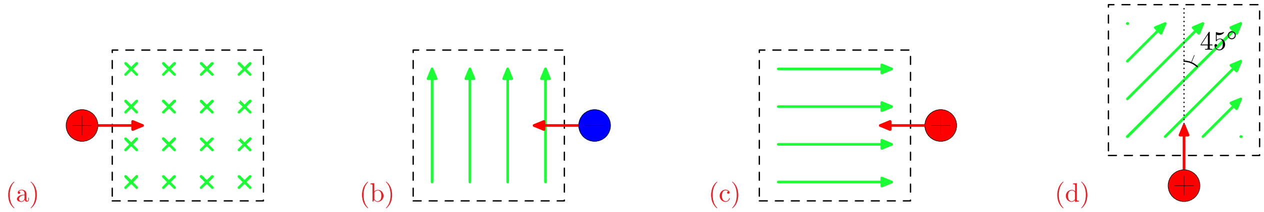
# Recitation 7
Chapter 22

**Problem 1.** Determine the initial direction of the deflection of charged particles as they enter magnetic fields as shown in Figure P22.1.




Using our right hand rule for the cross product, and $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$
(a) Force is up.
(b) Force is out of the page.
(c) No force.
(d) Force is into the page.

**Problem 3.** A proton travels with a speed of $v = 3.00 \cdot 10^6$ m/s at an angle of $\theta = 37.0°$ with the direction of a magnetic field of $B = 0.300$ T in the $+y$ direction. What are (a) the magnitude of the magnetic force on the proton and (b) its acceleration?

We'll pick the $\hat{\mathbf{i}}$ direction so that $\mathbf{v}$ has a positive $x$-component.
(a)

$$F_B = qvB\sin\theta = (1.60 \cdot 10^{-19}\ \text{C}) \cdot (3.00 \cdot 10^6\ \text{m/s}) \cdot (0.300\ \text{T}) \cdot \sin 37.0° = 8.67 \cdot 10^{-14}\ \text{N} \tag{1}$$

and the direction of the force is in the $\hat{\mathbf{k}}$ direction.
(b) Using $\mathbf{F} = m\mathbf{a}$ we have

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{8.67 \cdot 10^{-14}\ \text{N}}{1.67 \cdot 10^{-27}\ \text{kg}} = 5.19 \cdot 10^{13}\ \text{m/s}^2 \tag{2}$$

**Problem 4.** An electron is accelerated through $V = 2400$ V from rest and then enters a uniform $B = 1.70$ T magnetic field. What are (a) the maximum and (b) the minimum values of the magnetic force this charge can experience?

First we compute the electron's velocity $v$ upon entering the field. Conserving energy

$$qV = \frac{1}{2}mv^2 \tag{3}$$

$$v = \sqrt{\frac{2qV}{m}} = 29.0\ \text{Mm/s} \tag{4}$$

The magnetic force is given by $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, so it is maximized when $\mathbf{B}$ is perpendicular to $\mathbf{v}$, at which point

$$F = qvB = 7.90\ \text{pN} \tag{5}$$

The force is minimized then $\mathbf{B}$ is parallel (or anti-parallel) to $\mathbf{v}$, at which point $F = 0$.

**Problem 10.** A velocity selector consists of electric and magnetic fields described by the expressions $\mathbf{E} = E\hat{\mathbf{k}}$ and $\mathbf{B} = B\hat{\mathbf{j}}$, with $B = 15.0$ mT. Find the value of $E$ such that a $K = 750$ eV electron moving in the $\hat{\mathbf{i}}$ direction is undeflected.

The force from the magnetic field is in the $-\hat{\mathbf{k}}$ direction (right hand rule), so the sign of $E$ must be negative (to push the electron in the $\hat{\mathbf{k}}$ direction).
The velocity of the electron is given by

$$K = \frac{1}{2}mv^2 \tag{6}$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \cdot (750 \cdot 1.60 \cdot 10^{-19}\ \text{J})}{9.11 \cdot 10^{-31}\ \text{kg}}} = 16.2\ \text{Mm/s} \tag{7}$$

Balancing the magnitudes of the two forces

$$F_e = qE = F_B = qvB \tag{8}$$

$$E = vB = 243\ \text{kV/m} \tag{9}$$

So $\mathbf{E} = -243\hat{\mathbf{k}}$ kV/m.

**Problem 12.** A cyclotron designed to accelerate protons has an outer radius of $R = 0.350$ m. The protons are emitted nearly at rest from a source at the center and are accelerated through $V = 600$ V each time they cross the gap between the dees. The dees are between the poles of an electromagnet where the field is $B = 0.800$ T. (a) Find the cyclotron frequency $f$. (b) Find the speed $v_e$ at which the protons exit the cyclotron and (c) their kinetic energy $K$. (d) How many revolutions $N$ does a proton make in the cyclotron? (e) For what time $\Delta t$ interval does one proton accelerate?

