

Recitation Week 10

Chapter 29

Problem 29.1. A flat, rectangular coil consisting of 50 turns measures 25.0 cm by 30.0 cm. It is in a uniform, 1.20 T, magnetic field, with the plane of the coil parallel to the field. In 0.222 s, it is rotated so that the plane of the coil is perpendicular to the field. (a) What is the change in the magnetic flux through the coil due to this rotation? (b) Find the magnitude of the average emf induced in the coil during this rotation.

(a) Neither the area nor the magnetic field changes, only the angle between them, so

$$\Delta\Phi = AB[\cos(\theta_f) - \cos(\theta_i)] = AB[\cos(0^\circ) - \cos(90^\circ)] = AB = NLWB = 50 \cdot 0.250 \text{ m} \cdot 0.300 \text{ m} \cdot 1.20 \text{ T} = 4.50 \text{ Wb} . \quad (1)$$

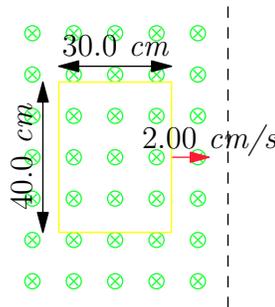
(b) From Faraday's law of inductance (Eq. 29.3, p. 996)

$$\mathcal{E} = -\frac{\partial\Phi_B}{\partial t} , \quad (2)$$

so the magnitude of the average \mathcal{E} is

$$|\mathcal{E}| = \frac{|\Delta\Phi|}{\Delta t} = \frac{2.50 \text{ Wb}}{0.222 \text{ s}} = 20.3 \text{ V} . \quad (3)$$

Problem 29.10. A rectangle measuring 30.0 cm by 40.0 cm is located inside a region of a spatially uniform magnetic field of 1.25 T, with the field perpendicular to the plane of the coil (Fig. 29.29). The coil is pulled out at a steady rate of 2.00 cm/s traveling perpendicular to the field lines. The region of field ends abruptly as shown. Find the emf induced in this coil when it is (a) all inside the field; (b) partly inside the field; (c) all outside the field.



(a) The area, flux, and angle between them are all constant while the coil remains entirely within the B -field region, so the flux is constant. Therefore

$$\mathcal{E} = -\frac{\partial\Phi_B}{\partial t} = 0 . \quad (4)$$

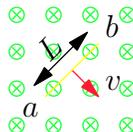
(b) As the coil leaves the field, the area with non-zero magnetic field decreases, so

$$\mathcal{E} = -\frac{\partial\Phi_B}{\partial t} = -\frac{\partial}{\partial t}(HWB) = -HB\frac{\partial W}{\partial t} = HBv = 0.400 \text{ m} \cdot 1.25 \text{ T} \cdot 2.00 \cdot 10^{-2} \text{ m/s} = 10.0 \text{ mV} . \quad (5)$$

(c) As in (a), the flux is constant, so

$$\mathcal{E} = -\frac{\partial\Phi_B}{\partial t} = 0 . \quad (6)$$

Problem 29.21. In Fig. 29.37 a conducting rod of length $L = 30.0 \text{ cm}$ moves in a magnetic field \mathbf{B} of magnitude 0.450 T directed into the plane of the figure. The rod moves with speed $v = 5.00 \text{ m/s}$ in the direction shown. (a) What is the potential difference between the ends of the rod? (b) Which point, a or b, is at higher potential? (c) When the charges in the rod are in equilibrium, what are the magnitude and direction of the electric field within the rod? (d) When the charges in the rod are in equilibrium, which point, a or b, has an excess of positive charge? (e) What is the potential difference across the rod if it moves (i) parallel to ab and (ii) directly out of the page?



(a) The conducting rod has mobile charges of charge q . These charges feel a magnetic force from passing through the magnetic field

$$F_B = |\mathbf{F}_B| = |q\mathbf{v} \times \mathbf{B}| = qvB, \quad (7)$$

because \mathbf{v} and \mathbf{B} are perpendicular.

This force pushes the charges from one end of the rod to the other which causes $Q = n|q|$ to build up on one side of the rod (b) and $-Q$ to be exposed on the other side (a). The charge separation increases until the electric force

$$F_E = qE \quad (8)$$

balances the magnetic force

$$qE = qvB \quad (9)$$

$$E = vB. \quad (10)$$

Because the electric field in the bar is constant, the electric potential difference V_{ab} is given by

$$V_{ab} = EL = vBL = 5.00 \text{ m/s} \cdot 0.450 \text{ T} \cdot 0.300 \text{ m} = 675 \text{ mV}. \quad (11)$$

The potential energy difference for moving a single charge between a and b (considering both B and E) is zero, because the net external force on the charge is zero.

(b) The positive charge build up at b gives a higher electric potential at b .

(c) The electric field points from b to a , resisting further charge separation. As we showed above, $E = vB = 2.25 \text{ V/m}$

(d) As shown above, b has the excess positive charge.

(e) If the rod moves parallel to ab

$$V_{ab} = EL = 0, \quad (12)$$

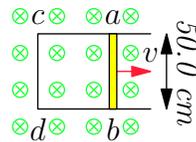
because the rod is thin ($L \approx 0$).

If the rod moves directly out of the page

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} = 0, \quad (13)$$

because \mathbf{v} and \mathbf{B} would be in the same direction, and no charge separation occurs, so $V_{ab} = 0$.

