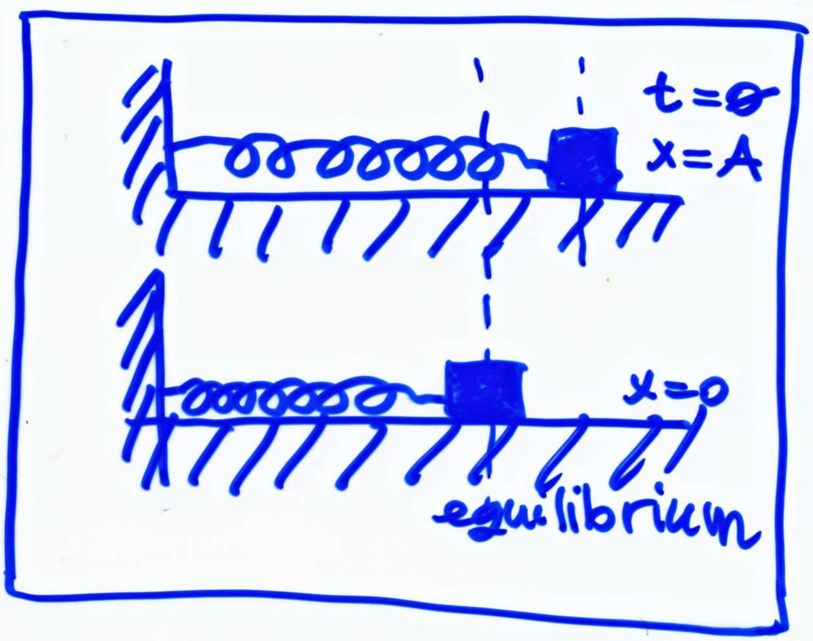
Quiz: What is the total distance the block travels during one full cycle of motion?

0 (a) A

11 (b) 2A

2 (c) 3A

15 (d) 4A



Example: A particle oscillates as a simple harmonic oscillator along x-axis according to the equation

$$x(t) = (0.05\,m) \cdot \cos\left(\pi t + \frac{\pi}{2}\right)$$

(a) $A = ?$ & $f = ?$ & $T = ?$

$$A = 5\,cm$$

$$f = \frac{1}{2\pi}\omega = \frac{1}{2\pi}\overset{s^{-1}}{\pi} = \frac{1}{2}\,s^{-1}$$

$$T = f^{-1} = 2\,s$$

(b) $v = ?$ $\quad a = ?$

$$v = \frac{dx}{dt} = -\pi\,(0.05\,m) \cdot \sin\left(\pi t + \frac{\pi}{2}\right)$$

$$a = \frac{dv}{dt} = -\pi^2(0.05\,m)\cos\left(\pi t + \frac{\pi}{2}\right)$$

(c) initial conditions $x(t=0) = ?$ & $v(t=0) = ?$

$$x(t=0) = (5\,cm) \cdot \cos\left(\frac{\pi}{2}\right) = 0$$

$$v(t=0) = -\overset{rad\ s^{-1}}{\pi} \cdot (5\,cm) \cdot \cancel{\sin}\left(\frac{\pi}{2}\right) \rightarrow 1 =$$

$$= -3.14 \times 5\,\frac{cm}{s^{-1}} = \underline{15.7\,cm\,s^{-1}}$$

Energy in Simple Harmonic Oscillator [SHO]

total energy = kinetic e. + potential e.

→ kinetic energy K

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi)$$

→ elastic potential energy U

$$U = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$$

→ total energy $E = K + U$

$$E = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \phi) +$$
$$+ \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$$

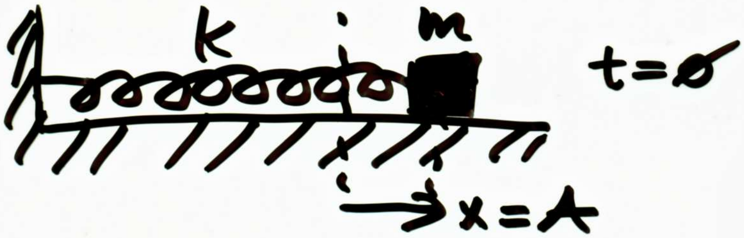
consider $\omega^2 = k/m \Rightarrow m\omega^2 = k$

$$E = \frac{1}{2} k A^2 \underbrace{\left[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)\right]}_{= 1}$$

Total energy of a SHO

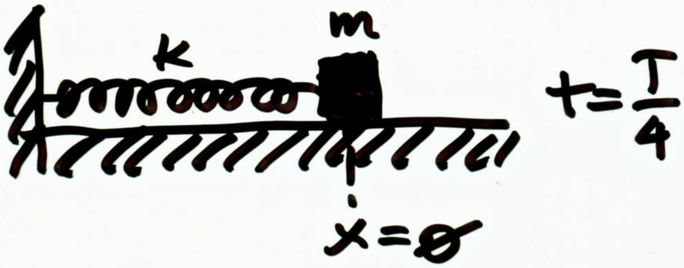
$$E = \frac{1}{2}kA^2 = \text{const.}$$

SHO is conserving the total energy!



