

# Recitation 1

## Chapter 19

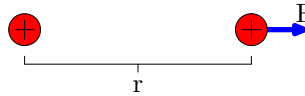
**Problem 3.** Nobel laureate Richard Feynman once said that if two persons stood at arm's length from each other and each person had  $p = 1\%$  more electrons than protons, the force of repulsion between them would be enough to lift a "weight" equal to that of the entire Earth. Carry out an order of magnitude calculation to substantiate this assertion.

Let  $m = 70$  kg be the mass of one person, and  $q_e$  be the charge of one electron. Assume that there are approximately equal numbers of protons, electrons, and neutrons in a person. Electrons have much less mass than protons or neutrons, so we ignore their mass contribution. Protons and neutrons have very similar masses, so  $N = (m/2)/m_p$  is the number of protons, and  $N_q = N \cdot p$  is the number of extra electrons in each person. Assume they are separated by  $r = 1$  m. The force of repulsion  $F$  is given by

$$F = k_e \frac{q^2}{r^2} = k_e \left( \frac{mpq_e}{2m_p r} \right)^2 = 9.0 \cdot 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \left( \frac{70 \text{ kg} \cdot 0.01 \cdot 1.6 \cdot 10^{-19} \text{ C}}{2 \cdot 1.7 \cdot 10^{-27} \text{ kg} \cdot 1 \text{ m}} \right)^2 \approx 1 \cdot 10^{10} \left( \frac{0.3 \cdot 10^{-19}}{1 \cdot 10^{-27}} \right)^2 \text{ N} = 1 \cdot 10^{25} \text{ N} \quad (1)$$

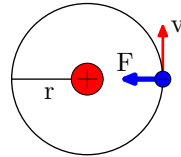
And a "weight" the mass of the earth would be  $F_g = Mg \approx 6 \cdot 10^{24} \text{ kg} \cdot 9.8 \text{ m/s}^2 \approx 6 \cdot 10^{25} \text{ N} \sim F$ .

**Problem 4.** Two protons in an atomic nucleus are typically separated by a distance of  $r = 2.00 \cdot 10^{-15}$  m. The electric repulsion force  $F$  between the protons is huge, but the attractive nuclear force is even stronger and keeps the nucleus from bursting apart. What is the magnitude of  $F$ ?



$$F = k_e \frac{q^2}{r^2} = 8.99 \cdot 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \left( \frac{1.60 \cdot 10^{-19} \text{ C}}{2.00 \cdot 10^{-15} \text{ m}} \right)^2 = 57.7 \text{ N} \quad (2)$$

**Problem 9.** In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is  $r = 0.529 \cdot 10^{-10}$  m. (a) Find the magnitude of the electric force each exerts on the other. (b) If this force causes the centripetal acceleration of the electron, what is the speed of the electron?



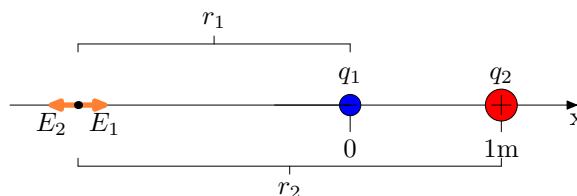
(a)

$$F = k_e \frac{q^2}{r^2} = 8.99 \cdot 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \left( \frac{1.60 \cdot 10^{-19} \text{ C}}{0.529 \cdot 10^{-10} \text{ m}} \right)^2 = 8.22 \cdot 10^{-8} \text{ N} \quad (3)$$

(b) Using  $F_c = ma_c = mv^2/r$

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{8.24 \cdot 10^{-8} \text{ N} \cdot 0.529 \cdot 10^{-10} \text{ m}}{9.11 \cdot 10^{-31} \text{ kg}}} = 2.19 \cdot 10^6 \text{ m/s} \quad (4)$$

**Problem 11.** In Figure P19.11, determine the point (other than infinity) at which the electric field is zero.  $q_1 = -2.50 \mu\text{C}$  and  $q_2 = 6.00 \mu\text{C}$ .



First, we need a coordinate system. Let  $q_1$  be the origin ( $x_1 = 0$ ), and  $q_2$  be at  $x_2 = 1.00$  m. The electric field of a finite number of point charge is given by (p. 612, 19.6)

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i \quad (5)$$

For any point off the  $x$  axis, there would be some force moving the charge in the vertical  $y$  direction, so we only need to look at positions on the  $x$  axis.

