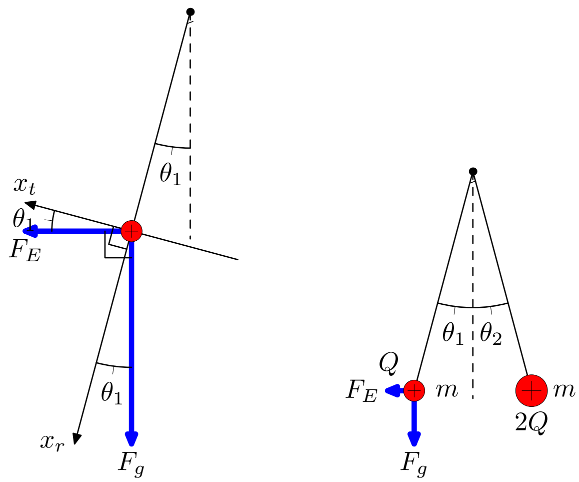
Problem 59. Two small spheres of mass m are suspended from strings of length l that are connected at a common point. One sphere has charge Q, and the other has charge $2Q$. The strings make angles θ_1 and θ_2 with the vertical. (a) How are θ_1 and θ_2 related? (b) Assume that θ_1 and θ_2 are small. Show that the distance r between the spheres is given by

$$r \approx \left(\frac{4k_eQ^2l}{mg}\right)^{1/3} \tag{1}$$



(a) Assuming that the charges are not rotating about each other, the forces on each charge must cancel. The forces on each sphere are gravity $F_g = mg$, electrostatic $F_E = k_e 2Q^2/r^2$, and tension T. The tension will automatically handle canceling forces in the radial direction, so we need only consider the tangential direction. Let us assume that F_E is purely in the horizontal direction (see (Note)). Summing the tangential forces on the first sphere

$$0 = F_E \cos\theta_1 - F_g \sin\theta_1 \tag{2}$$

$$\tan\theta_1 = \frac{F_E}{F_g} \tag{3}$$

And on the second sphere $\tan\theta_2 = \frac{F_E}{F_g}$ so $\theta_1 = \theta_2 = \theta$.

(b)

$$r = 2l\sin\theta \approx 2l\tan\theta = 2l\frac{F_E}{F_g} = 2l\frac{k_e 2Q^2/r^2}{mg} \tag{4}$$

$$r \approx \left(\frac{4lk_eQ^2}{mg}\right)^{1/3}, \tag{5}$$

where we used the small angle approximation $\sin\theta \approx \tan\theta$ for small θ.

(Note) Why $\mathbf{F}_E$ is horizontal.

Let q be the charge on the first mass and Q be the charge on the second. The force of 1 on 2 is given by $F_{12} = k_e qQ\hat{\mathbf{r}}_{12}/r^2$. This is identical to the force of 1 on 2 that we would get if we had put Q on 1 and q on 2 (let us say "the electric force does not care about which mass has which charge"). The only difference between the two masses is the charge, and the only effect of that difference (the electrostatic force) does not care about the difference, so the final situation must be symmetric ($\theta_1 = \theta_2$ [no calculation required :p] and $\mathbf{r}$ is horizontal). Because $\mathbf{F_E} \propto \hat{\mathbf{r}}_{12}$ it must also be horizontal.