

## Recitation Week 2

### Chapter 22

**Problem 16.** A solid metal sphere with radius 0.450 m carries a net charge of 0.250 nC. Find the magnitude of the electric field (a) at a point 0.100 m outside the surface of the sphere and (b) at a point inside the sphere, 0.100 m below the surface.

(a) From Gauss' law

$$\Phi_E = \oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q_e}{\epsilon_0} \quad (1)$$

we have (in cases of spherical symmetry)

$$\Phi_E = EA = 4\pi r^2 E = \frac{q_e}{\epsilon_0} \quad (2)$$

$$E = \frac{q_e}{4\pi\epsilon_0 r^2} = \frac{kq_e}{r^2} \quad (3)$$

which looks just like the electric field from a point charge, except the  $q_e$  is only the charge enclosed by the gaussian sphere of radius  $r$ .

In the case of  $r_a = R + 0.1$  m outside the sphere, all of the charge is enclosed ( $q_e = q$ ), so

$$E_a = \frac{kq}{r_a^2} = \frac{8.99\text{e9 Nm}^2/\text{C}^2 \cdot 0.250 \cdot 10^{-9} \text{ C}}{(0.550 \text{ m})^2} = 7.43 \text{ N/C} \quad (4)$$

(b) Inside a steady-state conductor,  $E = 0$ , otherwise charges would be moving around, and the system would not actually be in a steady state.

Another way to think about it is that all the excess positive charges in the sphere want to get as far away from each other as possible, so they end up evenly distributed in a thin shell around the outside of the sphere. This leaves no net charge inside any gaussian surface of  $r < R$ , so  $q_e = 0$ , and  $E = 0$ .

**Problem 18.** Some planetary scientists have suggested that the planet Mars has an electric field somewhat similar to that of the earth, producing a net electric flux of  $3.63 \cdot 10^{16} \text{ N}\cdot\text{m}^2/\text{C}$  at the planet's surface. Calculate: (a) the total electric charge on the planet; (b) the electric field at the planet's surface (refer to the astronomical data inside the back cover); (c) the charge density on Mars, assuming all the charge is uniformly distributed over the planet's surface.

(a) Normally Gauss' law problems involve some sort of symmetry argument to make the integral in

$$\Phi_E = \oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q_e}{\epsilon_0} \quad (5)$$

easy to deal with. In (a) though, we don't need any of that, since they give you the result of the integral  $\Phi_E$  directly. Finding  $q$  is simple algebra

$$q = \epsilon_0 \Phi_E = 8.885 \text{ C}^2/\text{Nm}^2 \cdot 3.63 \cdot 10^{16} \text{ Nm}^2/\text{C} = 3.21 \cdot 10^5 \text{ C} . \quad (6)$$

(b) Now the spherical symmetry comes in, and we use the formula for electric field outside a spherically symmetric body (with  $q_e = q$ ).

$$E = \frac{kq}{R^2} = \frac{\Phi_E}{A} = \frac{\Phi_E}{4\pi R^2} = 250 \text{ N/C} , \quad (7)$$

where we looked up the radius of Mars  $R = 3.40 \cdot 10^6$  m.

(c) The surface charge density  $\sigma$  is the charge per unit area. This is the same as the total charge on Mars divided by its total surface area.

$$\sigma = \frac{q}{A} = \frac{q}{4\pi R^2} = 2.21 \cdot 10^{-9} \text{ C/m}^2 . \quad (8)$$

**Problem 23.** A hollow, conducting sphere with an outer radius of 0.250 m and an inner radius of 0.200 m has a uniform surface charge density of  $+6.37 \cdot 10^{-6} \text{ C/m}^2$ . A charge of  $-0.500 \mu\text{C}$  is now introduced into the cavity inside the sphere. (a) What is the new charge density on the outside of the sphere? (b) Calculate the strength of the electric field just outside the sphere. (c) What is the electric flux through the spherical surface just inside the inner surface of the sphere?

(a) The total charge on the shell is  $Q = 4\pi R_o^2 \sigma = 5.00 \mu\text{C}$ .

Once the  $q = -0.500 \mu\text{C}$  charge is placed inside the shell,  $q_i = -q$  accumulates on the shell's inner wall. This ensures that the enclosed charge  $q_e = 0$  for any gaussian surface lying inside the body of the conducting shell, which must be the case, because  $E = 0$  inside steady-state conductors. The left over outer-wall charge

$$q_o = Q - q_i = 4.50 \mu\text{C} \quad (9)$$

is distributed uniformly over the outer surface, so

$$\sigma_o = \frac{q_o}{4\pi R_o^2} = \sigma - \frac{-q}{4\pi R_o^2} = 5.73 \mu\text{C} . \quad (10)$$

(b) Now that we know the charge distribution, we can calculate electric fields with Gauss' law for spherically symmetric charge distributions. For any point outside the sphere, the total charged enclosed will be  $q_e = q_o + q_i + q = q_o$ , so

$$E_o = \frac{kq_o}{R_o^2} = 647 \text{ kN/C} . \quad (11)$$

(c) At any point inside a steady-state conductor,  $E = 0$ , so  $\Phi_E = AE = 0$ .