A positive test charge placed between the two charges would be pulled to the left by  $q_1$  and pushed to the left by  $q_2$ . A positive test charge placed to the right of  $q_2$  would be pushed to the right by  $q_2$  more strongly (because  $q_2 > q_1$  and  $r_2 < r_1$ ) than it would be pulled to the left by  $q_1$ . So the only place to look for equilibrium is to the left of  $q_1$ , ( $x < 0$ , where  $r_2 = r_1 + x_2$ ).

$$\mathbf{E} = k_e \left( -\frac{q_1}{r_1^2} - \frac{q_2}{(r_1 + x_2)^2} \right) = 0 \quad (6)$$

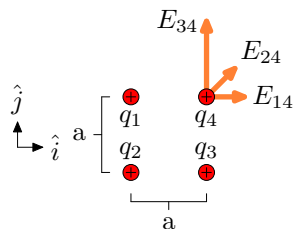
$$\frac{q_1}{r_1^2} = -\frac{q_2}{(r_1 + x_2)^2} \quad (7)$$

$$\frac{r_1 + x_2}{r_1} = 1 + \frac{x_2}{r_1} = \pm \sqrt{\frac{-q_2}{q_1}} \quad (8)$$

$$r_1 = \frac{x_2}{\sqrt{\pm \frac{-q_2}{q_1}} - 1} = 1.82 \text{ m}, -0.392 \text{ m} \quad (9)$$

But  $r_1 = -0.392$  m is between the two charges (where our assumption about the electric fields opposing each other doesn't hold), so  $\mathbf{E} = 0$  only at a  $r_1 = 1.82$  m ( $x = -1.82$  m).

**Problem 15.** Four point charges are at the corners of a square of side  $a$  as shown in Figure P19.15, with  $q_1 = 2q$ ,  $q_2 = 3q$ ,  $q_3 = 4q$ , and  $q_4 = q$ . (a) Determine the magnitude and direction of the electric field at the location of charge  $q_4$ . (b) What is the resultant force on  $q_4$ ?



Let  $\hat{\mathbf{i}}$  point to the right and  $\hat{\mathbf{j}}$  point up.

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i = k_e \left( \frac{2q}{a^2} \hat{\mathbf{i}} + \frac{3q}{(\sqrt{2}a)^2} \frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}} + \frac{4q}{a^2} \hat{\mathbf{j}} \right) = k_e \frac{q}{a^2} \left( 2\hat{\mathbf{i}} + \frac{3}{2\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}}) + 4\hat{\mathbf{j}} \right) \quad (10)$$

So the magnitude of  $\mathbf{E}$  is given by

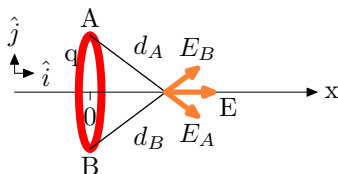
$$E = k_e \frac{q}{a^2} \sqrt{\left( 2 + \frac{3}{2\sqrt{2}} \right)^2 + \left( 4 + \frac{3}{2\sqrt{2}} \right)^2} = 5.91 k_e \frac{q}{a^2} \quad (11)$$

And the direction  $\theta$  (measured counter clockwise from  $\hat{\mathbf{i}}$ ) of  $\mathbf{E}$  is given by

$$\theta = \arctan \left( \frac{4 + \frac{3}{2\sqrt{2}}}{2 + \frac{3}{2\sqrt{2}}} \right) = 58.8^\circ \quad (12)$$

(b)  $\mathbf{F} = q\mathbf{E}$  so the direction of  $\mathbf{F}$  is the same as the direction of  $\mathbf{E}$ . The magnitude of  $\mathbf{F}$  is given by  $F = 5.91 k_e q^2 / a^2$

**Problem 19.** A uniformly charged ring of radius  $r = 10.0$  cm has a total charge of  $q = 75.0 \mu\text{C}$ . Find the electric field on the axis of the ring at (a)  $x_a = 1.00$  cm, (b)  $x_b = 5.00$  cm, (c)  $x_c = 30.0$  cm, and (d)  $x_d = 100$  cm from the center of the ring.



From Example 19.5 (p. 616) we see the electric field along the axis ( $\hat{\mathbf{i}}$ ) of a uniformly charged ring is given by

$$E = \frac{k_e x q}{(x^2 + r^2)^{3/2}} \hat{\mathbf{i}} \quad (13)$$

So applying this to our 4 distances (remembering to convert the distances to meters), we have

$$E_a = 6.64 \cdot 10^6 \text{ N/C } \hat{\mathbf{i}} \quad (14)$$

$$E_b = 24.1 \cdot 10^6 \text{ N/C } \hat{\mathbf{i}} \quad (15)$$

$$E_c = 6.40 \cdot 10^6 \text{ N/C } \hat{\mathbf{i}} \quad (16)$$

$$E_d = 0.664 \cdot 10^6 \text{ N/C } \hat{\mathbf{i}} \quad (17)$$