Recitation 5

Chapter 21

Problem 1. In a particular cathode-ray tube, the measured beam current is $I = 30.0\ \mu$A. How many electrons strike the tube screen every $\Delta t = 40.0$ s

Current is defined as *charge passing through a given surface per unit time* or in SI units:

$$1 \text{ A} = \frac{1 \text{ C}}{1 \text{ s}} . \tag{1}$$

So

$$\Delta Q = I\Delta t = 1.20 \text{ mC} \tag{2}$$

$$N_e = \frac{\Delta Q}{e} = 7.50 \cdot 10^{15} , \tag{3}$$

where $e = 1.60 \cdot 10^{-19}$ C is the charge on one electron.

Problem 4. The quantity of charge q (in coulombs) that has passed through a surface of area $A = 2.00 \text{ cm}^2$ varies with time according to the equation $q = 4t^3 + 5t + 6$, where t is in seconds. (a) What is the instantaneous current across the surface at $t_a = 1.00$ s? (b) What is the value of the current density?

(a)

$$I(t) = \frac{dQ}{dt} = 12t^2 + 5 \tag{4}$$

$$I(t_a) = 17.0 \text{ A} \tag{5}$$

(b)

$$j(t_a) = I(t_a)/A = 8.50 \text{ A/cm}^2 \tag{6}$$

Problem 14. A toaster is rated at $P = 600$ W when connected to a $V = 120$ V source. What current I does the toaster carry, and what is its resistance R?

(Assuming the voltage is DC). The power through a resistor is given by $P = IV$ so

$$I = \frac{P}{V} = \frac{600 \text{ W}}{120 \text{ V}} = 5 \text{ A} \tag{7}$$

The voltage across a resistor is given by $V = IR$ so

$$R = \frac{V}{I} = \frac{120 \text{ V}}{5 \text{ A}} = 24\ \Omega \tag{8}$$

Problem 17. Suppose a voltage surge produces $V_s = 140$ V for a moment. By what percentage p does the power output of a $V = 120$ V, $P = 100$ W lightbulb increase? Assume the resistance does not change.

The voltage across a resistor is

$$V = IR \tag{9}$$

So power absorbed by a resistor is

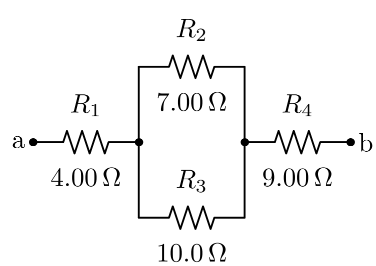
$$P = IV = \frac{V^2}{R} \tag{10}$$

And the fractional change in power f is given by

$$f = \frac{P_s}{P} = \frac{V_s^2/R}{V^2/R} = \left(\frac{V_s}{V}\right)^2 = 1.361 \tag{11}$$

So $p = 36.1$ %.

Problem 27. (a) Find the equivalent resistance between points a and b in Figure P21.27. (b) A potential difference of $V = 34.0$ V is applied between points a and b. Calculate the current in each resistor.



(a) First, we calculate the equivalent resistance to the two resistors in parallel

$$R_p = \left(\frac{1}{R_2} + \frac{1}{R_1}\right)^{-1} = 4.12\ \Omega \tag{12}$$

Then we calculate the equivalent resistance of the three resistors in series

$$R_{ab} = R_1 + R_p + R_4 = 17.1\ \Omega \tag{13}$$

(b) Now applying $V = IR$ to the equivalent system

$$I_{ab} = I_1 = I_4 = I_p = \frac{V}{R_{ab}} = 1.99\ \text{A} \tag{14}$$

From which we can compute the voltage across the parallel resistors

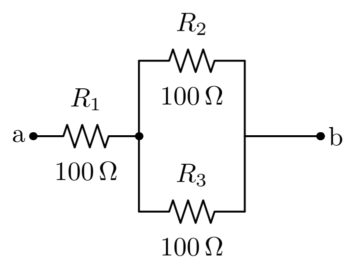
$$V_p = I_p R_p = 8.18\ \text{V} \tag{15}$$

Giving us currents of

$$I_2 = \frac{V_p}{R_2} = 1.17\ \text{A} \tag{16}$$

$$I_3 = \frac{V_p}{R_3} = 0.818\ \text{A} \tag{17}$$

Problem 30. Three $R = 100\ \Omega$ resistors are connected as shown in Figure P21.30. The maximum power that can safely be delivered to any one resistor is $P_{max} = 25.0$ W. (a) What is the maximum voltage that van be applies to the terminals a and b? (b) For the voltage determined in (a), what is the power delivered to each resistor? What is the total power delivered?