**Problem 25.** The electric field at a distance of 0.145 m from the surface of a solid insulating sphere with a radius 0.355 m is 1750 N/C. (a) Assuming the sphere's charge is uniformly distributed, what is the charge density inside it? (b) Calculate the electric field inside the sphere at a distance of 0.200 m from the center.

(a) Using Gauss' law for spherically symmetric charge distributions

$$E = \frac{kq}{r^2} \quad (12)$$

$$q = \frac{Er^2}{k} = \frac{1750 \text{ N/C} \cdot (0.145 \text{ m} + 0.355 \text{ m})^2}{8.99 \cdot 10^9 \text{ Nm}^2/\text{C}^2} = 48.7 \text{ nC} . \quad (13)$$

For a uniform distribution, the charge density (per unit volume)  $\rho$  is the total charge divided by the total volume of the sphere.

$$\rho = \frac{q}{\frac{4}{3}\pi R^3} = 2.60 \cdot 10^{-7} \text{ C/m}^3 \quad (14)$$

(b) Once you know the charge density, we can find the charge enclosed by a gaussian surface of radius  $r = 0.200 \text{ m}$ .

$$q_e = \frac{4}{3}\pi r^3 \rho = q \frac{r^3}{R^3} = 8.70 \text{ nC} . \quad (15)$$

Then back to our Gauss' law formula for the electric field

$$E = \frac{kq_e}{r^2} = \frac{kq \frac{r^3}{R^3}}{r^2} = \frac{kqr}{R^3} = \frac{8.99 \text{ Nm}^2/\text{C}^2 \cdot 48.7 \text{ nC} \cdot 0.200 \text{ m}}{(0.355 \text{ m})^3} = 1960 \text{ N/C} \quad (16)$$

**Problem 37.** A long coaxial cable consists of an inner cylindrical conductor with a radius  $a$  and an outer coaxial cylinder with inner radius  $b$  and outer radius  $c$ . The outer cylinder is mounted on insulating supports and has no net charge. The inner cylinder has a uniform positive charge per unit length  $\lambda$ . Calculate the electric field (a) at any point between the cylinders a distance  $r$  from the axis and (b) at any point outside the outer cylinder. (c) Graph the magnitude of the electric field as a function of the distance  $r$  from the axis of the cable, from  $r = 0$  to  $r = 2c$ . (d) Find the charge per unit length on the inner surface and on the outer surface of the outer cylinder.

For this question, we need to apply Gauss' law to cylindrically symmetric charge distributions. Take a look at Example 22.6 in the text to see their solution for an infinitely long, charged wire. Eventually you end up with

$$E = \frac{\lambda_e}{2\pi\epsilon_0 r} \quad (17)$$

for the electric field  $E$  a distance  $r$  from a long, straight, cylindrically symmetric charge distribution carrying a charge density (per unit length) of  $\lambda_e$ . That's really all we need for this problem.

(a) Between the two cylinders, the enclosed charge density  $\lambda_e = \lambda$ , the charge density of the inner cylinder, so

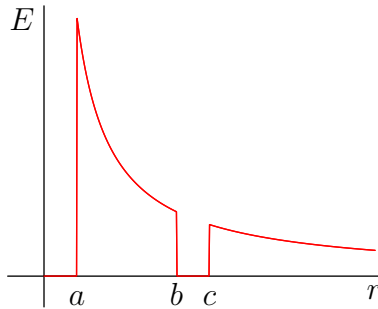
$$E = \frac{\lambda}{2\pi\epsilon_0 r} . \quad (18)$$

(b) Outside the outer cylinder, the enclosed charge density is still given by  $\lambda_e = \lambda$ , because the outer cylinder carries no net charge. Therefore

$$E = \frac{\lambda}{2\pi\epsilon_0 r} , \quad (19)$$

which is the same formula as in (a).

(c) We've already determined the electric field in all non-conductor regions. Inside any steady-state conductor,  $E = 0$ , otherwise charges would be moving, and the conductor would not be in a steady-state. Therefore,  $E(r)$  looks like



(d) The inner surface of the outer cylinder must carry  $\lambda_i = -\lambda$  so that the total charge enclosed by any gaussian surface embedded inside the outer cylinder is 0 (as it must be, because  $E = 0$  inside a conductor). Because the outer cylinder carries no net charge,  $\lambda_o = -\lambda_i = \lambda$ .