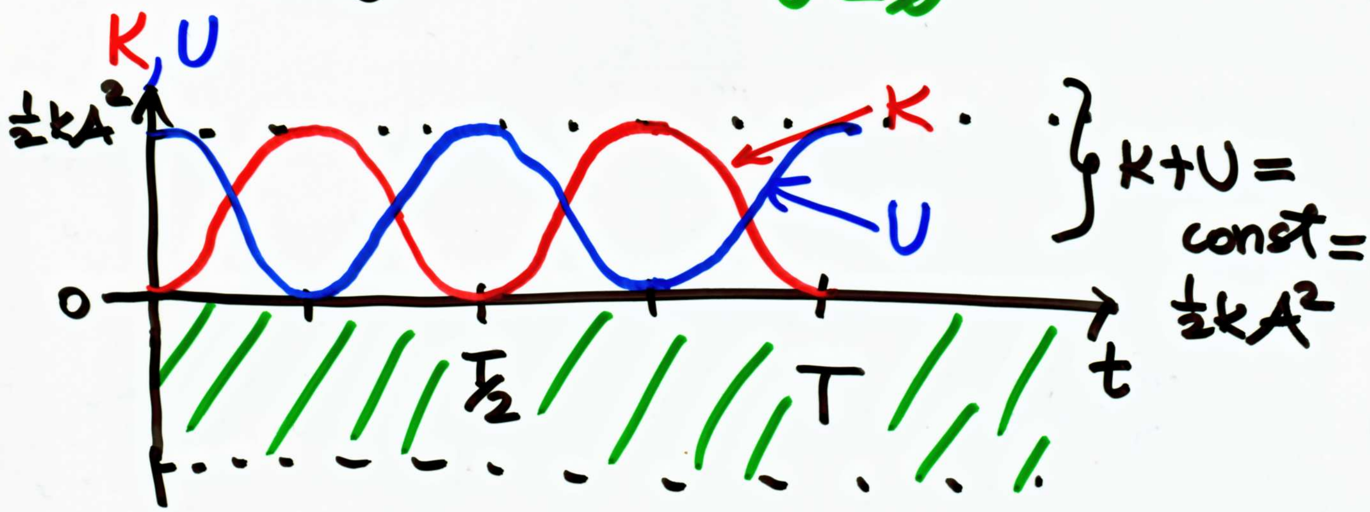
$K = 0$

$U = \frac{1}{2}kA^2 = \max$



$K = \frac{1}{2}m\omega^2 A^2 =$
$= \frac{1}{2}kA^2 = \max$

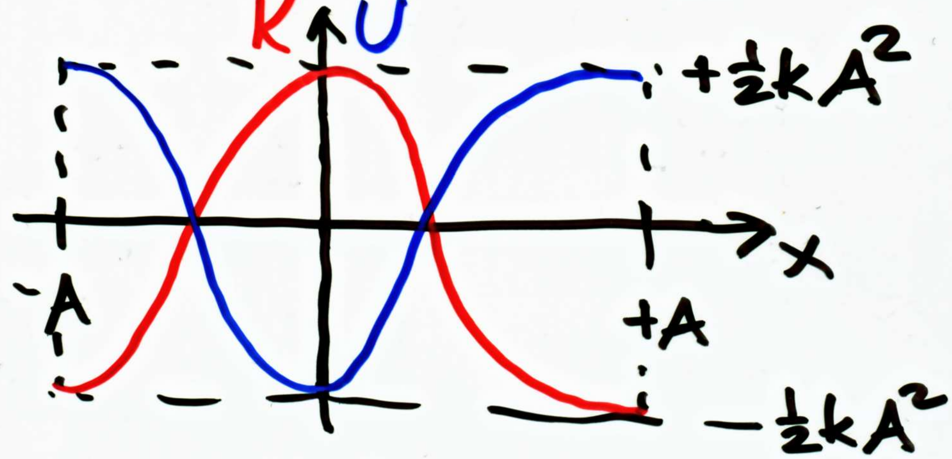
$U = 0$



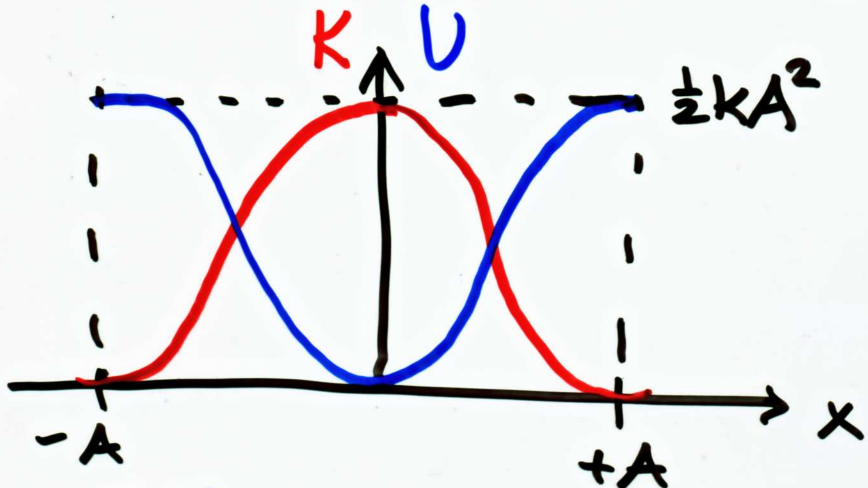
K & U are oscillating with a period T/2! K & U are positive at all times!