(a) Protons with velocities $v$ in a constant magntic field will move in circles of radius $r$ in the plane perpendicular to the magnetic field. The centerward acceleration is given by

$$F_c = m\frac{v^2}{r} = F_B = qvB \tag{10}$$

$$v = \frac{qrB}{m} \tag{11}$$

Their velocity can also be related to their period $T = 1/f$ by

$$v = \frac{dr}{dt} = \frac{2\pi r}{T} = 2\pi r f \tag{12}$$

So

$$2\pi r f = \frac{qrB}{m} \tag{13}$$

$$f = \frac{qB}{2\pi m} = 12.2 \text{ MHz} \tag{14}$$

This is the frequency of revolution for a proton *anywhere* inside the cyclotron.
(b) Using our $v(r)$ equation from (a) when $r = R$, we have

$$v_e = \frac{qRB}{m} = \frac{(1.60 \cdot 10^{-19} \text{ C}) \cdot (0.350 \text{ m}) \cdot (0.800 \text{ T})}{1.69 \cdot 10^{-27} \text{ kg}} = 26.8 \text{ Mm/s} \tag{15}$$

(c)

$$K = \frac{1}{2}mv^2 = \frac{(qrB)^2}{2m} = 6.01 \cdot 10^{-13} \text{ J} \tag{16}$$

(d) The kinetic energy $K$ is built up from $2N$ passes through $V$ (twice per revolution).

$$N = \frac{K}{2qV} = \frac{6.01 \cdot 10^{-13} \text{ J}}{2 \cdot (1.60 \cdot 10^{-19}) \cdot (600 \text{ V})} = 3130 \tag{17}$$

(e)

$$\Delta t = TN = N/f = 257\ \mu\text{s} \tag{18}$$

**Problem 15.** A wire carries a steady current of $A = 2.40$ A. A straight section of the wire is $l = 0.750$ m long and lies in the $\hat{\mathbf{i}}$ direction within a uniform magbnetic field, $\mathbf{B} = 1.60\hat{\mathbf{k}}$ T. What is the magnetic force on the section of wire?

We can find the magnetic force on a wire using $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. Consider an infinitesimal bit of wire of length $d\mathbf{s}$, a current $I = dq/dt$ means that $dq$ will move through this bit of wire in time $dt$. So the force on the bit of wire is

$$\mathbf{dF} = dq\frac{\mathbf{ds}}{dt} \times \mathbf{B} = \frac{dq}{dt}\mathbf{ds} \times \mathbf{B} = I\mathbf{ds} \times \mathbf{B} \tag{19}$$

If the wire is straight, we can integrate easily to find the total force on the whole segment

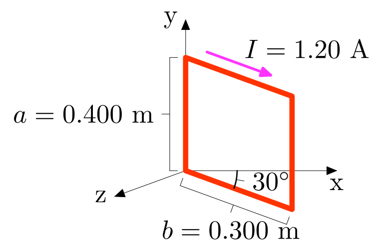
$$\mathbf{F} = \int_0^l \mathbf{dF} = \int_0^l I\mathbf{ds} \times \mathbf{B} = IB\sin\theta \int_0^l ds = IlB\sin\theta = I\mathbf{l} \times \mathbf{B} \tag{20}$$

Plugging in for our specific case we get a force in the $-\hat{\mathbf{j}}$ direction from the right-hand-rule, with a magnitude of

$$F = IlB = (2.40 \text{ A}) \cdot (0.750 \text{ m}) \cdot (1.60 \text{ T}) = 2.88 \text{ N} \tag{21}$$

So $\mathbf{F} = -2.88\hat{\mathbf{j}}$ N

**Problem 21.** A rectangular coil consists of $N = 100$ closely wrapped turns and has dimensions $a = 0.400$ m and $b = 0.300$ m. The coil is hinged along the $y$ axis, and its plane makes an angle $\theta = 30.0°$ with the $x$ axis (Fig. P22.21). What is the magnitude of the torque exerted on the coil by a uniform magnetic field $B = 0.800$ T directed along the x axis whwn the current is $I = 1.20$ A in the direction shown? What is the expected direction of motion of the coil?




Using our formula for force on a wire segment $\mathbf{F} = I\mathbf{l} \times \mathbf{B}$, and recalling that torque is defined $\tau = \mathbf{r} \times \mathbf{F}$, we can use the right hand rule to find the direction of motion.
The torque from the portion of the coil lying on the $y$ axis is zero, because the lever arm is zero ($r$ in the torque equation). The torque from the top portion is also zero, because the force is in the $\hat{\mathbf{j}}$ direction and the coil is not free to rotate in that direction. Similarly the torque from the bottom portion is zero, because the force is in the $-\hat{\mathbf{j}}$ direction. All the torque comes from the force on the outer leg, giving a force in the $\hat{\mathbf{k}}$ direction. So we expect the angle $\theta$ to increase.
To find the magnitude of the torque, we simply plug in

$$\tau = \mathbf{r} \times \mathbf{F} = bF\cos\theta = b\cos\theta \cdot (NIaB) = IabB\cos\theta = 9.98 \text{ J} \tag{22}$$

Where we multiplied the force from a single wire by $N$ because there are $N$ wraps, and took $\cos\theta$ to get the perpendicular force because $\theta$ is the complement of the angle between $r$ and $F$.