Recitation 8

Chapter 19

Problem Q4. Two lightbulbs of resistance R_1 and R_2 ($R_2 > R_1$) are connected in series. Which is brighter? What if they are connected in parallel? Explain.

(a) In series, the same current I flows through both bulbs, so the power (proporional to the brightness) can be found via

$$\begin{aligned} R_1 &< R_2 \\ I^2 R_1 &< I^2 R_2 \\ P_1 = I^2 R_1 &< I^2 R_2 = P_2 \\ P_1 &< P_2 \end{aligned}$$

(a) In series, both bulbs see the same voltage V, so the power (proporional to the brightness) can be found via

$$\begin{aligned} R_1 &< R_2 \\ \frac{1}{R_1} &> \frac{1}{R_2} \\ \frac{V^2}{R_1} &> \frac{V^2}{R_2} \\ P_1 = \frac{V^2}{R_1} &> \frac{V^2}{R_2} = P_2 \\ P_1 &> P_2 \end{aligned}$$

Problem Q7. If two identical resistors are connected in series to a battery, does the battery have to supply more power or less power than when only one of the resistors is connected? Explain.

We know that the power provided by the battery is given by

$$P = IV$$

so the power supplied increases if the current I increases (because V remains constant for batteries).
From Kirchoff's loop rule, we know the voltage drop across the resistors is the same as the voltage gain across the battery.

$$V_b = V_R$$

We also know that the voltage across the resistors relates to the current via Ohm's law

$$V_R = IR$$

Finally, we know that the effective resistance of two identical resistors in parallel is given by

$$R_2 = R_1 + R_1 = 2R_1$$

Putting these together in the case of a single resistor, we find a current of

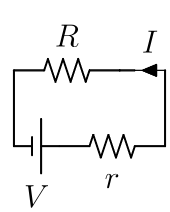
$$I_1 = \frac{V_R}{R_1} = \frac{V_b}{R_1}$$

and in the case of the two resistors in series

$$I_2 = \frac{V_R}{R_2} = \frac{V_b}{2R_1} = \frac{I_1}{2}$$

So with two resistors in series, we have less current and need less power.

Problem Q13. Explain in detail how you could measure the internal resistance of a battery.



Make a circuit using a known resistance R to connect the two terminals of the battery, and measure the current I. From Kirchoff's loop rule

$$
\begin{aligned}
V - Ir - IR &= 0 \\
Ir &= V - IR \\
r &= \frac{V}{I} - R
\end{aligned}
$$

Problem 2. Four 1.5 V cells are connected in series to a 12 Ω lightbulb. If the resulting current is 0.45 A, what is the internal resistance of each cell, assuming they are identical and neglecting the wires.

This is simply an application of the procedure outlined in Question 13. The external resistance is the lightbulb $R_{ext} = 12\ \Omega$. The total internal resistance is the sum of all the individual cell resistances $r_{int} = 6r$. The total voltage is the sum of all the individual cell voltages $V = 6 \cdot 1.5\text{ V} = 9\text{ V}$. Putting these together we have

$$
\begin{aligned}
r_{int} = 6r &= \frac{V}{I} - R_{ext} = \frac{9\text{ V}}{0.45\text{ A}} - 12\ \Omega \\
r &= 1.3\ \Omega
\end{aligned}
$$

Problem 7. A 650 Ω and a 2200 Ω resistor are connected in series with a 12 V battery. What is the voltage across the 2200 Ω0 resistor?

First we find the total current in the circuit. The two resistances, $R_1 = 650\ \Omega$ and $R_2 = 2200\ \Omega$, in series provide an effective resistance of $R_e = R_1 + R_2$. By Kirchoff's loop rule

$$
\begin{aligned}
V - IR_e &= 0 \\
I &= \frac{V}{R_e} = \frac{V}{R_1 + R_2}
\end{aligned}
$$

And applying Ohm's law to the second resistor

$$
V_2 = IR_2 = \frac{VR_2}{R_1 + R_2} = 9.3\text{ V}
$$

Problem 15. Eight 7.0 W Christmas tree lights are connected in series to each other and to a 110 V source. What is the resistance of each bulb.

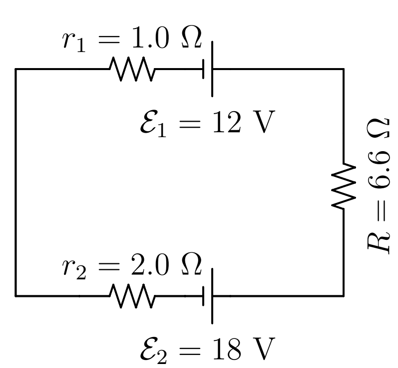
Let $V = 110$ V be the source voltage, $P_1 = 7.0$ W be the power of one bulb, and R_1 be the resistance of one bulb. By Kirchoff's loop rule

$$
\begin{aligned}
V - 8IR_1 &= 0 \\
I &= \frac{V}{8R_1}
\end{aligned}
$$

And we can find R_1 by considering the power dissipated by the bulb

$$
\begin{aligned}
P = IV_1 = I^2 R_1 &= \left(\frac{V}{8}\right)^2 \frac{1}{R_1} \\
R_1 &= \left(\frac{V}{8}\right)^2 \frac{1}{P} = 27\ \Omega
\end{aligned}
$$

Problem 24. Determine the terminal voltage of each battery in Fig. 19-44.