Quiz: Consider a block on a "massless" spring. At $t=0$, $x=A$, then we let it go. What is the correct dependence of kinetic e. K & potential e. U on x:

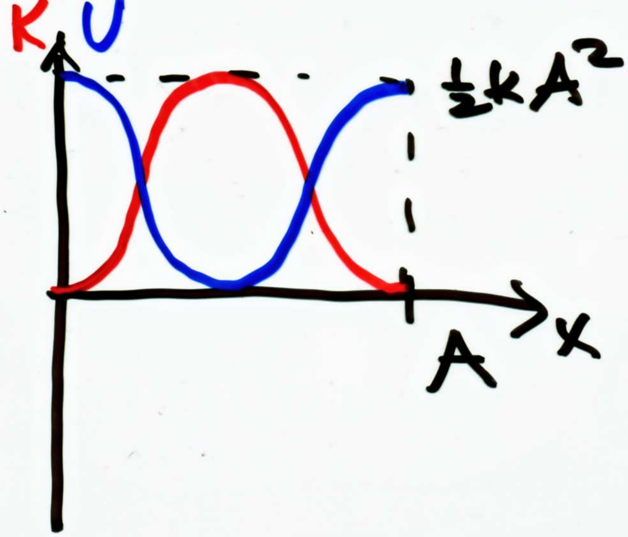
(a)



~~(b)~~

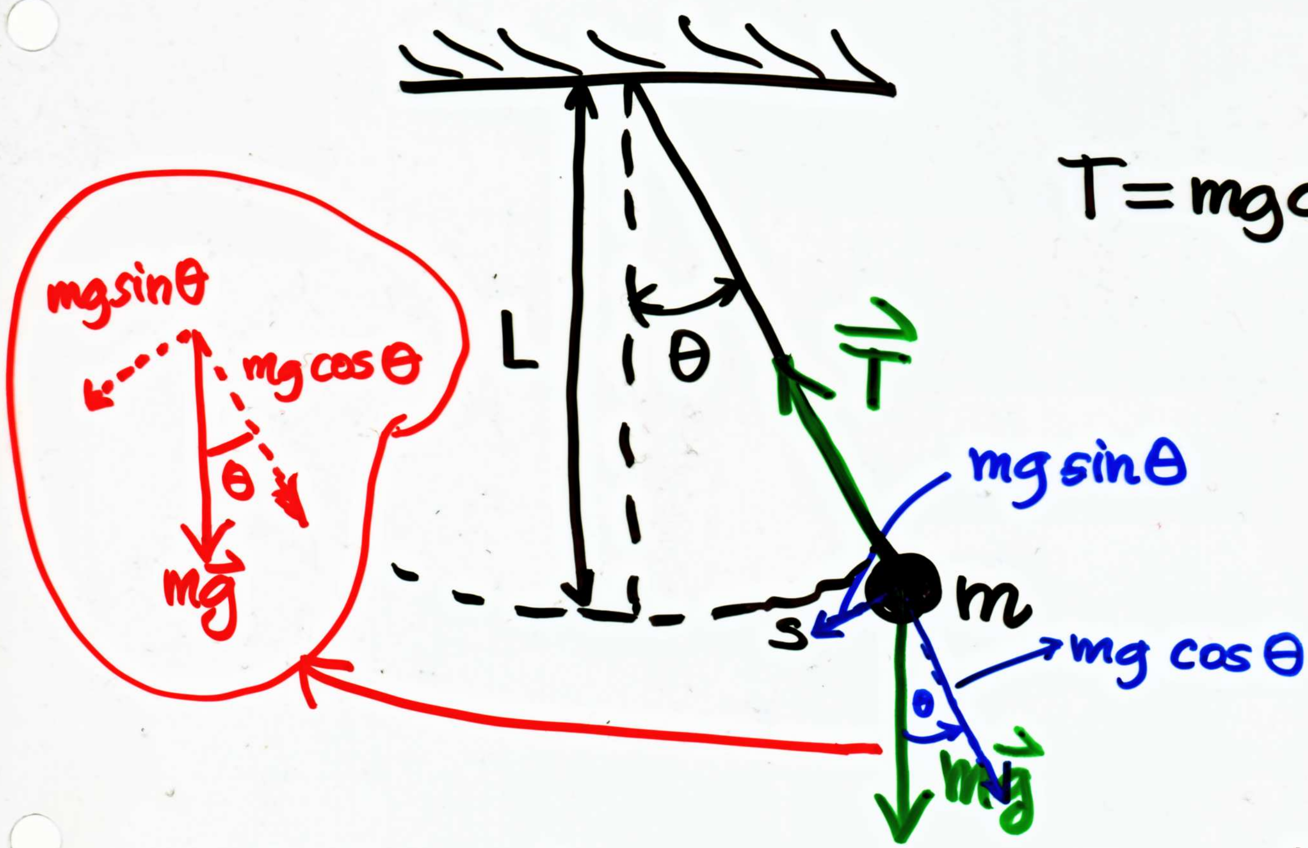


(c)



(2) The Simple Pendulum

$T = mg\cos\theta$



→ restoring force : along the path s

$-\boxed{mg\sin\theta}$

→ Newton 2nd Law : $m\frac{d^2s}{dt^2} = -mg\sin\theta$

→ s ... circular arc : $s = L\theta$

$$\frac{d^2s}{dt^2} = L\cdot\frac{d^2\theta}{dt^2}$$

⇓

$$\cancel{m}L\frac{d^2\theta}{dt^2} = -\cancel{m}g\sin\theta \;\Big/ : m$$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L} \cdot \sin\theta}$$

Because of $\sin\theta$ (NOT θ), the simple pendulum does not experience simple harmonic motion in general.

BUT, for small angles θ, we can approximate

$\sin\theta \sim \theta$ (where θ is measured in radians!)

