Homework 6

Chapter 21

Problem 40. A dead battery is charged by connecting it to the live battery of another car with jumper cables (Fig. P21.40). Determine the current in the starter and in the dead battery.

Let $V_L = 12$ V and $R_L = 0.01\Omega$ be the parameters of the live battery, $V_D = 10$ V and $R_D = 1.00\Omega$ be the parameters of the dead battery, and $R_S = 0.06\Omega$ be the resistance of the starter. Let I_L be the current going upward in the left branch, I_D be the current going upward in the middle branch, and I_S be the current going downward in the right branch.
Applying Kirchoff's junction rule to the top node, we have

$$I_L + I_D - I_S = 0 \tag{1}$$

Applying Kirchoff's loop rule to the outer and right loops respectively, we have

$$V_L - I_L R_L - I_S R_S = 0 \tag{2}$$

$$V_D - I_D R_D - I_S R_S = 0 \tag{3}$$

Solving these using linear algebra (or your method of choice)

$$\begin{pmatrix} 0 \\ V_L \\ V_D \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ R_L & 0 & R_S \\ 0 & R_D & R_S \end{pmatrix} \begin{pmatrix} I_L \\ I_D \\ I_S \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ 0.01 & 0 & 0.06 \\ 0 & 1.00 & 0.06 \end{pmatrix} \begin{pmatrix} I_L \\ I_D \\ I_S \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} I_L \\ I_D \\ I_S \end{pmatrix} = \begin{pmatrix} 0.850 & 15.0 & -0.850 \\ 0.0085 & -0.850 & 0.992 \\ -0.142 & 14.2 & 0.142 \end{pmatrix} \begin{pmatrix} 0 \\ 12 \\ 10 \end{pmatrix} = \begin{pmatrix} 172 \\ -0.283 \\ 171 \end{pmatrix} \text{ A} \tag{5}$$

Where $I_2 < 0$ indicates that the current in the middle branch actually flows downward, recharging the dead battery.

Problem 46. A $C = 10.0\ \mu$F capacitor is charged by a $\epsilon = 10.0$ V battery through a resistance R. The capacitor reaches a potential difference of $V_C(t_f) = 4.00$ V at the instant $t_f = 3.00$ s after the charging begins. Find R.

Applying Kirchoff's loop rule,

$$\epsilon - V_C - V_R = \epsilon - \frac{q}{C} - IR = 0 \tag{6}$$

$$R\frac{dq}{dt} = \epsilon - \frac{q}{C} \tag{7}$$

$$\frac{dq}{dt} = \frac{C\epsilon - q}{RC} \tag{8}$$

$$\frac{dq}{C\epsilon - q} = \frac{dt}{RC} \tag{9}$$

$$\int \frac{dq}{C\epsilon - q} = \int \frac{dt}{RC} \tag{10}$$

$$-\ln(C\epsilon - q) = t/RC + A \tag{11}$$

$$C\epsilon - q = Be^{-t/RC} \tag{12}$$

$$q = C\epsilon - Be^{-t/RC} \tag{13}$$

$$q = C\epsilon(1 - e^{-t/RC}) \tag{14}$$

Where A is a constant of integration, $B = e^{-A}$ is another way of writing that constant, $C\epsilon = Q$ (because as $t \to \infty$, $q \to Q$), and $B = Q = C\epsilon$ (because at $t = 0$, $q = 0$).
Note: The book derives this on pages 709-710 if you want more details.
Now applying this to our particular problem,

$$V_C(t_f) = \frac{q(t_f)}{C} = \frac{C\epsilon}{C}(1 - e^{-t_f/RC}) \tag{15}$$

$$\frac{V_C(t_f)}{\epsilon} = 1 - e^{-t_f/RC} \tag{16}$$

$$e^{-t_f/RC} = 1 - \frac{V_C(t_f)}{\epsilon} \tag{17}$$

$$\frac{-t_f}{RC} = \ln\left(1 - \frac{V_C(t_f)}{\epsilon}\right) \tag{18}$$

$$R = \frac{-t_f}{C\ln(1 - V_C(t_f)/\epsilon)} = \frac{-3.00 \text{ s}}{10.0 \cdot 10^{-6} \text{ F} \cdot \ln(1 - 4.00 \text{ V}/10.0 \text{ V})} = 587 \text{ k}\Omega \tag{19}$$

Problem 55. Four $V = 1.50$ V AA batteries in series are used to power a transistor radio. If the batteries can move a charge of $\Delta Q = 240$ C, how long will they last if the radio has a resistance of $R = 200\Omega$?

Using Kirchoff's loop rule,

$$V + V + V + V - IR = 0 \tag{20}$$

$$I = \frac{4V}{R} = \frac{\Delta q}{\Delta t} \tag{21}$$

$$\Delta t = \frac{\Delta q R}{4V} = \frac{240 \text{ C} \cdot 200\Omega}{4 \cdot 1.50 \text{ V}} = 8000 \text{ s} = 2.22 \text{ hours} \tag{22}$$