Useful Equations

1 Chapter 19: Electric Forces and Electric Fields

1.1 Coulomb's law

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{12} \tag{1}$$

Where $k_e \approx 8.988 \cdot 10^9$ N·m^2/C^2 is the *Coulomb constant*, $\mathbf{F}_{12}$ is the force on q_1 due to q_2, and $\hat{\mathbf{r}}_{12}$ is a unit vector pointing from q_1 to q_2.

1.2 Electric field

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0} \tag{2}$$

So for a point charge, the electric field at a point $\mathbf{r}$ is given by

$$\mathbf{E}(\mathbf{r}) = k_e \frac{q}{r^2} \hat{\mathbf{r}} \tag{3}$$

And for a group of charges, the electric field is given by

$$\mathbf{E}(\mathbf{r}) = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}} \tag{4}$$

1.3 Electric flux

The electric flux is the amount of electric field "flowing" through a surface S:

$$\Phi_E \equiv \int_S \mathbf{E} \cdot d\mathbf{A} \tag{5}$$

When you know something about the symmetry of the problem, you can often use *Gauss's law*

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{in}}{\epsilon_0} \tag{6}$$

Where the $\oint$ symbol reminds us that the surface must be closed, q_{in} is the enclosed charge, and $\epsilon_0 \approx 8.854 \cdot 10^{-12}$ C^2/N·m^2 is the *permittivity of free space.*

2 Chapter 20: Electric Potential and Capacitance

2.1 Potential

The change in potential ΔU of a charge q_0 moving from A to B in an electric field $\mathbf{E}(\mathbf{r})$ is given by

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \tag{7}$$

Where $d\mathbf{s}$ is an infinitesimal displacement vector.
The electric potential is defined as

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \tag{8}$$

Or, taking the derivative of both sides in the x direction

$$E_x = -\frac{dV}{dx} \tag{9}$$

With similar cases in the y and z directions. For those of you with vector calculus who recognize the gradient ∇ you can express the above formula as:

$$\mathbf{E} = -\nabla V \tag{10}$$

The voltage a distance r from a point charge q is given by

$$V = -\int_A^B \frac{k_e q}{r^2} dr = k_e \frac{q}{r} \tag{11}$$

Which we can combine with the definition of the electric potential to find the potential energy of two point charges seperated by a distance r_{12}:

$$U = q_1 V = k_e \frac{q_1 q_2}{r_{12}} \tag{12}$$

Just as we could integrate over a charge distribution to find the electric field at a given point, we can find electric potential with

$$V = k_e \int \frac{dq}{r} \tag{13}$$

Which is usually an easier integral because the integrand is a *scalar*.

2.2 Capacitance

The capacitance C of a pair of conductors (plates, other shapes...) is defined as

$$C \equiv \frac{Q}{\Delta V} \tag{14}$$

Where Q is the charge on one of two oppositely charged conductors, and ΔV is the voltage difference between the conductors. The capacitance of a capacitor consisting of two parallel plates of area A seperated by a distance d is given by

$$C = \frac{\epsilon_0 A}{d} \tag{15}$$

where ϵ_0 is the permittivity of free space. (Which comes from using the definition of capacitance, and the electric field between two charged plates that we calculated in recitation 2, problem P19.62.)
The equivalent capacitance of several capacitors in parallel is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \tag{16}$$

and in series is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \tag{17}$$

The electric potential energy U stored in a capacitor is

$$U = \frac{1}{2} C (\Delta V)^2 \tag{18}$$

3 Chapter 21: Current and Direct Current Circuits

3.1 Current

The electrical current I through a conductor is defined by

$$I \equiv \frac{dQ}{dt} , \tag{19}$$

the amount of charge passing through some surface per unit time.

3.2 Resistance

Resistance R of a conductor is given by Ohm's law

$$V = IR , \tag{20}$$

where I is the current through the conductor, and V is the electric potential difference across the resistor.