Taylor series

Thus, for $\theta \ll 1$: $\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$

$$\Rightarrow \boxed{\theta = \theta_0 \cdot \cos(\omega t + \phi)}$$

$$\omega = \sqrt{\frac{g}{L}}$$

ω does not depend on the mass of the pendulum

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

All simple pendulums of equal length oscillate with equal periods.

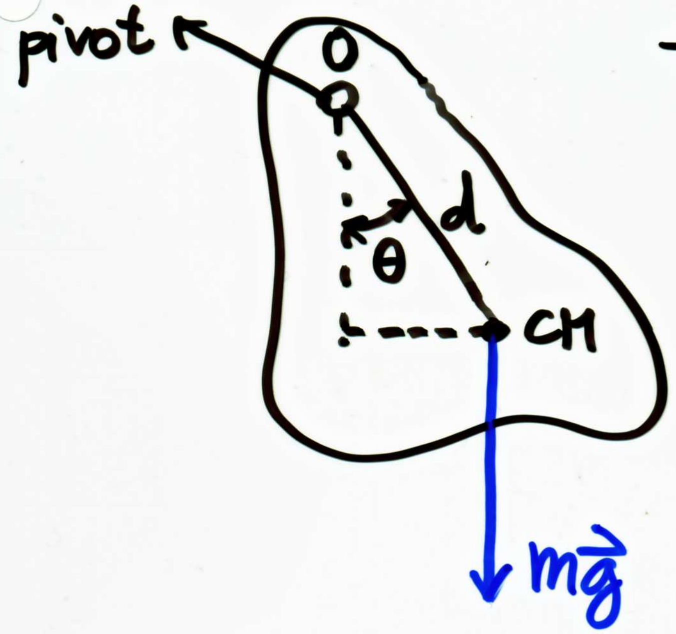
Quiz: Both the block-spring system and the simple pendulum are transported to the Moon. Which of the two will oscillate with a different ω on the Moon as compared to the Earth?

(a) the block-spring system

~~(b)~~ the simple pendulum

(c) both

(3) The Physical Pendulum



→ hanging object that oscillates about a fixed axis that does not pass through its CM

→ torque due to gravitational force about the axis O:

$$\vec{\tau} = \vec{r} \times \vec{F}_g \longmapsto -mgd\sin\theta$$

→ Newton's 2^{nd} law:

$$\sum \vec{\tau} = I\vec{\alpha}$$, I moment of inertia

$\vec{\alpha} = \dfrac{d^2\theta}{dt^2}$ angular acceleration

$\Downarrow$

$$I\frac{d^2\theta}{dt^2} = -mgd\sin\theta$$

→ small angle approximation

$$\frac{d^2\theta}{dt^2} = -\frac{mgd}{I}\theta$$

Angular frequency ω & period T

$$\left.\begin{aligned} \omega &= \sqrt{\frac{mgd}{I}} \\ T &= 2\pi\sqrt{\frac{I}{mgd}} \end{aligned}\right\} *$$

The physical pendulum is a generalized pendulum. You can use the above Eqs. * to find the simple pendulum ω & T.

For simple pendulum:

$$I = md^2 \Rightarrow \omega = \sqrt{\frac{g}{d}}$$

$$T = 2\pi\sqrt{\frac{d}{g}}$$

(4) Damped Oscillations

SHO was considered to occur in ideal frictionless conditions.

In a more realistic description, we need to take into account FRICTION, a resistive force which opposes motion

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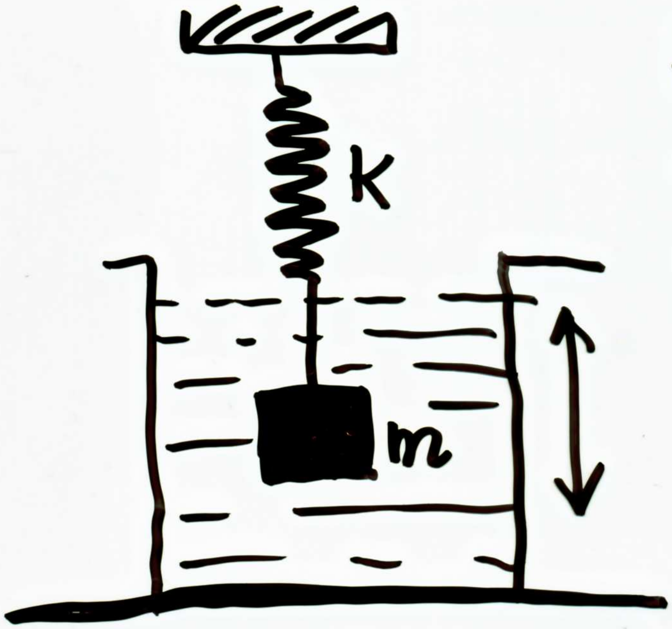
Total energy of a damped oscillator decreases with time.



