Chapter 19: DC Circuits

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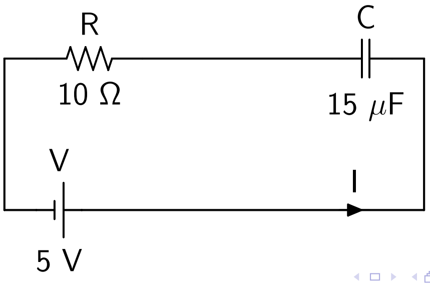
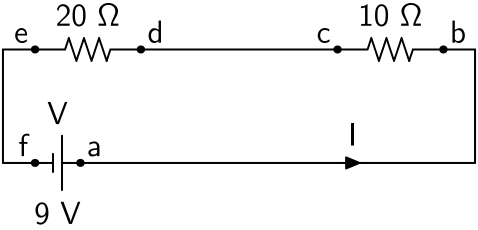
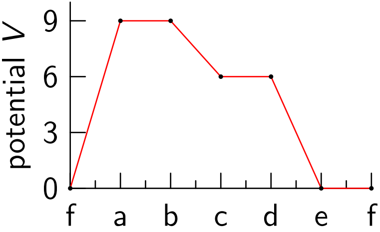
DC circuits

- *Direct Current* (DC) circuits are important
  - often used in digital electronic devices and almost anything that is battery powered.
- The physics of DC currents is straightforward
  - Kirchoff's loop rule
  - Kirchoff's junction rule
  - Understanding the individual components
    - Resistors
    - Capacitors
    - ...



Kirchoff's loop rule

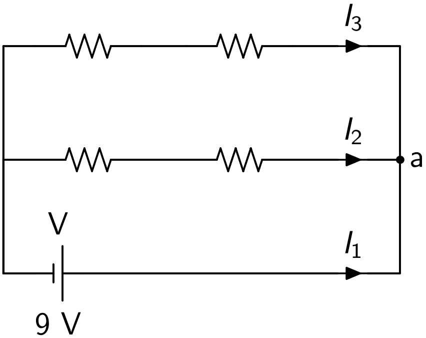




$$\sum_{\text{loop}} \Delta V_i = 0$$

Conserving energy

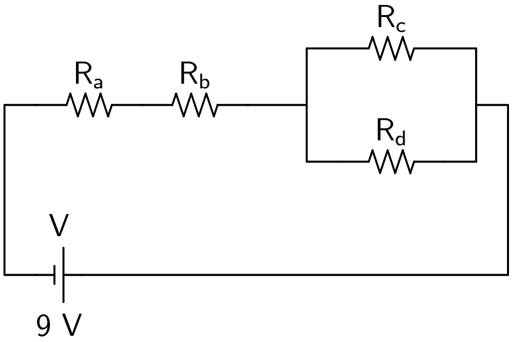
Kirchoff's junction rule



$$\sum_{\text{junction}} I_i = 0$$

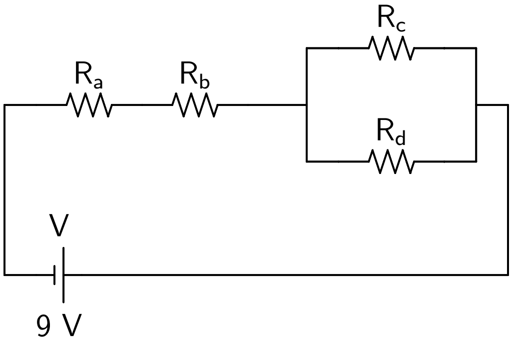
Conserving charge, steady state

Elements in series



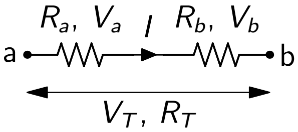
Two elements are in series if you cannot make a loop crossing one of them without also crossing the other.

Elements in parallel



Two elements are in parallel if you can make a loop crossing both of them which crosses no other elements.

Resistors in series



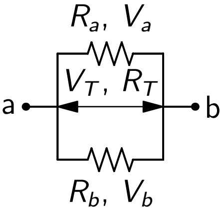
By Kirchoff's junction rule, $I_a = I_b$ (because there are no intervening junctions).

$$V_T = V_a + V_b = IR_a + IR_b = I(R_a + R_b)$$
$$V_T = IR_T$$

So the effective resistance is give by

$$R_T = R_a + R_b$$

Resistors in parallel



By Kirchoff's loop rule, $V_a - V_b = 0$ so $V_a = V_b = V_T$.
By Kirchoff's junction rule, $I_T = I_a + I_b$

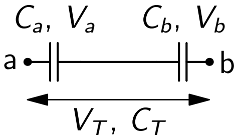
$$V_T = I_a R_a = I_b R_b \qquad I_b = I_a \frac{R_a}{R_b}$$

$$I_T = I_a + I_b = I_a \left(1 + R_a/R_b\right)$$

$$V_T = I_a R_a = \frac{I_T}{1 + R_a/R_b} R_a = I_T \left(1/R_a + 1/R_b\right)^{-1} = I_T R_T$$

So the effective resistance is give by $R_T = \left(1/R_a + 1/R_b\right)^{-1}$

Capacitors in series



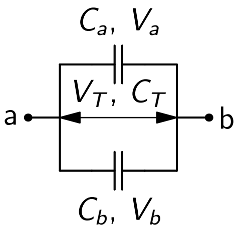
By Kirchoff's junction rule, $Q_a = Q_b = Q_T$ (because there are no intervening junctions).

$$V_T = V_a + V_b = Q_a/C_a + Q_b/C_b = Q_T(1/C_a + 1/C_b)$$
$$V_T = Q_T/C_T$$

So the effective capacitance is give by

$$C_T = \left(\frac{1}{C_a} + \frac{1}{C_b}\right)^{-1}$$

Capacitors in parallel

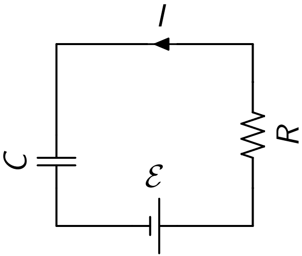


By Kirchoff's loop rule, $V_a - V_b = 0$ so $V_a = V_b = V_T$.
By Kirchoff's junction rule, $Q_T = Q_a + Q_b$

$$Q_T = Q_a + Q_b = C_a V_T + C_b V_T = (C_a + C_b) V_T$$
$$Q_T = C_T V_T$$

So the effective capacitance is give by $C_T = C_a + C_b$.

RC circuits



From Kirchoff's loop rule $\sum_{loop} V_i = \mathcal{E} - IR - Q/C = 0$.
From the definition of current $I = \frac{\mathrm{d}Q}{\mathrm{d}t}$.
So

$$\mathcal{E} - Q/C = R\frac{Q}{t}$$

$$\mathrm{d}t = (\mathcal{E} - Q/C)^{-1} R\mathrm{d}Q$$

$$Q = C\mathcal{E}\left(1 - e^{-t/RC}\right) \qquad V_C = \mathcal{E}\left(1 - e^{-t/RC}\right)$$