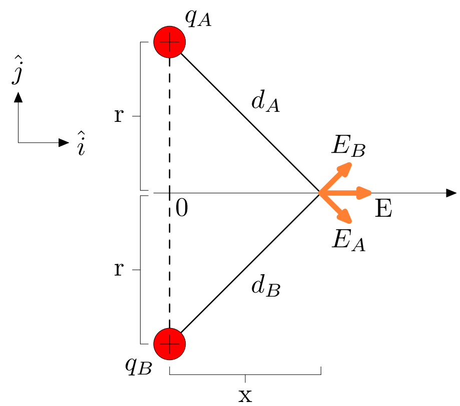
Recitation 1, additions

Ring electric field derivation

I said in my Wednesday class that I'd post a simple, non-calculus derivation of the formula giving the electric field along the axis of a charged ring, so here it is.

First, consider the situation where we have two charges $q_A = q_B = Q/2$ instead of a ring.



The magnitude of the electric field generated by q_A at x is given by

$$E_A = k_e \frac{q_A}{d_A^2} \, , \tag{1}$$

and that of q_B by

$$E_A = k_e \frac{q_A}{d_B^2} \, . \tag{2}$$

From the Pythagorean theorem we have

$$d_A = d_B = \sqrt{r^2 + x^2}. \tag{3}$$

From symmetry, we can see the electric field in the y direction cancels out, so our total electric field is given by

$$\mathbf{E} = 2k_e \frac{q_A}{d_A^2} \cos\theta \hat{\mathbf{i}} = 2k_e \frac{q_A}{d_B^2} \cos\theta \hat{\mathbf{i}} = k_e \frac{Q}{r^2 + x^2} \cdot \frac{x}{\sqrt{r^2 + x^2}} \hat{\mathbf{i}} = \frac{k_e Q x}{(r^2 + x^2)^{3/2}} \hat{\mathbf{i}} \, , \tag{4}$$

where Q is the sum of the two charges.

Moving back to the ring, we see the same situation, with each little charge dq having it's $\mathbf{E}_y$ cancled out by an equal dq directly across the ring. All that's left is the sum of the $\mathbf{E}_x$, which by the same argument as above is

$$\mathbf{E} = \frac{k_e Q x}{(r^2 + x^2)^{3/2}} \hat{\mathbf{i}} \, , \tag{5}$$

where Q is the total charge on the ring.

This matches up with the formula obtained following the derivation in Example 19.5 (p. 616).