Recitation 3

Chapter 20

Problem 3. A uniform electric field of magnitude $E = 250$ V/m is directed in the positive x direction ($\hat{\mathbf{i}}$). A $q = +12.0$ μC charge moves from the origin to the point $(x, y) = (20.0 \text{ cm}, 50.0 \text{ cm})$. (a) What is the change in the potential energy ΔU of the charge-field system? (b) Through what potential difference ΔV does the charge move?

(a) From the text Equation 20.1 (page 643) we see

$$\Delta U = -q \int_A^B \mathbf{E} \cdot d\mathbf{s} = -q \int_A^B E\hat{\mathbf{i}} \cdot d\mathbf{s} = -qE \int_{x_1}^{x_2} dx = -qE\Delta x \tag{1}$$

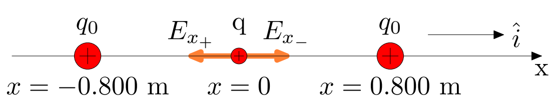
Which is the same process the book used to get to their Equation 20.9. Plugging in our numbers

$$\Delta U = -12.0 \cdot 10^{-6} \text{ C} \cdot 250 \text{ V/m} \cdot 0.200 \text{ m} = -6.00 \cdot 10^{-4} \text{ J} \tag{2}$$

(b) The change in electric potential is given by

$$\Delta V = \frac{\Delta U}{q} = -50.0 \text{ V} \tag{3}$$

Problem 8. Given two $q_0 = 2.00$ μC charges as shown in Figure P20.8 and a positive test charge of $q = 1.28 \cdot 10^{-18}$ C at the origin, (a) what is the net force exerted by the two q_0 charges on the test charge q? (b) What is the electric field at the origin due to the two q_0 charges? (c) What is the electrical potential at the origin due to the two q_0 charges?



(a) Letting $a = 0.800$ m and summing the forces from Coulomb's law

$$\mathbf{F} = \mathbf{F}_A + \mathbf{F}_B = k_e \left[\frac{q_0 q}{a^2} \hat{\mathbf{i}} + \frac{q_0 q}{a^2}(-\hat{\mathbf{i}}) \right] = 0 \tag{4}$$

Which makes sense because the situation is symmetric.
(b) Summing the electric fields

$$\mathbf{E}(0) = \mathbf{E}_A + \mathbf{E}_B = k_e \left[\frac{q_0}{a^2} \hat{\mathbf{i}} + \frac{q_0}{a^2}(-\hat{\mathbf{i}}) \right] = \frac{\mathbf{F}}{q} = 0 \tag{5}$$

(c) Summing the potentials

$$V(0) = V_A + V_B = k_e \frac{q_0}{a} + k_e \frac{q_0}{a} = 2k_e \frac{q_0}{a} = 2 \cdot 8.99 \cdot 10^9 Vm/C \frac{2.00 \cdot 10^{-6} \text{ C}}{0.800 \text{ m}} = 45.0UkV \tag{6}$$

Problem 19. A light, unstressed spring has a length d. Two identical particles, each with charge q, are connected to the opposite ends of the spring. The particles are held stationary a distance d apart and are then released at the same time. The spring has a bit of internal kinetic friction, so the oscillation is damped. The particles eventually stop vibrating when the distance between them is $3d$. Find the increase in internal energy ΔE_i that appears in the spring during the oscillations. Assume that the system of the spring and the two charges is isolated.

From last quarter, we remember that spring potential energy is given by $U_s = 1/2 \cdot kx^2$. To plug in for k, we balance the forces at equilibrium

$$F_e = k_e \left(\frac{q}{3d} \right)^2 = F_s = k \cdot 2d \tag{7}$$

$$k = k_e \frac{q^2}{9 \cdot 2 \cdot d^3} \tag{8}$$

From this chapter (Equation 20.13), we see that the electrical potential energy of two charges is given by

$$U_e = k_e \frac{q_1 q_2}{r_{12}} \tag{9}$$

So the total potential energy of the system in it's final state is given by the sum of the electric U_e and spring U_s potentials.

$$U = U_e + U_s = \frac{1}{2} k(r - d)^2 + k_e \frac{q^2}{r} \tag{10}$$

The total energy at a point in time is the sum of potential and internal energies

$$E_t = U_t + E_{it} \ . \tag{11}$$

Since we were only interested in the change in internal energy, we can set the initial internal energy to 0, and call the final internal energy E_i.
Conserving energy $E_0 = E_1$ (because the system is isolated)

$$E_0 = U_0 = E_1 = U_1 + E_i \tag{12}$$

$$E_i = U_0 - U_1 = k_e \frac{q^2}{d} - \frac{1}{2}k(2d)^2 - k_e \frac{q^2}{3d} = k_e \frac{q^2}{d}\left(1 - \frac{1}{3}\right) - 2kd^2 \tag{13}$$

$$= k_e \frac{2q^2}{3d} - 2\left(k_e \frac{q^2}{9 \cdot 2 \cdot d^3}\right) d^2 = k_e \frac{2q^2}{3d} - k_e \frac{q^2}{9d} = \frac{5k_e q^2}{9d} \ , \tag{14}$$