3.3 Power

The power dissipated by a current I dropping through an electric potential difference V is given by

$$P = IV , \tag{21}$$

so using Ohm's law, the power dissipated by a resistor is

$$P_R = I(IR) = I^2 R = \left(\frac{V}{R}\right)^2 R = \frac{V^2}{R} . \tag{22}$$

3.4 Kirchhoff's rules

Complicated steady-state circuits can be analyzed using Kirchhoff's two rules. The loop rule conserves energy by forcing the electric potential change about a loop to be 0.

$$\sum_{\text{elements in a loop}} \Delta V = 0 \; . \tag{23}$$

The junction rule conserves charge by forcing the current into a node to balance the current out of a node.

$$\sum_{\text{branches from a node}} I = 0 \; . \tag{24}$$

By combining these two equations, we can derive equivalent capacitances and resistances for circuit elements in series or in parallel, and, more generally, find the steady-state behavior to any steady-state circuit's problem.

4 Chapter 22: Magnetic Forces and Magnetic Fields

4.1 Cross products

The magnitude of $\mathbf{A} = \mathbf{B} \times \mathbf{C}$ is given by

$$|\mathbf{A}| = |\mathbf{B}| \cdot |\mathbf{C}| \cdot \sin\theta \; , \tag{25}$$

where θ is the angle between $\mathbf{B}$ and $\mathbf{C}$. In terms of (x, y, z) components, $\mathbf{A}$ is given by

$$\mathbf{A} = \mathbf{B} \times \mathbf{C} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \hat{\mathbf{i}}(B_yC_z - B_zC_y) - \hat{\mathbf{j}}(B_xC_z - B_zC_x) + \hat{\mathbf{k}}(B_xC_y - B_yC_x) = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}} \; , \tag{26}$$

so

$$A_x = (B_yC_z - B_zC_y) \qquad A_y = (B_zC_x - B_xC_z) \qquad A_z = (B_xC_y - B_yC_x) \tag{27}$$

The direction of $\mathbf{A}$ is given either by solving Eqn. 27 explicitly or by using Eqn. 25 and using a *right hand rule*.

4.2 Magnetic force

The magnetic force on a moving charged particle is given by

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} \; , \tag{28}$$

where q is the charge on the particle, $\mathbf{v}$ is the particle's velocity, and $\mathbf{B}$ is the external magnetic field.
The magnetic force on a current carrying wire is given by

$$\mathbf{F}_B = I\mathbf{l} \times \mathbf{B} \; , \tag{29}$$

where I is the current in the wire, $\mathbf{l}$ is the length of the wire in the direction of the current, and $\mathbf{B}$ is the external magnetic field.

4.3 Magnetic dipole moments

The magnetic dipole moment μ of a loop carrying current I is

$$\mu = I\mathbf{A} \; , \tag{30}$$

where $\mathbf{A}$ is the area of the loop. The torque on such a loop placed in an external magnetic field $\mathbf{B}$ is

$$\tau = \mu \times \mathbf{B} \; . \tag{31}$$

4.4 Generating magnetic fields

The Biot-Savart law determines the magnetic field generated by an infinitesimal current (much like Coulomb's law determined the electric field generated by a point charge).

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \; , \tag{32}$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ T·m/A is the permeability of free space, $d\mathbf{s}$ is the infinitesimal length of the current I, and $\mathbf{r}$ is the vector from the location of the current to the location of the magnetic field.
Ampère's law determines the magnetic field generated by a current, in a manner suitable for symmetric current distributions (much like Gauss's law determined the electric field for symmetric charge distributions).

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{in}} \; , \tag{33}$$

where $\oint$ signifies integration about a closed loop, and I_{in} is the total current penetrating the loop.
You can use Ampère's law to determine the magnetic field from some standard current distributions as follows

$$B = \frac{\mu_0 I}{2\pi r} \qquad \text{Distance } r \text{ from a long, straight wire} \tag{34}$$

$$B = \frac{\mu_0 N I}{2\pi r} \qquad \text{Inside a torroid of radius } r \tag{35}$$

$$B = \mu_0 n I \qquad \text{Inside a solenoind with } n \text{ turns per unit length} \tag{36}$$