

Homework 4

Chapter 20

Problem 41. Four capacitors are connected as shown in Figure P20.41. (a) Find the equivalent capacitance between points a and b . (b) Calculate the charge on each capacitor, taking $\Delta V_{ab} = 15.0 \text{ V}$

(a) First consider the top two capacitors, $C_1 = 15.0 \mu\text{F}$ and $C_2 = 3.00 \mu\text{F}$. They are in series, so the effective capacitance of the top line is given by

$$C_t = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = 2.50 \mu\text{F} \quad (1)$$

We can find the effective capacitance of the box, because C_t is in parallel with $C_3 = 6.00 \mu\text{F}$.

$$C_b = C_t + C_3 = 8.50 \mu\text{F} \quad (2)$$

We can find the total equivalent capacitance, because C_b is in series with $C_4 = 20.0 \mu\text{F}$.

$$C_{eq} = \left(\frac{1}{C_b} + \frac{1}{C_4} \right)^{-1} = 5.96 \mu\text{F} \quad (3)$$

(b) Working backwards to find the charges, using $Q = CV$, we have

$$Q_4 = Q_b = C_{eq}V_{ab} = 89.5 \mu\text{C} \quad (4)$$

So the voltage across the box is

$$V_b = \frac{Q_b}{C_b} = 10.5 \text{ V} \quad (5)$$

So

$$Q_3 = C_3V_b = 63.2 \mu\text{C} \quad (6)$$

and

$$Q_1 = Q_2 = C_tV_b = 26.3 \mu\text{C} \quad (7)$$

Problem 49. Two capacitors, $C_1 = 25.0 \mu\text{F}$ and $C_2 = 5.00 \mu\text{F}$, are connected in parallel and charged with a $\Delta V = 100 \text{ V}$ power supply. (a) Draw a circuit diagram and calculate the total energy stored in the two capacitors. (b) What potential difference would be required across the same two capacitors connected in series so that the combination stores the same energy as in (a)? Draw a circuit diagram for this circuit.

The diagrams are given in the back of the book.

(a) The equivalent capacitance is $C_{eq} = C_1 + C_2 = 30.0 \mu\text{F}$, so the stored energy is

$$U = \frac{1}{2}C_{eq}(\Delta V)^2 = 0.150 \text{ J} \quad (8)$$

(b) The equivalent capacitance is now

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = 4.17 \mu\text{F} \quad (9)$$

So the necessary voltage is given by

$$U = \frac{1}{2}C_{eq}(\Delta V)^2 \quad (10)$$

$$\Delta V = \sqrt{\frac{2U}{C_{eq}}} = 268 \text{ V} \quad (11)$$

Problem 69. The x axis is the symmetry axis of a stationary, uniformly charged ring of radius R and charge Q (Fig. P20.69). A particle with charge q and mass M is located at the center of the ring. When it is displaced slightly, the point charge accelerates along the x axis to infinity. Show that the ultimate speed of the point charge is

$$v = \left(\frac{2k_e q Q^2}{MR} \right)^{1/2} \quad (12)$$

Conserving energy, the initial energy is entirely electric,

$$E_i = U_e = k_e \frac{Q^2}{R} \quad (13)$$

because all the ring charge is a distance R from the particle.

The final energy is entirely kinetic

$$E_f = K = \frac{1}{2} M v^2 \quad (14)$$

So

$$k_e \frac{Q^2}{R} = E_i = E_f = \frac{1}{2} M v^2 \quad (15)$$

$$v = \sqrt{\frac{2k_e Q^2}{MR}} \quad (16)$$