Recitation 4

Chapter 20

Problem 40. Two capacitors, $C_1 = 5.00\ \mu$F and $C_2 = 12.0\ \mu$F, are connected in series, and the resulting combination is connected to a $\Delta V = 9.00$ V battery. Find (a) the equivalent capacitance of the combination, (b) the potential difference across each capacitor, and (c) the charge on each capacitor.

(a) The wire connecting the inner plates of C_1 and C_2 contains no net charge, so we know that any charge on the inner plate of C_1 must have come from the inner plate of C_2. Because these charges are equal and opposite, the total charge Q on each capacitor seperately is the same for both ($Q_1 = Q_2$). So using the definition of capacitance for both cases we have

$$\Delta V_1 = Q/C_1 \tag{1}$$
$$\Delta V_2 = Q/C_2 \tag{2}$$
$$\Delta V = \Delta V_1 + \Delta V_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{Q}{C_{eq}} \tag{3}$$

So

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{5.00 \cdot 10^{-6}\ \text{F}} + \frac{1}{12.0 \cdot 10^{-6}\ \text{F}}\right)^{-1} = 3.53\ \mu\text{F} \tag{4}$$

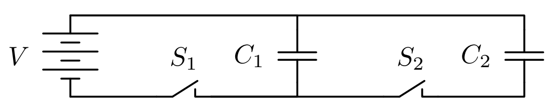
(c) Plugging back into equation 3 we have

$$Q = \Delta V \cdot C_{eq} = 3.53\ \mu\text{F} \cdot 9.00\ \text{V} = 31.8\ \mu\text{C} \tag{5}$$

(b) And plugging into equations 1 and 2 we have

$$\Delta V_1 = \frac{31.8 \cdot 10^{-6}\ \text{C}}{5.00 \cdot 10^{-6}\ \text{F}} = 6.35\ \text{V} \tag{6}$$
$$\Delta V_2 = \frac{31.8 \cdot 10^{-6}\ \text{C}}{12.0 \cdot 10^{-6}\ \text{F}} = 2.65\ \text{V} \tag{7}$$
$$\tag{8}$$

Problem 43. Consider the circuit shown in Figure P20.43, where $C_1 = 6.00\ \mu$F, $C_2 = 3.00\ \mu$F, and $\Delta V = 20.0$ V. Capacitor C_1 is first charged with Q_1 by the closing of switch S_1. Switch S_1 is then opened, and the charged capacitor is connected to the uncharged capacitor by the closing of S_2. Calculate Q_1 and the final charge on each capacitor (Q_1' and Q_2').



The first situation with S_1 closed and S_2 open is just a standard capacitor charging problem. Using the definition of capacitance

$$Q_1 = C_1 \Delta V = 6.00 \cdot 10^{-6}\ \text{F} \cdot 20.0\ \text{V} = 120\ \mu\text{C} \tag{9}$$

After disconnecting the battery and connecting the two capacitors, we have a net charge of Q_1 in the upper wire that we can distribute as we desire between C_1 and C_2. Because charge is conserved, we know

$$Q_1 = Q_1' + Q_2' \tag{10}$$

We also know that at equilibrium the voltage across each capacitor must be equal (because if there was a voltage difference beween the upper plates of the two capacitors, it would push current through the upper wire until the voltage difference dissapeared, etc.). So

$$\Delta V_1' = \frac{Q_1'}{C_1} = \Delta V_2' = \frac{Q_2'}{C_2} \tag{11}$$

Now we have two equations relating our two unknowns Q_1' and Q_2'. Solving equation 11 for Q_2' and plugging into equation 10 we get

$$Q_2' = \frac{C_2}{C_1} Q_1' \tag{12}$$
$$Q_1 = \left(1 + \frac{C_2}{C_1}\right) Q_1' \tag{13}$$
$$Q_1' = \frac{Q_2}{1 + C_2/C_1} = \frac{120\ \mu\text{C}}{1.5} = 80\ \mu\text{C} \tag{14}$$
$$Q_2' = 0.5 \cdot 80\ \mu\text{C} = 40\ \mu\text{C} \tag{15}$$

Problem 47. (a) A $C = 3.00\ \mu$F capacitor is connected to a $\Delta V_a = 12.0$ V battery. How much energy U_a is stored in the capacitor? (b) If the capacitor had been connected to a $\Delta V_b = 6.00$ V battery, how much energy would have been stored?