Problem 20. In 1911, Ernest Rutherford and his assistants Hans Geiger and Ernest Mardsen conducted an experiment in which they scattered alpha particles from thin sheets of gold. An alpha particle, having a charge of $q_\alpha = +2e$ and a mass of $m = 6.64 \cdot 10^{-27}$ kg is a product of certain radioactive decays. The results of the experiment lead Rutherford to the idea that most of the mass of an atom is in a very small nucleus, whith electrons in orbit around it, in his planetary model of the atom. Assume that an alpha particle, initially very far from a gold nucleus, is fired with a velocity $v = 2.00 \cdot 10^7$ m/s directly toward the nucleus (charge $Q = +79e$). How close does the alpha particle get to the nucleus before turning around? Asume that the gold nucleus remains stationary.

Let r be the distance between the alpha particle and the gold nucleus. Conserving energy between the initial point at $r = \infty$ where the energy is all kinetic

$$E_0 = \frac{1}{2}mv_0^2 \tag{15}$$

And the point of closest approach where the energy is all electric potential

$$E_1 = k_e \frac{(2e)(79e)}{r} \tag{16}$$

We have

$$E_0 = \frac{1}{2}mv^2 = E_1 = k_e \frac{158e^2}{r} \tag{17}$$

$$r = \frac{2 \cdot 158 \cdot k_e e^2}{mv^2} = \frac{316 \cdot 8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2 \cdot (1.60 \cdot 10^{-19} \text{ C})^2}{6.64 \cdot 10^{-27} \text{ kg} \cdot (2.00 \cdot 10^7 \text{ m/s})^2} = 2.74 \cdot 10^{-14} \text{ m} \tag{18}$$

Which is significantly less than the $r_e \sim 10^{-10}$ m radius of the gold atom.

Problem 21. The potential in a region between $x = 0$ and $x = 6.00$ m is $V = a + bx$, where $a = 10.0$ V and $b = -7.00$ V/m. Determine (a) the potential at $x = 0$, 3.00 m, and 6.00 m; and (b) the magnitude and direction of the electric field at $x = 0$, 3.00 m, and 6.00 m.

(a) Simply plugging into their $V(x)$ formula

$$V(0 \text{ m}) = 10.0 \text{ V} \tag{19}$$

$$V(3.00 \text{ m}) = 10.0 \text{ V} - 21.0 \text{ V} = -11 \text{ V} \tag{20}$$

$$V(6.00 \text{ m}) = 10.0 \text{ V} - 42.0 \text{ V} = -32 \text{ V} \tag{21}$$

(b) Using $E_x = -dV/dx$ we have

$$E = -\frac{d}{dx}(a + bx) = -b = 7.00 \text{ V/m} \tag{22}$$

At any point for $0 \leq x \leq 6.00$ m.

Problem 22. The electric potential insize a charged spherical conductor of radius R is given by $V_i = k_e Q/R$, and the outside potential is given by $V_o = k_e Q/r$. Using $E_r = -dV/dx$, determine the electric field (a) inside and (b) outside this charge distribution.

(a)

$$E_i = -\frac{d}{dx}\left(\frac{k_e Q}{R}\right) = 0 \tag{23}$$

Because V_i is constant with respect to r.
(b)

$$E_o = -\frac{d}{dx}\left(\frac{k_e Q}{r}\right) = -k_e Q \frac{d}{dx}\left(\frac{1}{r}\right) = -k_e Q \frac{-1}{r^2} = \frac{k_e Q}{r^2} \tag{24}$$

Problem 24. Consider a ring of radius R with the total charge Q spread uniformly over its perimeter. What is the potential difference between the point at the center of the ring and a point on its axis a distance $d = 2R$ from the center?

From the first week's recitation (P19.19), we have the electric field along the axis due to the ring as

$$\mathbf{E} = \frac{k_e x Q}{(x^2 + R^2)^{3/2}}\hat{\mathbf{i}} \tag{25}$$

So the potential drop from 0 to d is given by

$$\Delta V = -\int_0^d E_x dx = -k_e Q \int_0^d \frac{x \cdot dx}{(x^2 + R^2)^{3/2}} \tag{26}$$

Substituting $u = x^2 + R^2$ so $du = 2xdx$ we have

$$\Delta V = -k_e Q \int \frac{1/2 \cdot du}{u^{3/2}} = -\frac{1}{2} k_e Q \frac{-2}{\sqrt{u}} = \frac{k_e Q}{\sqrt{u}} \tag{27}$$

And plugging back in in terms of x

$$\Delta V = \left. \frac{k_e Q}{\sqrt{x^2 + R^2}} \right|_0^d = \frac{k_e Q}{\sqrt{d^2 + R^2}} - \frac{k_e Q}{R} = k_e Q \left(\frac{1}{\sqrt{(4+1)R^2}} - \frac{1}{R} \right) = \frac{k_e Q}{R} \left(\frac{1}{\sqrt{5}} - 1 \right) = -0.533 \frac{k_e Q}{R} \tag{28}$$