Example of a damped oscillator

Mathematical description

→ linear resistive force : $-b\vec{v} = -b\dfrac{d\vec{x}}{dt}$

→ Hooke's force : $-kx$

→ Newton's 2nd law: $\sum \vec{F} = m\vec{a}$

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} \qquad (1D)$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \omega_0^2 x = 0}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

underdamped osc. $b < \sqrt{4mk}$

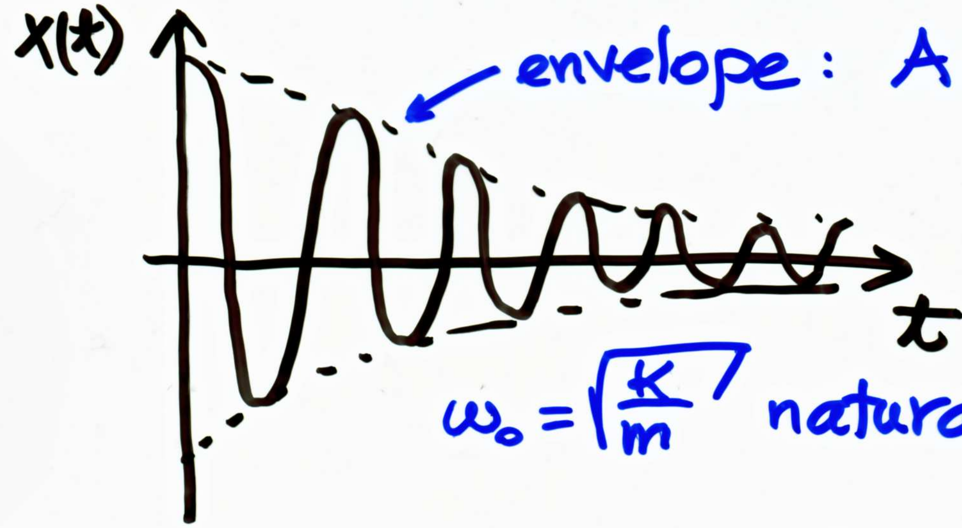
critically damped osc. $b = \sqrt{4mk}$

overdamped osc. $b > \sqrt{4mk}$

$$x(t) = A_0 e^{-(b/2m)t} \cos(\omega t + \phi)$$

$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

Underdamped oscillator:



$\omega_0 = \sqrt{\frac{k}{m}}$ natural frequency

→ envelope exponential decay (the larger the b, the faster the decay)

→ frequency ω DEPENDS on b:

$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

⇓

Oscillations slower is a more viscous liquid (Larger b)

Critically damped & overdamped
⇒ no oscillations,
only exponential decay of x(t)

Example: Calculate how the total E of a damped oscillator decreases with time. Assume that at $t=0$, $x=A$.

(5) Forced Oscillations & Resonance

→ to avoid loss of mechanical E with time in a damped oscillator, apply an external force to provide positive work on the system:

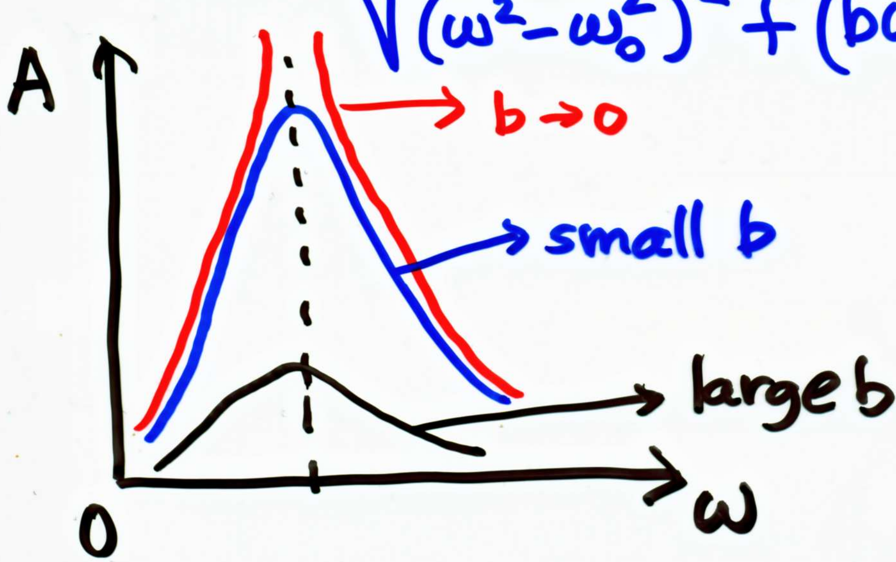
$$F = F_0 \sin \omega t$$

→ after some initial time, the system driven by the external force gets into a steady state:

$$x(t) = A \cos(\omega t + \phi)$$

where:

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega/m)^2}}$$



when $\omega = \omega_0$
⇓
resonance

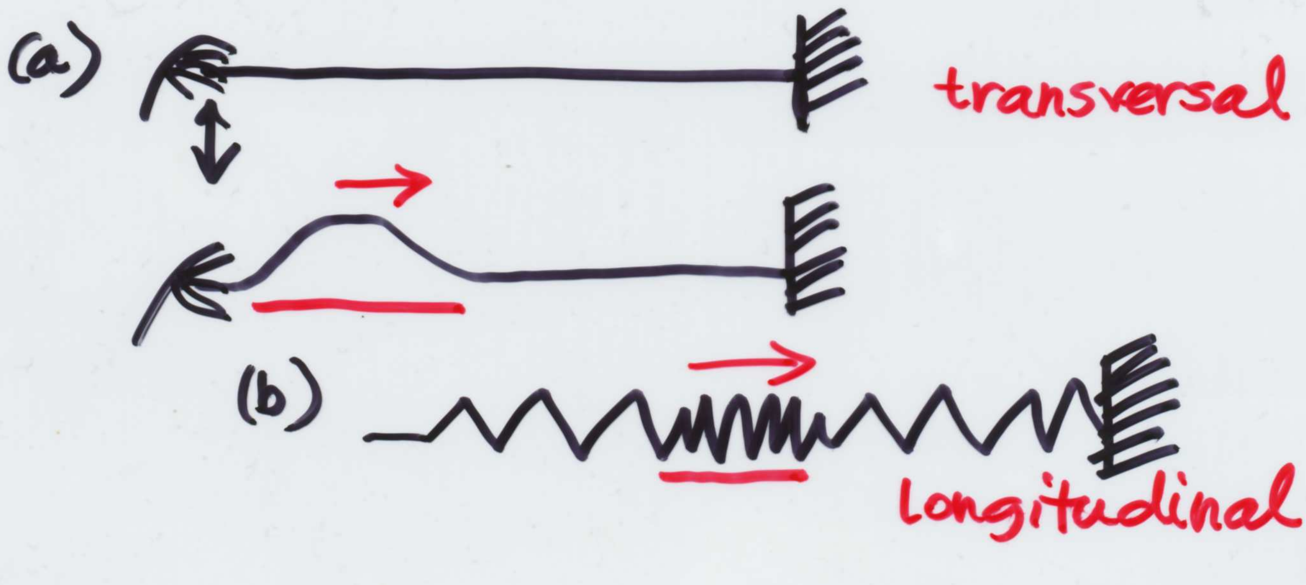
Ch. 13: Mechanical Waves

a pebble dropped into a pond
⇛ disturbance ⇛ water waves

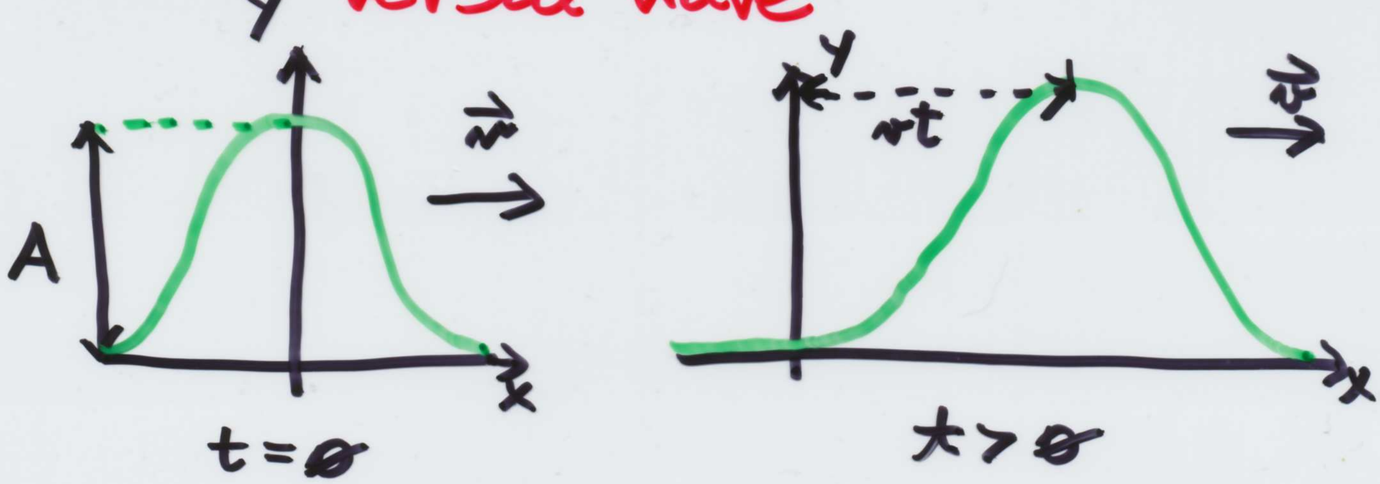
Mechanical waves:
(a) → transversal (metal string)
(b) → logitudinal (sound)

Requirements for creating the mechanical waves

(1) source of disturbance
(2) medium that can be disturbed
(3) physical mechanism through which elements of media influence each other



Mathematical description of a transversal wave



→ consider a wave that does not change the shape with time
⇒ no dispersion

→ mathematical formulation:

$$y(x,t) = y(x - vt, 0)$$

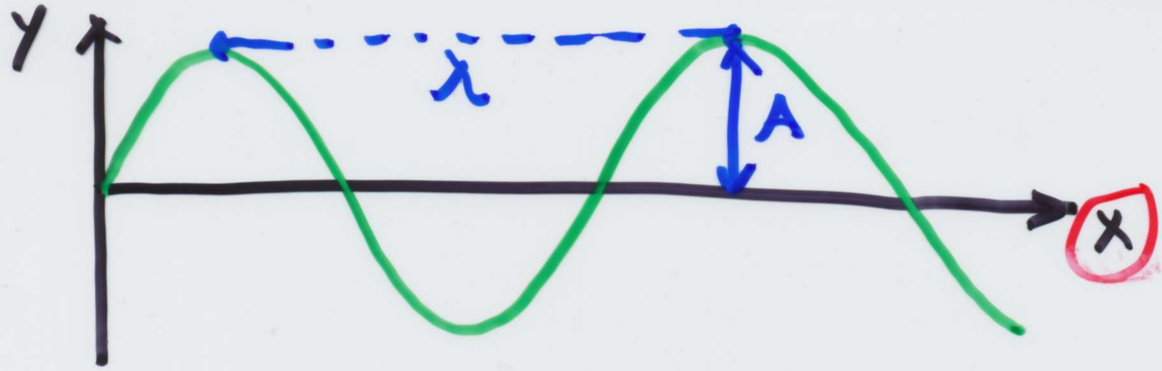
for a wave moving L → R

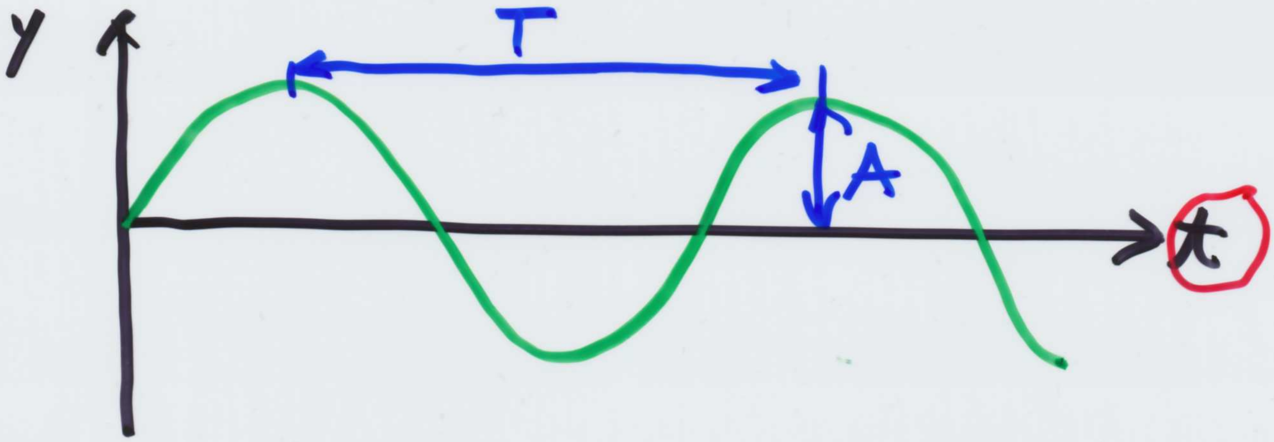
$$y(x,t) = y(x + vt, 0)$$

for a wave moving R → L

→ $y(x,t)$ represents the y-position of any element of the string located at x at time t

Wave Model $y = y(x, t)$





λ ... wavelength

A ... amplitude

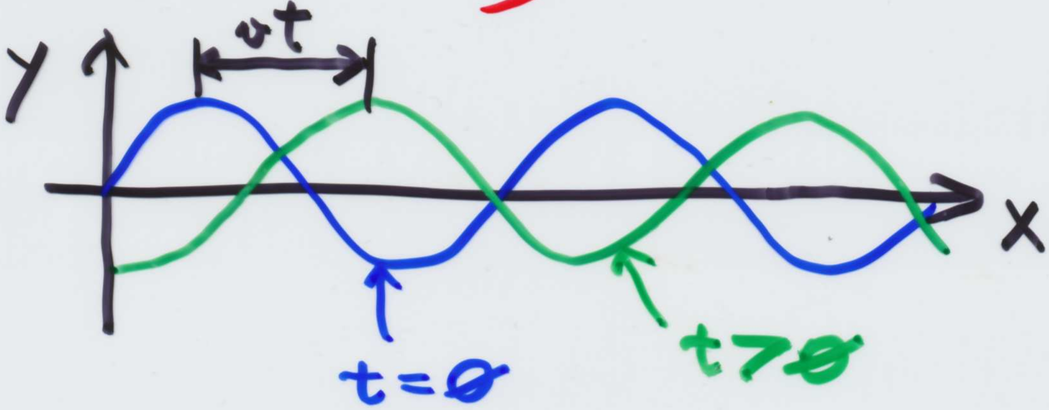
T ... period

$$T = \frac{1}{f}$$

$$\omega = 2\pi f$$

Each element (of a string/water) oscillates vertically in the y direction with simple harmonic motion.

The Travelling Wave



→ $t=0$: $y = A \sin\left(\frac{2\pi}{\lambda}x\right)$

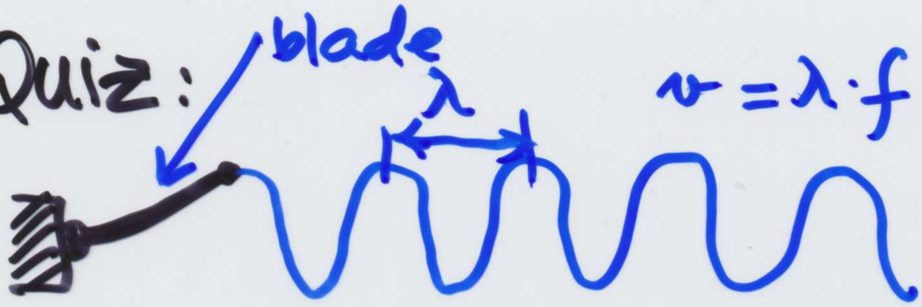
→ $t>0$: $y = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$

$v = \frac{\lambda}{T}$ (T...period)

Most common form of a travelling wave :

$$\boxed{\begin{aligned} &y(x,t) = A \cdot \sin(kx - \omega t) \\ &k = \frac{2\pi}{\lambda} \quad \text{wave number} \\ &\omega = \frac{2\pi}{T} = 2\pi f \quad \text{angular frequency} \\ &v = \frac{\omega}{k} \ \text{OR} \ v = \lambda f = \frac{\lambda}{T} \end{aligned}}$$

Quiz:



A sinusoidal wave with frequency f is traveling along a stretched string. Then, we replace the blade such that the new one is oscillating with $2f$, producing a wave with frequency $2f$, traveling on the same string.