(a) The current through the entire setup I_{ab} all goes through R_1, so $I_{ab} = I_1$. Then it splits 50/50, so $I_{ab} = 2I_2 = 2I_3$. (R_1 and R_2 each get half the current going through R_1). Because it gets the most current, the maximum current I_{ab} is when the power P_1 absorbed by R_1 is P_{max}.

$$P_{max} = \frac{V_1^2}{R_1} \tag{18}$$

$$V_1 = \sqrt{R_1 P_{max}} = 50\ \text{V} \tag{19}$$

So $I_1 = I_{ab} = V_1/R_1 = 0.500$ A.
The equivalent resistance of the two parallel resistors is

$$R_p = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = 50\ \Omega \tag{20}$$

So the voltage drop over them is $V_p = I_{ab} R_p = 25.0$ V.
Adding the two voltages together

$$V_{ab} = V_1 + V_p = 75.0\ \text{V} \tag{21}$$

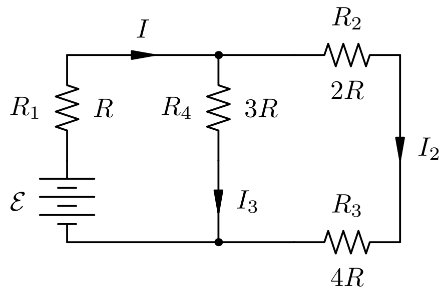
(b) The power absorbed by the other two resistors is then

$$P_2 = P_3 = I_2 V_p = 0.250\ \text{A} \cdot 25.0\ \text{V} = 6.25\ \text{W}\ , \tag{22}$$

and the total power delivered is

$$P = P_1 + P_2 + P_3 = (25 + 2 \cdot 6.25)\ \text{W} = 37.5\ \text{W}\ . \tag{23}$$

Problem 32. Four resistors are connected to a battery as shown in Figure P21.32. The current in the battery is I, the battery emf is ϵ, and the resistor values are $R_1 = R$, $R_2 = 2R$, $R_3 = 4R$, and $R_4 = 3R$. (a) Rank the resistors according to the potential difference across them, form largest to smallest. Note any cases of equal potential difference. (b) Determine the potential difference across each resistor in terms of ϵ. (c) Rank the resistors according to the current in them, from largest to smallest. Note any cases of equal current. (d) Determine the current in each resistor in terms of I. (e) If R_3 is increased, what happens to the current in each of the resistors? (f) In the limit that $R_3 \to \infty$, what are the new values of the current in each resistor in terms of I, the original current in the battery?



(a) R_2 and R_3 both have I_2 passing through them, so from Ohm's law we know $V_2 = I_2 R_2 < V_3 = I_2 R_3$, because $R_2 < R_4$. R_4 and the equivalent resistance $R_s = R_2 + R_3$ are in parallel, so they have the same voltage across them. Because $V_4 = V_s = V_2 + V_4$, the voltage V_4 across R_4 is greater than either V_2 or V_3. Finally, the equivalent resistance of R_4 and R_s in parallel is given by

$$R_p = \left(\frac{1}{R_4} + \frac{1}{R_s}\right)^{-1} = \left(\frac{1}{3R} + \frac{1}{6R}\right)^{-1} = 3R\left(1 + \frac{1}{2}\right)^{-1} = 3R \cdot \frac{2}{3} = 2R \, ,$$

so $R_p > R_1$. Since both R_p and R_1 have I going through them, and $V_p = V_4 = V_s > V_1$. We still need to place V_1 relative to V_2 and V_3, so we use the formula for voltage across series resistors

$$I = \frac{V_A}{R_A} = \frac{V_B}{R_B} \, .$$

$R_p = 2R_1$, so $V_1 = V_p/2$, and $R_3 = 2R_2$, so $V_3 = 2V_2$. $V_3 + V_2 = V_p$, so $V_3 = 2/3 \cdot V_p$ and $V_2 = V_p/3$. The final ranking is therefore $V_4 = V_p > V_3 = 2/3 \cdot V_p > V_1 = V_p/2 > V_2 = V_p/3$.
(b) We've done most of the work in (a).

$$\mathcal{E} = V_1 + V_p = \frac{3V_p}{2} \, ,$$

so

$$V_4 = V_p = \frac{2\mathcal{E}}{3} \tag{24}$$
$$V_1 = \frac{V_p}{2} = \frac{\mathcal{E}}{3} \tag{25}$$
$$V_2 = \frac{V_p}{3} = \frac{2\mathcal{E}}{9} \tag{26}$$
$$V_3 = \frac{2V_p}{3} = \frac{4\mathcal{E}}{9} \tag{27}$$

(c) $I = I_2 + I_3$, and all our currents are positive as we've labled them, so I is greater than I_2 and I_3. $R_4 = 3R < R_s = 6R$, so $I_3 > I_2$. The final ranking is therefore $I > I_3 > I_2$, with I_2 passing through both R_2 and R_3.
(d) To be quantitative about (c), we can use Ohm's law for each current:

$$I = \frac{V_1}{R_1} = \frac{\mathcal{E}}{3} \cdot \frac{1}{R} = \frac{\mathcal{E}}{3R} \tag{28}$$
$$I_3 = \frac{V_p}{R_4} = \frac{2\mathcal{E}}{3} \cdot \frac{1}{3R} = \frac{2}{3} \cdot \frac{\mathcal{E}}{3R} = \frac{2I}{3} \tag{29}$$
$$I_2 = \frac{V_p}{R_s} = \frac{2\mathcal{E}}{3} \cdot \frac{1}{6R} = \frac{1}{3} \cdot \frac{\mathcal{R}}{3R} = \frac{I}{3} \, . \tag{30}$$

We see that $I = I_2 + I_3$, as it should by Kirchoff's junction rule.
(e) If R_3 increases, R_s increases and R_p increases, so I_2 and I decrease. The change in I_3 is a balance of increased flow relative to I_2 and decreased overall I. We see that less current through I drops V_1, but $V_1 + V_4 = \mathcal{E}$ which doesn't change, so V_4 increases, so I_3 increases.
(f) As $R_3 \to \infty$, I_2 is choked off entirely, so $I = I_3$. So I flows through R_1 and R_4, and nothing flows through R_2 and R_3.