Problem 29.25. The conducting rod ab shown in Fig. 29.38 makes contact with metal rails ca and db . The apparatus is in a uniform magnetic field of 0.800 T , perpendicular to the plane of the figure. (a) Find the magnitude of the emf induced in the rod when it is moved toward the right with speed 7.50 m/s . (b) In what direction does the current flow in the rod? (c) If the resistance of the circuit $abcd$ is 1.50Ω (assumed to be constant), find the force (magnitude and direction) required to keep the rod moving to the right with a constant speed of 7.50 m/s . You can ignore friction. (d) Compare the rate at which mechanical work is done by the force (Fv) with the rate at which thermal energy is developed in the circuit (I^2R).



(a) As the bar moves to the right, the flux increases

$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t} = -\frac{\partial}{\partial t}(HWB) = -HB\frac{\partial W}{\partial t} = -HBv = -0.500 \text{ m} \cdot 0.800 \text{ T} \cdot 7.50 \text{ m/s} = -3.00 \text{ V} \quad (14)$$

$$|\mathcal{E}| = 3.00 \text{ V}. \quad (15)$$

(b) From (a), we see that $\mathcal{E} < 0$ when we pick coordinates such that $B > 0$. Thus the current direction is counter-clockwise ($b \rightarrow a$).

(c) The current in the circuit will be

$$V = IR \quad (16)$$

$$I = \frac{V}{R} = \frac{3.00 \text{ V}}{1.50 \Omega} = 2.00 \text{ A} \quad (17)$$

You must then pull the bar with enough force to balance the magnetic force

$$F = IHB = \frac{V}{R}HB = \frac{HBv}{R}HB = \frac{H^2B^2v}{R} = 0.800 \text{ N} \quad (18)$$

(d) The mechanical power you put into the bar is given by

$$P_M = Fv = \frac{H^2B^2v^2}{R} . \quad (19)$$

The electrical power dissipated by the resistance is

$$P_R = IV = \frac{V^2}{R} = \frac{H^2B^2v^2}{R} = P_M . \quad (20)$$

This is good, because we expect energy to be conserved, and the circuit contains no capacitors or other means of storing energy. Therefore, what goes in must come out.

Problem 29.28. A long, thin solenoid has 900 turns per meter and radius 2.50 cm. The current in the solenoid is increasing at a uniform rate of 60.0 A/s. What is the magnitude of the induced electric field at a point near the center of the solenoid and (a) 0.500 cm from the axis of the solenoid; (b) 1.00 cm from the axis of the solenoid?

For any position inside the solenoid ($r < R = 2.50 \text{ cm}$), the magnetic field inside a solenoid is

$$B = \mu_0 n I \quad (21)$$

neglecting fringe effects, which we can do because the solenoid is long and thin.

Because the system has spherical symmetry, we know that the electric field must also have cylindrical symmetry, so \mathbf{E} can only be a function of the distance r from the solenoid's axis.

From Gauss' law

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\epsilon_0} , \quad (22)$$

we know that the radial component of the electric field must be zero, since there is no excess of charge in the system, and any radial component to the \mathbf{E} -field would lead to a net flux through a Gaussian cylinder.

We can find the tangential component of the electric field from Faraday's law

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} . \quad (23)$$

We choose the path to be a circle of radius r centered on the cylinder axis. Because all the requested radii are inside the solenoid, the flux-area is the entire πr^2 area within the loop.

$$E \oint dl = -\frac{d}{dt}(\pi r^2 B) \quad (24)$$

$$2\pi r E = -\pi r^2 \frac{dB}{dt} = -\pi r^2 \frac{d}{dt}(\mu_0 n I) \quad (25)$$

$$2E = -r \mu_0 n \frac{dI}{dt} \quad (26)$$

$$E = -\frac{1}{2} r \mu_0 n \frac{dI}{dt} = -\frac{1}{2} \cdot 4\pi \cdot 10^{-7} \text{ Wb/A}\cdot\text{m} \cdot 900 \cdot 60.0 \text{ A/s} \cdot r = -0.0339 \text{ V/m}^2 \cdot r . \quad (27)$$

Plugging in the three requested radii

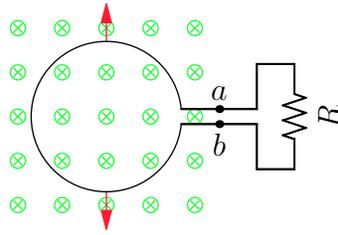
$$E(0) = 0 \quad (28)$$

$$E(0.500 \text{ cm}) = -170 \mu\text{V/m} \quad (29)$$

$$E(1.00 \text{ cm}) = -339 \mu\text{V/m} , \quad (30)$$

where the $-$ sign denotes counter-clockwise electric field when looking along the direction of the solenoid's magnetic field.

Problem 29.53. A flexible, circular loop 6.50 cm in diameter lies in a magnetic field with magnitude 0.950 T, directed into the plane of the page, as shown in Fig. 29.46. The loop is pulled at the points indicated by the arrows, forming a loop of zero area in 0.250 s. (a) Find the average induced emf in the circuit. (b) What is the direction of the current in R: from a to b or from b to a? Explain your reasoning.



(a)

$$\Delta\Phi = \Delta A \cdot B = (A_f - A_i)B = -A_i B, \quad (31)$$

so the average \mathcal{E} is

$$\mathcal{E} = -\frac{\Delta\Phi}{\Delta t} = \frac{A_i B}{\Delta t} = \frac{\pi(d/2)^2 B}{\Delta t} = \frac{\pi \cdot (3.25 \cdot 10^{-2} \text{ m})^2 \cdot 0.950 \text{ T}}{0.250 \text{ s}} = 12.6 \text{ mV}. \quad (32)$$

(b) The positive emf drives a current in the B -field direction: clockwise, so current passes through the resistor **from a to b** .