What is the wavelength of the new wave?

(a) $\lambda_N = 2\lambda$

(b) $\lambda_N = \lambda$

(c) $\lambda_N = \frac{1}{2}\lambda$

(d) impossible to determine

The Linear Wave Equation

Consider transverse traveling wave

$$y(x,t) = A \cdot \sin(kx - \omega t)$$

Transverse velocity & acceleration

$$\rightarrow v_y = \frac{\partial y}{\partial t} = \left.\frac{dy}{dt}\right|_{x=const} = -\omega A \cos(kx - \omega t)$$

$$\rightarrow a_y = \frac{\partial^2 y}{\partial t^2} = \left.\frac{d^2 y}{dt^2}\right|_{x=const} = -\omega^2 A \cdot \sin(kx - \omega t)$$

are periodic functions, out of phase

We obtained: $$\frac{\partial^2 y}{\partial t^2} = -\omega^2 y(x,t)$$

Second derivative with respect to x,

$\frac{\partial^2 y}{\partial x^2}$: $$\frac{\partial^2 y}{\partial x^2} = -k^2 y(x,t)$$

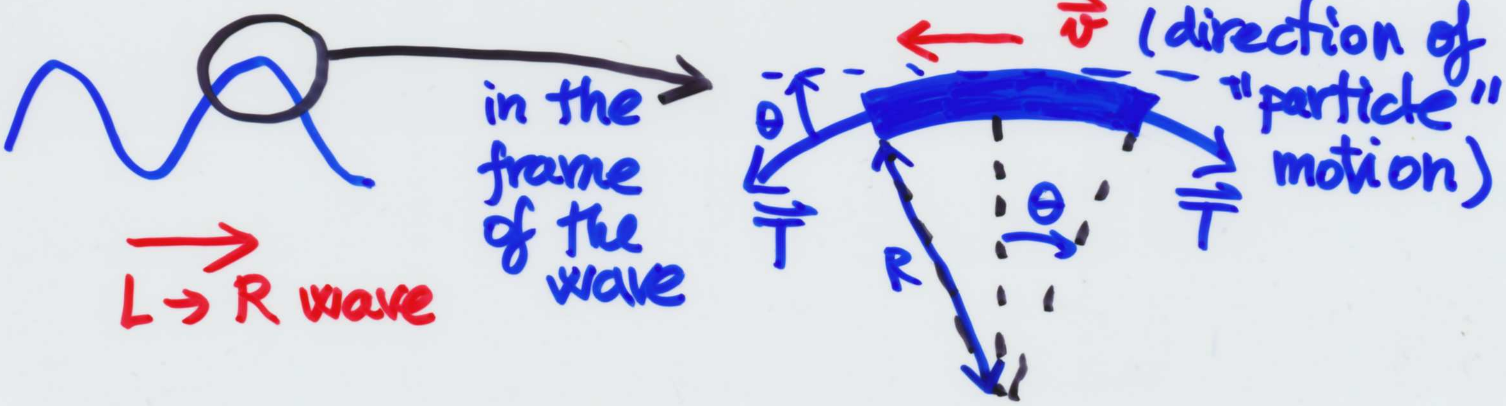
Take into account $v = \omega / k$

$\Downarrow$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = -\frac{k^2}{\omega^2} \cdot \omega^2 y(x,t) = +\frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}}$$

The Speed of Transverse Waves on Strings

Important: the wave speed depends only on the properties of the medium through which the wave travels



radial force $F_T = 2T\sin\theta \approx 2T\theta$ (SAA)

$$\left.\begin{aligned} m &= \mu \cdot \Delta s \\ \Delta s &= R(2\theta) \end{aligned}\right\} \Rightarrow$$

μ [$g\,m^{-1}$] mass/unit length

$$\Rightarrow m = 2\mu R\theta$$

$F_T = \dfrac{mv^2}{R}$ (Newton's 2nd law) centripetal force

$$\cancel{2}T\cancel{\theta} = \frac{\cancel{2}\mu\cancel{R}\cancel{\theta}v^2}{\cancel{R}} \Rightarrow v = \sqrt{\frac{T}{\mu}}$$

The Rate of Energy Transfer

$$\left.\begin{aligned}\Delta K &= \tfrac{1}{2}(\Delta m)v_y^2 = \\ &= \tfrac{1}{2}(\mu\,\Delta x)v_y^2\end{aligned}\right\} dK = \tfrac{1}{2}\mu\omega^2A^2\cos^2(kx-\omega t)$$

$$\left.\begin{aligned}K_\lambda &= \int_0^\lambda dK = \tfrac{1}{4}\mu\omega^2A^2\lambda \\ U_\lambda &= K_\lambda\end{aligned}\right\} E_\lambda = \tfrac{1}{2}\mu\omega^2A^2\lambda$$

$$\mathcal{P} = \frac{E_\lambda}{T} = \tfrac{1}{2}\mu\omega^2A^2\frac{\lambda}{T}, \quad \frac{\lambda}{T} = v$$

$$\boxed{\mathcal{P} = \tfrac{1}{2}\mu\omega^2A^2v}$$

The rate of energy transfer $\propto$ square of the angular frequency ω and the square of amplitude A!

Quiz: How would you increase the rate of energy transfer THE MOST?

(a) reduce the linear mass density μ

(b) doubling λ

(c) doubling T (tension in the string)

(d) doubling the amplitude