From Kirchoff's loop rule

$$\begin{aligned}\mathcal{E}_1 - IR - \mathcal{E}_2 - Ir_2 - Ir_1 &= 0\\ I(R + r_1 + R_2) &= \mathcal{E}_1 - \mathcal{E}_2\\ I &= \frac{\mathcal{E}_1 - \mathcal{E}_2}{R + r_1 + r_2} = 0.625 \text{ A}\end{aligned}$$

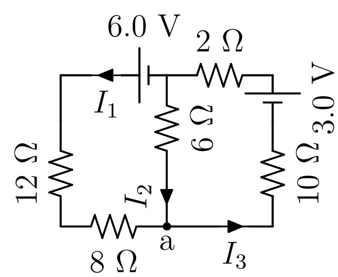
So the voltage across the top battery is

$$V_1 = \mathcal{E}_1 - Ir_1 = 17 \text{ V} \qquad (17.375 \text{ V})$$

and the voltage across the bottom battery is

$$V_2 = \mathcal{E}_2 - Ir_2 = 11 \text{ V} \qquad (10.75 \text{ V})$$

Problem 31. Calculate the currents in each resistor of Fig. 19-49.



Label the resistors from left to right: $R_1 = 12\ \Omega$, $R_2 = 8\ \Omega$, $R_3 = 6\ \Omega$, $R_4 = 2\ \Omega$, and $R_5 = 10\ \Omega$.
Label the batteries from left to right: $V_1 = 6.0$ V and $V_2 = 3.0$ V.
Applying Kirchoff's junction rule to junction a we have

$$I_1 + I_2 - I_3 = 0$$

Applying Kirchoff's loop rule to the left-hand loop we have

$$V_1 - I_1(R_1 + R_2) + R_3 I_2 = 0$$

where we *add* the voltage change over R_3 because we cross it *against* the direction of the current I_2.
Applying Kirchoff's loop rule to the right-hand loop we have

$$V_2 - R_4 I_3 - R_3 I_2 - R_5 I_5 = V_2 - I_3(R_4 + R_5) - R_3 I_2 = 0$$

We now have three equations for three unknowns (the I_i). Solving the loop rools for I_1 and I_3 we have

$$\begin{aligned}I_1 &= \frac{V_1 + R_3 I_2}{R_1 + R_2} = \frac{V_1 + R_3 I_2}{R_{12}}\\ I_3 &= \frac{V_2 - R_3 I_2}{R_4 + R_5} = \frac{V_2 - R_3 I_2}{R_{45}}\end{aligned}$$

where we have used the equivalent resistances $R_{12} \equiv R_1 + R_2$ and $R_{45} \equiv R_4 + R_5$ to save writing later. We can then plug those currents into the junction rule and solve for I_2

$$\begin{aligned}\frac{V_1 + R_3 I_2}{R_{12}} + I_2 - \frac{V_2 - R_3 I_2}{R_{45}} &= 0\\ \frac{V_1}{R_{12}} + \frac{R_3}{R_{12}} I_2 + I_2 - \frac{V_2}{R_{45}} + \frac{R_3}{R_{45}} I_2 &= 0\\ \left(\frac{R_3}{R_{12}} + 1 + \frac{R_3}{R_{45}}\right) \cdot I_2 &= \frac{V_2}{R_{45}} - \frac{V_1}{R_{12}}\\ I_2 &= \frac{\frac{V_2}{R_{45}} - \frac{V_1}{R_{12}}}{\frac{R_3}{R_{12}} + 1 + \frac{R_3}{R_{45}}}\\ I_2 &= -28 \text{ mA}\end{aligned}$$

Where the $-$ sign means the true current is in the opposite direction to the one we have assigned (so the true current flows upward in the figure). We can now plug this current in to find I_1 and I_3.

$$I_1 = \frac{V_1 + R_3 I_2}{R_{12}} = 292 \text{ mA}$$
$$I_3 = \frac{V_2 - R_3 I_2}{R_{45}} = 264 \text{ mA}$$

Double-checking our algebra, we see $I_1 + I_2 - I_3 = 292 - 27 - 264 = -1$ mA ≈ 0 where difference of 1 mA is due to rounding errors from forcing our answers to milli-Volt precision.