Simply plugging into the formula for energy stored in a capacitor we have

$$U_a = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00 \cdot 10^{-6}\ \text{F}) \cdot (12.0\ \text{V})^2 = 216\ \mu\text{J} \tag{16}$$

$$U_b = \frac{1}{2}(3.00 \cdot 10^{-6}\ \text{F}) \cdot (6.0\ \text{V})^2 = 54\ \mu\text{J} \tag{17}$$

Problem 51. Show that the force between two plates of a parallel-plate capacitor each have an attractive force given by

$$F = \frac{Q^2}{2\epsilon_0 A} \tag{18}$$

The electric field generated by the plate A is given by $E_A = Q/2\epsilon_0 A$ (which we derived for P19.62, along with $\sigma = Q/A$). So the force on plate B due to plate A is given by

$$F = QE_A = \frac{Q^2}{2\epsilon_0 A} \tag{19}$$

Problem 54. (a) How much charge Q_c can be placed on a capacitor with air between the plates before it breaks down if the area of each plate is $A = 5.00\ \text{cm}^2$? (b) Find the maximum charge assuming polystyrene is used between the plates instead of air.

From Chapter 19, the voltage difference due to a constant electric field $\mathbf{E}$ over a displacement $\mathbf{d}$ is given by $\Delta V = \mathbf{E} \cdot \mathbf{d}$. So for two plates a distance d apart, the breakdown voltage is given by

$$V_c = E_c d \tag{20}$$

where E_c is the dielectric strength of the material.
The capacitance of a parallel-plate capacitor is given by

$$C = \frac{\kappa\epsilon_0 A}{d} \tag{21}$$

Combining these two formula with the definition of capacitance we have

$$E_c d = V = \frac{Q}{C} = \frac{Qd}{\kappa\epsilon_0 A} Q = \kappa E_c \epsilon_0 A \tag{22}$$

Looking up the values for air and polystyrene in Table 20.1 on page 699 of the text we see:

Name	Dielectric constant κ	Dielectric strength E_c
Air	1.00059	$3 \cdot 10^6$ V/m
Polystyrene	2.56	$24 \cdot 10^6$ V/m

So plugging into our formula for the charge

$$Q_a = 1.00 \cdot (3 \cdot 10^6\ \text{V/m}) \cdot (8.85 \cdot 10^{-12}\ \text{C}^2/\text{N m}^2) \cdot 5 \cdot 10^{-4}\ \text{m}^2 = 1.33 \cdot 10^{-8}\ \text{C} \tag{23}$$

$$Q_b = 2.56 \cdot (24 \cdot 10^6\ \text{V/m}) \cdot (8.85 \cdot 10^{-12}\ \text{C}^2/\text{N m}^2) \cdot 5 \cdot 10^{-4}\ \text{m}^2 = 2.72 \cdot 10^{-7}\ \text{C} \tag{24}$$

Problem 73. A parallel-plate capacitor is constructed using a dielectric material whose dielectric constant is $\kappa = 3.00$ and whose dielectric strength is $E_c = 2.00 \cdot 10^8$ V/m. The desired capacitance is $C = 0.250\ \mu$F, and the capacitor must withstand a maximum potential difference of $V_c = 4000$ V. Find the minimum area A of the capacitor plates.

Using equation 20, we have

$$d \geq \frac{V_c}{E_c} \tag{25}$$

Where equality represents a breakdown at V_c and larger d give us more protection with larger breakdown voltages.
From equation 21 we have

$$A = \frac{dC}{\kappa\epsilon_0} \tag{26}$$

From which we can see that the smaller d is, the smaller A can be, and we pick $d = V_c/E_c$, the smallest possible value we can. Then the smallest area is given by

$$A = \frac{V_c C}{E_c \kappa \epsilon_0} = \frac{(4000\ \text{V}) \cdot (0.25 \cdot 10^{-6}\ \text{F})}{(2.00 \cdot 10^8\ \text{V/m}) \cdot 3.00 \cdot (8.85 \cdot 10^{-12}\ \text{C}^2/\text{Nm}^2)} = 0.188\ \text{m}^2 \tag{27}$$